PHYS 501
HANDOUT 1 - *Questions on Vectors*

1.1 *Direction of a vector*: Given a vector **A**= −**i**+ 2**j** – 2**k** in Cartesian coordinates, find the expression for the unit vector in the direction of **A**.

1.2 *Relation between two vectors*: Show that, if **A**·**B** = **A**·**C** and **A**×**B**=**A**×**C**, where **A** is not a null vector, then **B**=**C**.

1.3 *Multiple applications of · and ×* : Consider arbitrary vectors **A**, **B**, **C** and **D**.

(a) Is $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{C}\times\mathbf{B})\times\mathbf{A}$? Explain.

(b) Is $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=(\mathbf{C}\times\mathbf{B})\cdot\mathbf{A}$? Explain.

(c) Does $\mathbf{A}\times\mathbf{B}=\mathbf{A}\times\mathbf{C}$, implies $\mathbf{B}=\mathbf{C}$? Explain.

1.4 *Vector components and directions*: Find the relative position vector **R** of the point *P*(2,-2,3) with respect to *Q*(-3,1,4). What are the direction angles of **R**? (The direction angles are the angles between the vector **R** and the axes *x*, *y* and *z* respectively).

1.5 *Vector components and directions*: Given that **A**= **i**+ 2**j** +3**k** and **B**= 4**i**-5**j** +6**k**. Find the angle between the two vectors. Find the component of **A** in the direction of **B**.

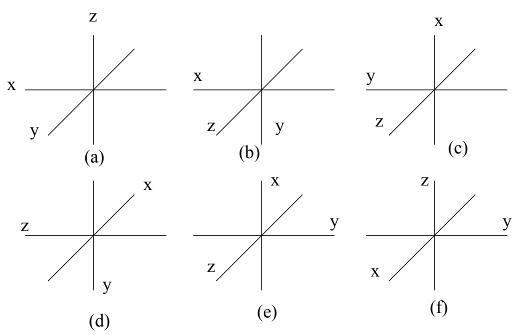
1.6 What could we deduce for the vectors **a**, **b** if;

a) **a**+**b**=**c** and *a*+*b*=*c*
b) **a**+**b**=**a**-**b**
c) **a**+**b**=**c** and $a^2+b^2=c^2$

1.7 (a) Using unit vectors, with expressions for the body diagonals (the straight lines from one corner to another through the center) of a cube in terms of its edges, which have length *a*. (b) determine the angles that the body diagonals make with adjacent edges. (c) Determine the length of the body diagonals.

1.8 Can the magnitude of the difference between two vectors ever be greater than (a) the magnitude of one of the vectors, (b) the magnitudes of both vectors, and (c) the magnitude of their sum?

1.9 If **A** and **B** are the adjacent sides of a parallelogram, **C**=**A**+**B** and **D**=**A**-**B** are the diagonals, and θ is the angle between **A** and **B**, show that $(C^2+D^2)=2(A^2+B^2)$ and $(C^2-D^2)=4AB\cos\theta$.

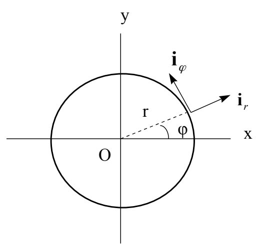


1.10 Which of the arrangements of axes in figure can be labeled "right handed coordinate system"? As usual, each axis label indicates the positive side of the axis.

1.11 If $\mathbf{a}\cdot\mathbf{b} = \mathbf{a}\cdot\mathbf{c}$, must **b** equal **c**?

1.12 Prove that two vectors must have equal magnitudes if their sum is perpendicular to their difference.

1.13 Two vectors are given by $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find (a) **a+b**, (b) **a-b**, and (c) a vector **c** such that **a-b+c=0**.



