اجابة واجب 3

1. **احسبي قيمة مرونة الاحلال لكل من دوال الانتاج الاتية..**

$$q = 5 L^{.25} K^{.75}$$
$$q = 2L + 4K$$
$$q = L^2 + LK + K^2$$
$$q = e^{L+K}$$
$$q = \text{Log}_e (LK)$$

الحل السؤال الاول

1) $q = 5 L^{.25} K^{.75}$

الدالة من نوع كوب دوجلاس دالة متجانسة من الدرجة الاولى ومرونة الاحلال دائما تساوي الواحد صحيح

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2) $q = 2L + 4K$

$$q = 2\lambda L + 4\lambda K$$
$$q = \lambda(2L + 4K)$$
$$q = \lambda q$$

اذن متجانسة من الدرجة الاولى

$$\sigma_{12} = \frac{\phi_1 \phi_2}{q \phi_{12}}$$
$$\phi_1 = 2$$
$$\phi_2 = 4$$
$$\phi_{12} = 0$$
$$\sigma_{12} = \frac{\phi_1 \phi_2}{q \phi_{12}} = \frac{(2)(4)}{q(0)} = \infty$$

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3) $q = L^2 + LK + K^2$

$$q = \lambda^2 L^2 + \lambda L \lambda K + \lambda^2 K^2$$
$$q = \lambda^2 (L^2 + LK + K^2)$$
$$q = \lambda^2 q$$

الدالة متجانسة من الدرجة الثانية

$$\sigma_{12} = \frac{\phi_1 \phi_2}{q \phi_{12}}$$
$$\phi_1 = (2L + K)$$
$$\phi_2 = (2K + L)$$
$$\phi_{12} = 1$$
$$\sigma_{12} = \frac{\phi_1 \phi_2}{q \phi_{12}} = \frac{(2L + K)(2K + L)}{(L^2 + LK + K^2)}$$

4) $q = e^{L+K}$

$$\sigma_{12} = \frac{\phi_1 L + \phi_2 K}{2\phi_1\phi_2\phi_{12} - \phi_1^{\,2}\phi_{22} - \phi_2^{\,2}\phi_{11}} \times \frac{\phi_1\phi_2}{LK}$$

$$\phi_1 = e^{L+K} = q$$

$$\phi_2 = e^{L+K} = q$$

$$\phi_{12} = e^{L+K} = q \qquad \phi_{11} = e^{L+K} = q \qquad \phi_{22} = e^{L+K} = q$$

$$\sigma_{12} = \frac{\phi_1 L + \phi_2 K}{0} \times \frac{\phi_1\phi_2}{LK} = \infty$$

5) $q = \text{Log}_e(LK)$

$$\mathbf{q = \ell n(LK)}$$

$$\boldsymbol{\sigma_{12} = \frac{\phi_1 L + \phi_2 K}{2\phi_1\phi_2\phi_{12} - \phi_1^{\,2}\phi_{22} - \phi_2^{\,2}\phi_{11}} \times \frac{\phi_1\phi_2}{LK}}$$

$$\boldsymbol{\phi_1 = \frac{1}{L}}$$

$$\boldsymbol{\phi_2 = \frac{1}{K} \qquad \phi_1\phi_2 = \frac{1}{LK}}$$

$$\boldsymbol{\phi_{12} = 0 \qquad \phi_{11} = \frac{-1}{L^2} \qquad \phi_{22} = \frac{-1}{K^2} \qquad 2\phi_1\phi_2\phi_{12} = 0}$$

$$\boldsymbol{\phi_1 L + \phi_2 K = \frac{1}{L}L + \frac{1}{K}K = 2}$$

$$\boldsymbol{\sigma_{12} = -\frac{2}{0 + \frac{2}{L^2K^2}} \times \frac{\frac{1}{LK}}{LK} = -1}$$

1. اذا كانت دالة الانتاج هي :

$$X = A a^{1/2} b^{1/4} c^{1/4}$$

اثبتي

$$X = \frac{\partial X}{\partial a}(a) + \frac{\partial X}{\partial b}(b) + \frac{\partial X}{\partial c}(c)$$

عندما

$$\frac{\partial X}{\partial a} = \frac{\partial X}{\partial b} = \frac{\partial X}{\partial c}$$

