

King Saud University

Department of Mathematics

Tutorial 2

ACTU 473 – Models of Financial Economics

Exercise 1.

For a one-period binomial model for stock prices, you are given

- (i) Each period is 3 months.
- (ii) The current price of a non-dividend paying stock is 100.
- (iii) The continuously compounded risk-free interest rate is 4%.
- (iv) The stock's volatility is 30%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period. Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

Exercise 2.

For a two-period binomial model for stock prices, you are given

- (v) Each period is 6 months.
- (vi) The current price of a non-dividend paying stock is 70.
- (vii) $u = 1.181$, where u is one plus the rate of capital gain on the stock per period if the price goes up.
- (viii) $d = 0.890$, where d is one plus the rate of capital loss on the stock per period if the price goes down.
- (ix) The continuously compounded risk-free interest rate is 5%.

Calculate the time-0 price of a one-year American put option on the stock with a strike price of 80.

Exercise 3.

You are to price a 2-year at-the money European call option on a stock with a binomial model. You are given:

- i. The current price of the stock is 100.
- ii. The stock pays dividends at a rate proportional to its price. The dividend yield is 3%.
- iii. The binomial tree consists of two-time steps of 1 year.
- iv. In each time step, the stock price either moves up by a proportional amount of 25% or moves down by a proportional amount of 25%.
- v. The continuously compounded risk-free interest rate is 3%.

Calculate the current price of the call option.

Exercise 4.

You are to price a 1-year straddle on a non-dividend paying stock with a two-period binomial tree. You are given:

- i. The straddle can only be exercised at the end of 1 year.
- ii. The current price of the stock is 60.
- iii. The binomial tree consists of two-time steps of 6 months.
- iv. In each time step, the stock price either moves up by a proportional amount of 25% or moves down by a proportional amount of 20%.
- v. The continuously compounded risk-free interest rate is 8%.
- vi. The strike price of the straddle is 50.

Calculate the current price of the straddle.

Exercise 5.

You are to price a 1-year at-the money European call option on a stock. You are given:

- a) The stock currently sells for 100.
- b) The stock's volatility is 30%.
- c) The stock pays dividends at a rate proportional to its price. The dividend yield is 3%.
- d) The continuously compounded risk-free interest rate is 3%.

Calculate the current price of the option using a forward tree that consists of two-time steps of 6 months.

Exercise 6.

You use a Cox-Ross-Rubinstein tree to price a strangle on a stock. You are given:

- (i) The strangle can only be exercised at the end of one year.
- (ii) Let $S(1)$ be the stock price at the end of one year. The payoff of the strangle is as follows:

Stock Price	Payoff
$S(1) \leq 70$	$70 - S(1)$
$70 < S(1) < 80$	0
$S(1) \geq 80$	$S(1) - 80$

- (iii) The stock pays no dividends and currently sells for 70.
- (iv) The continuously compounded risk-free interest rate is 5%.
- (v) The stock's volatility is 25%.
- (vi) The Cox-Ross-Rubinstein binomial tree consists of two-time steps of 6 months.

Calculate the current price of the strangle.

Exercise 7.

Consider a 9-month dollar-denominated European call option on British pounds. You are given that:

- (i) The current exchange rate is 1.43 US dollars per pound.
- (ii) The strike price of the call is 1.56 US dollar per pound.
- (iii) The volatility of the exchange rate is 0.3.
- (iv) The US dollar continuously compounded risk-free interest rate is 8%.
- (v) The British pound continuously compounded risk-free interest rate is 9%.

Using a three-period binomial model, calculate the price of the put.

Exercise 8.

You consider a single period binomial model for a 1-month at-the-money dollar denominated European put option on British pounds. You are given:

- (i) The current exchange rate is 0.66667 British pounds per US dollar.
- (ii) The volatility of the exchange rate is $\sigma = 0.4$.
- (iii) The US dollar continuously compounded risk-free interest rate is 7%.
- (iv) The British pound continuously compounded risk-free interest rate is 9%.
- (a) Construct a replicating portfolio for the put option.
- (b) Calculate the current price of the put option.

Exercise 9.

You consider a binomial tree for pricing a 2-year 110-strike call on a futures contract. You are given;

- (i) The period is 1 year.
- (ii) In one-time step, the futures price either moves up by a proportional amount of 25% or moves down by a proportional amount of 20%.
- (iii) The initial futures price is 100.
- (iv) The continuously compounded risk-free interest rate is 6%.

Calculate the difference between the values of Δ at the upper and lower nodes in one year.

Exercise 10.

You are to price options on a futures contract. The movement of the futures price are modeled by a binomial tree. You are given:

- (i) Each period is 1 year.
- (ii) $\frac{u}{d} = 3$, where u is one plus the rate of gain on the futures price if it goes up in one period, and d is one plus the rate of loss if it goes down in one period.
- (iii) The risk-neutral probability of an up move is 0.4
- (iv) The initial futures price is 100

Calculate u .

Exercise 11.

You use a one period binomial model to price a European call option on a non-dividend paying stock. You are given:

- (i) The period is one year.
- (ii) The replicating portfolio of the option consists of 0.5 purchased shares of the stock and an amount of 47 borrowed at the risk-free interest rate.
- (iii) The current price of the stock is 100.
- (iv) The continuously compounded annual expected return on the stock is 10%.
- (v) The continuously compounded risk-free interest rate is 5%.

Calculate the appropriate discount rate for the call option in the real world.

Exercise 12.

You use a one period binomial model to price a 1-year European call option on a non- dividend paying stock. You are given:

- (i) The period is one year.
- (ii) In the risk-neutral world, the expected value of the stock price in one year is 105.
- (iii) The current price is 100.
- (iv) In the risk-neutral world, the expected payoff from the option in one year is 30.

Calculate the price of the option