1.14 For objects that move in a circle about an origin O, it can be convenient touse the mutually peprpendicular unit vectors $\mathbf{i}_r$ and $\mathbf{i}_\varphi$ as shown in figure. Express $\mathbf{i}_r$ and $\mathbf{i}_\varphi$ as a combination of $\mathbf{i}$ and $\mathbf{j}$.

1.15 Two vectors **A**, **B** have precisely the same magnitudes. For the magnitude of **A+B** to be hundred times larger than the magnitude of **A-B** what must be the angle between them?

1.16 If the component of a vector **A** along the direction of a vector **B** is zero what can you conclude about these two vectors?

1.17 Show that the magnitude of a vector **A**, $A = \left(A_x^2 + A_y^2\right)^{1/2}$ is independent of the orientation of the rotated coordinate system i.e., $\left(A_x^2 + A_y^2\right)^{1/2} = \left(A_x^{'2} + A_y^{'2}\right)^{1/2}$.

1.18 Prove the orthogonality condition $\sum_i a_{ji}a_{ki} = \delta_{jk}$.

1.19 Show that the scalar product of two vectors **A** and **B** is invariant under rotations.

1.20 Find the vector $(\mathbf{A}-\mathbf{B})\times(\mathbf{A}+\mathbf{B})$.

1.21 Prove that $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{A}\times\mathbf{B}) = (AB)^2 - (\mathbf{A}\cdot\mathbf{B})^2$.

1.22 Using the vectors $\mathbf{P} = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$, $\mathbf{Q} = \mathbf{i}\cos\phi - \mathbf{j}\sin\phi$, $\mathbf{R} = \mathbf{i}\cos\phi + \mathbf{j}\sin\phi$, prove the familiar trigonometric identities

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi, \quad \cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi.$$

1.23 If four vectors **a**, **b**, **c**, and **d** all lie in the same plane, show that: $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) = 0$.

1.24 Derive the law of sines in a triangle.

1.25 Show that: $\mathbf{A}\times\left(\mathbf{B}\times\mathbf{C}\right) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$.

1.26 The angular momentum **L** of a particle is given by $\mathbf{L} = \mathbf{r}\times\mathbf{p} = m\mathbf{r}\times\mathbf{v}$, where **p** is the linear momentum. With the linear and angular velocity related by $\mathbf{v} = \vec{\omega}\times\mathbf{r}$ show that: $\mathbf{L} = mr^2\left[\vec{\omega} - \mathbf{r}_0\left(\mathbf{r}_0\cdot\vec{\omega}\right)\right]$. Where $\mathbf{r}_0$ is the unit vector in the **r** direction. What happens when $\mathbf{r}\cdot\vec{\omega} = 0$?

1.27 The kinetic energy of a particle is given by $K = \left(1/2\right)mv^2$. For rotational motion this becomes $\left(1/2\right)m\left(\vec{\omega}\times\mathbf{r}\right)^2$. Show that: $K = \left(1/2\right)m\left[r^2\omega^2 - \left(\mathbf{r}\cdot\vec{\omega}\right)^2\right]$. What happens when $\mathbf{r}\cdot\vec{\omega} = 0$?

1.28 Show that: $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) + \mathbf{b}\times(\mathbf{c}\times\mathbf{a}) + \mathbf{c}\times(\mathbf{a}\times\mathbf{b}) = 0$.

1.29 Show that $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$. Then prove the following relations:

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B} = -\mathbf{A}\cdot\mathbf{C}\times\mathbf{B} =$$
$$-\mathbf{C}\cdot\mathbf{B}\times\mathbf{A}$$

1.30 Show that $\left(\mathbf{A}\times\mathbf{B}\right)\cdot\left(\mathbf{C}\times\mathbf{D}\right) = \left(\mathbf{A}\cdot\mathbf{C}\right)\left(\mathbf{B}\cdot\mathbf{D}\right) - \left(\mathbf{A}\cdot\mathbf{D}\right)\left(\mathbf{B}\cdot\mathbf{C}\right)$.