فإن

$$X = \frac{Aa}{\sqrt{2}}$$

- دالة متجانسة من الدرجة الاولى تنطبق عليها نظرية اويلر

$$X = A a^{\frac{1}{2}} b^{\frac{1}{4}} c^{\frac{1}{4}}$$

$$X = \frac{\partial X}{\partial a}(a) + \frac{\partial X}{\partial b}(b) + \frac{\partial X}{\partial c}(c)$$

$$X = \left[\left(\frac{1}{2}\right) A a^{\frac{-1}{2}} b^{\frac{1}{4}} c^{\frac{1}{4}}\right](a) + \left[\left(\frac{1}{4}\right) A a^{\frac{1}{2}} b^{\frac{-3}{4}} c^{\frac{1}{4}}\right](b) + \left[\left(\frac{1}{4}\right) A a^{\frac{1}{2}} b^{\frac{1}{4}} c^{\frac{-3}{4}}\right](c)$$

$$X = \left[\left(\frac{1}{2}\right) A a^{\frac{1}{2}} b^{\frac{1}{4}} c^{\frac{1}{4}}\right] + \left[\left(\frac{1}{4}\right) A a^{\frac{1}{2}} b^{\frac{1}{4}} c^{\frac{1}{4}}\right] + \left[\left(\frac{1}{4}\right) A a^{\frac{1}{2}} b^{\frac{1}{4}} c^{\frac{1}{4}}\right]$$

$$X = \left[A a^{\frac{1}{2}} b^{\frac{1}{4}} c^{\frac{1}{4}}\right]$$

$$\frac{\partial X}{\partial a}=\frac{\partial X}{\partial b}=\frac{\partial X}{\partial c}$$

$$\left[\left(\frac{1}{2}\right)\ A\ a^{\frac{-1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}\right]=\left[\left(\frac{1}{4}\right)\ A\ a^{\frac{1}{2}}\ b^{\frac{-3}{4}}\ c^{\frac{1}{4}}\right]=\left[\left(\frac{1}{4}\right)\ A\ a^{\frac{1}{2}}\ b^{\frac{1}{4}}\ c^{\frac{-3}{4}}\right]$$

$$\frac{A\,b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{2\,a^{\frac{1}{2}}}=\frac{A\ a^{\frac{1}{2}}\ c^{\frac{1}{4}}}{4\ \ b^{\frac{3}{4}}}=\frac{A\ a^{\frac{1}{2}}\ b^{\frac{1}{4}}}{4\,c^{\frac{3}{4}}}$$

بالضرب $\times\ \dfrac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}$

$$\frac{A\,b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{2\,a^{\frac{1}{2}}}\times\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}=\frac{A\ a^{\frac{1}{2}}\ c^{\frac{1}{4}}}{4\,b^{\frac{3}{4}}}\times\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}=\frac{A\ a^{\frac{1}{2}}\ b^{\frac{1}{4}}}{4\,c^{\frac{3}{4}}}\times\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}$$

$$\frac{}{2\,a^{\frac{1}{2}}}\times\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{a^{\frac{1}{2}}}=\frac{}{4\,b^{\frac{3}{4}}}\times\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{b^{\frac{1}{4}}}=\frac{}{4\,c^{\frac{3}{4}}}\times\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{c^{\frac{1}{4}}}$$

$$\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{2\,a}=\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{4\,b}=\frac{A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}}{4\,c}$$

$$\frac{X}{2\,a}=\frac{X}{4\,b}=\frac{X}{4\,c}\quad\rightarrow\quad 2\,a=4\,b=4\,c\quad\rightarrow\quad\frac{1}{2}a=b=c$$

$$X=A\ a^{\frac{1}{2}}b^{\frac{1}{4}}\ c^{\frac{1}{4}}\quad\rightarrow\quad X=A\ a^{\frac{1}{2}}\left(\frac{1}{2}a\right)^{\frac{1}{4}}\ \left(\frac{1}{2}a\right)^{\frac{1}{4}}\quad\rightarrow\quad X=A\ a\left(\frac{1}{2}\right)^{\frac{1}{2}}$$

