

# King Saud University

## Department of Mathematics

### Tutorial 1

ACTU 473 – Models of Financial Economics

#### Exercise 1.

For a stock, you are given:

- (i) The current price of the stock is 50.
- (ii) At the end of 3 months, the stock price will be either 45 or 55
- (iii) The stock pays dividends at a rate proportional to its price. The dividend yield is 3%.
- (iv) The continuously compounded risk-free interest rate is 4%.

Consider a 3-month 53-strike European put on the stock.

- (a) Construct a replicating portfolio of the put option.
- (b) Calculate the time-0 price of the put option.
- (c) Suppose that the market price of the put is 4.60, describe an arbitrage opportunity and find the arbitrage profit.

#### Exercise 2.

For a non-dividend paying stock, you are given:

- i. The current price of the stock is 40.
- ii. At the end of 1 month, the stock price will be either 42 or 38
- iii. The continuously compounded risk-free interest rate is 8%.

- (a) Calculate the current price of a 1 -month 39-strike European call.
- (b) Identify an arbitrage strategy if the market price of the option is 1.5

### Exercise 3.

For a non-dividend paying stock, you are given:

- (i) The current stock price is 100
- (ii) It is known that at the end of 1 year, the stock price will be either  $100u$  or 92
- (iii) The continuously compounded risk-free interest rate is 5%
- (iv) The price of a 1-year at-the-money European call option on the stock is 1.78

Calculate  $u$

### Exercise 4.

For a non-dividend paying stock, you are given:

- (i) The current stock price is 100
- (ii) It is known that at the end of 1 year, the stock price will be either 105 or 95.
- (iii) The continuously compounded risk-free interest rate is 3%.

- (a) Calculate the risk-neutral probability of an increase in stock price in one year.
- (b) Calculate the time-0 price of a 1-year at-the-money European call on the stock.

### Exercise 5.

For a stock, you are given:

- (i) The current stock price is 100
- (ii) It is known that at the end of 1 year, the stock price will be either 90 or 110.
- (iii) The stock pays dividends at a rate proportional to its price. The dividend yield is 5%.
- (iv) The continuously compounded risk-free interest rate is 6%.

- (a) Calculate the price of a 1-year 105-strike European call on the stock.
- (b) Calculate the price of a 1-year 105-strike European put on the stock.
- (c) Show that the prices you found in (a) and (b) satisfy the put-call parity.

### Exercise 6.

For a 1-year strangle on a non-dividend paying stock, you are given:

- (i) The strangle can only be exercised at the end of one year.
- (ii) Let  $S(1)$  be the stock price at the end of one year. The payoff of the straddle is as follows:

Stock Price	Payoff
$S(1) \leq 60$	$60 - S(1)$
$60 < S(1) < 70$	0
$S(1) \geq 70$	$S(1) - 70$

- (iii) The stock currently sells for 60.
- (iv) The continuously compounded risk-free interest rate is 8%
- (v) In one year, the stock will either sell for 75 or 45

Calculate the current price of the strangle.

### Exercise 7.

For a 6-month butterfly spread on a stock, you are given:

- (i) The butterfly spread can only be exercised at the end of 6 month.
- (ii) Let  $S(0.5)$  be the stock price at the end of 6 months. The payoff from the butterfly spread is as follows:

Stock Price	Payoff
$S(0.5) \leq 80$	0
$80 \leq S(0.5) < 100$	$S(0.5) - 80$
$100 \leq S(0.5) < 120$	$120 - S(0.5)$
$S(0.5) \geq 120$	0

- (iii) The stock currently sells for 100.
- (iv) The continuously compounded risk-free interest rate is 5%.
- (v) In 6 months, the stock will either sell for 90 or 110.
- (vi) The stock pays dividends at a rate proportional to its price. The dividend yield is  $\delta$ .

Calculate the current price of the butterfly spread.