اذ ن

$$X=\frac{Aa}{\sqrt{2}}$$

3 - اذا كانت دالة الانتاج تساوي

$$Z=A\left(e^{\frac{-a}{xy}}+e^{\frac{-b}{x}}+e^{\frac{-c}{y}}\right)$$

اثبتي :

$$\frac{\partial Z}{\partial X}>0$$

$$\frac{\partial Z}{\partial X\partial Y}>0$$

$$Z = A\left(e^{\frac{-a}{xy}} + e^{\frac{-b}{x}} + e^{\frac{-c}{y}} \right)$$

$$\frac{\partial Z}{\partial X} = A\left[\frac{a\, e^{\frac{-a}{XY}}}{X^2 Y} + \frac{be^{\frac{-b}{x}}}{X^2} \right] > 0 \qquad \rightarrow \text{ if } \qquad \rightarrow A, a, b \; > 0$$

$$\frac{\partial Z}{\partial X \partial Y} = \frac{\partial A\left[\frac{a\, e^{\frac{-a}{XY}}}{X^2 Y} + \frac{be^{\frac{-b}{x}}}{X^2} \right]}{\partial Y} = \frac{\partial}{\partial Y}\left[A\left(a\, e^{\frac{-a}{XY}} X^{-2} Y^{-1} + be^{\frac{-b}{x}} X^{-2} \right) \right]$$

$$= Aa\, e^{\frac{-a}{XY}} X^{-2} Y^{-1} \left(aX^{-1} Y^{-2} \right) - Aae^{\frac{-a}{XY}} X^{-2} Y^{-2}$$

$$= Aa^2\, e^{\frac{-a}{XY}} X^{-3} Y^{-3} - Aae^{\frac{-a}{XY}} X^{-2} Y^{-2} \qquad \rightarrow \quad = Aa\, e^{\frac{-a}{XY}} X^{-3} Y^{-3} \left(a - XY \right)$$

$$= \frac{Aa\, e^{\frac{-a}{XY}}}{X^3 Y^3} \left(a - XY \right) \qquad \rightarrow \quad \frac{\partial Z}{\partial X \partial Y} > 0 \quad \text{if} \quad a > XY$$

4 - وضحي ما اذا كانت دوال الانتاج الاتية ، تخضع لثبات ، تناقص ، ام تزايد الغلة :

$$q = 2L + 4K$$
$$q = 2\lambda L + 4\lambda K$$
$$q = \lambda (2L + 4K)$$
$$q = \lambda q$$

متجانسة من الدرجة الاولى فهي تخضع لثبات الغلة

$$q = L^{.5} K^{.5}$$
$$q = \lambda^{.5} L^{.5} \lambda^{.5} K^{.5}$$
$$q = \lambda(L^{.5} K^{.5})$$
$$q = \lambda q$$

متجانسة من الدرجة الاولى فهي تخضع لثبات الغلة

$$q = 2 L^{1/2} K^{1/3}$$
$$q = 2 \lambda^{1/2} L^{1/2} \lambda^{1/3} K^{1/3}$$
$$q = \lambda^{5/6} (2 L^{1/2} K^{1/3})$$
$$q = \lambda^{5/6} q$$

متجانسة من درجة 5/6 فهي تخضع لتناقص الغلة

$$q = 4 L^{.75} K^{.5}$$
$$q = 4 \lambda^{.75} L^{.75} \lambda^{.5} K^{.5}$$
$$q = \lambda^{1.25} (4 L^{.75} K^{.5})$$
$$q = \lambda^{1.25} q$$

متجانسة من درجة 1.25 فهي تخضع لتزايد الغلة

$$q = .2(L^{.5} + K^{.25})$$
$$q_1 = .2(\lambda^{.5} L^{.5} + \lambda^{.25} K^{.25})$$
$$q_1 < \lambda q$$

حيث

$$\lambda^{.5} \le \lambda$$

و

$$\lambda^{.25} \le \lambda$$

وعلية فان الدالة تخضع لتناقص الغلة

$$q = 2 L^2 + LK + K^2$$
$$q = 2 \lambda^2 L^2 + \lambda L \lambda K + \lambda^2 K^2$$
$$q = \lambda^2 (L^2 + LK + K^2)$$
$$q = \lambda^2 (L^2 + LK + K^2)$$
$$q = \lambda^2 q$$

متجانسة من درجة الثانية فهى تخضع لتزايد الغلة

5 - وضحي ما اذا كانت دوال الانتاج الاتية متجانسة ، وحددي درجة التجانس ، ثم اثبتي نظرية اويلر بالنسبة لكل دالة متجانسة .

$$Z = 3X^3 + 5XY^2 + Y^3$$
$$Z = \frac{14}{X} - \frac{20}{Y}$$
$$Z = 25Y^6 - X^2Y^4$$
$$Z = \frac{3}{X^2} + \frac{25}{XY} + \frac{6}{Y^2}$$

‘‘

-1

$$Z = 3X^3 + 5XY^2 + Y^3$$
$$Z = 3\lambda^3X^3 + 5\lambda X\lambda^2 Y^2 + \lambda^3Y^3$$
$$Z = \lambda^3 \left(3X^3 + 5XY^2 + Y^3\right)$$
$$Z = \lambda^3 Z$$

$$Z = \frac{\partial Z}{\partial X} X + \frac{\partial Z}{\partial Y} Y$$
$$Z = \left(9X^2 + 5Y^2\right)X + \left(10XY + 3Y^2\right)Y$$
$$Z = 9X^3 + 5XY^2 + 10XY^2 + 3Y^3$$
$$Z = 9X^3 + 15XY^2 + 3Y^3$$
$$Z = 3\left(3X^3 + 5XY^2 + Y^3\right)$$
$$Z = 3Z$$

متجانسة من درجة الثالثة

-2

$$Z = \frac{14}{X} - \frac{20}{Y}$$

$$Z = \frac{14}{\lambda X} - \frac{20}{\lambda Y}$$

$$Z = \frac{1}{\lambda}\left(\frac{14}{X} - \frac{20}{Y}\right) \quad \rightarrow \quad Z = \lambda^{-1}\left(\frac{14}{X} - \frac{20}{Y}\right)$$

$$Z = \frac{\partial Z}{\partial X}X + \frac{\partial Z}{\partial Y}Y$$

$$Z = \frac{-14}{X} - \frac{20}{Y}$$

$$Z = -1\left(\frac{-14}{X} + \frac{20}{Y}\right) \quad \rightarrow \quad Z = -Z$$

متجانسة من درجة سالب واحد

-3

$$Z = 25Y^6 - X^2Y^4$$

$$Z = 25\lambda^6Y^6 - \lambda^2X^2\lambda^4Y^4$$

$$Z = \lambda^6\left(25Y^6 - X^2Y^4\right)$$

$$Z = \frac{\partial Z}{\partial X}X + \frac{\partial Z}{\partial Y}Y$$

$$Z = \left(-2X^2Y^4\right) + \left(-4X^2Y^4 + 150Y^6\right)$$

$$Z = -2X^2Y^4 - 4X^2Y^4 + 150Y^6 \quad \rightarrow \quad = -6X^2Y^4 + 150Y^6$$

$$Z = 6\left(25Y^6 - X^2Y^4\right) \quad \rightarrow \quad Z = 6Z$$

متجانسة من درجة السادسة

-4

$$Z = \frac{3}{\lambda^2X^2} + \frac{25}{\lambda X\lambda Y} + \frac{6}{\lambda^2Y^2}$$

$$Z = \frac{1}{\lambda^2}\left(\frac{3}{X^2} + \frac{25}{XY} + \frac{6}{^2Y^2}\right) \quad \rightarrow\rightarrow \quad Z = \frac{1}{\lambda^2}Z \quad \rightarrow\rightarrow \quad = \lambda^{-2}Z$$

$$Z = \frac{3}{X^2} + \frac{25}{XY} + \frac{6}{Y^2}$$

$$Z = \frac{\partial Z}{\partial X}X + \frac{\partial Z}{\partial Y}Y \quad \rightarrow \quad Z = -\frac{6}{X^2} - \frac{25}{XY} - \frac{25}{XY} - \frac{12}{Y^2}$$

$$Z = \frac{-6}{X^2} - \frac{-50}{XY} - \frac{6}{Y^2} \quad \rightarrow\rightarrow Z = -2\left(\frac{3}{X^2} + \frac{25}{XY} + \frac{6}{Y^2}\right)$$

$$Z = -2Z$$

متجانسة من درجة سالب اثنان (-2)
