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Tutorial Session (2)' تابع

***pb 1.3.3 p.25 Textbook**

Let X be a poisson random variable with parameter λ. Determine the probability that X is odd.

Ans:

$pr\{X \text{ is odd}\}$

$$= \sum_{k=1,3,5,\ldots} \frac{\lambda^k e^{-\lambda}}{k!}$$

$$= e^{-\lambda}\left[\lambda + \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \cdots\right]$$

$$= e^{-\lambda} \sinh(\lambda)$$

$$= e^{-\lambda} \cdot \frac{1}{2}(e^{\lambda} - e^{-\lambda})$$

$$= \frac{1}{2}(1 - e^{-2\lambda})$$

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Remember

Maclaurin Series of fns e^{λ}, $e^{-\lambda}$, $\sinh\lambda$ and $\cosh\lambda$

$$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots$$

$$e^{-\lambda} = 1 - \lambda + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \cdots$$

$$\sinh\lambda = \frac{1}{2}(e^{\lambda} - e^{-\lambda}) = \lambda + \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \cdots$$

$$\cosh\lambda = \frac{1}{2}(e^{\lambda} + e^{-\lambda}) = 1 + \frac{\lambda^2}{2!} + \frac{\lambda^4}{4!} + \cdots$$

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Let X and Y be independent poisson distributed random variables having means μ and υ, respectively. Evaluate the convolution of their mass functions to determine the probability distribution of their sum $Z = X + Y$.

Ans: let $Z = X + Y = n$ where X and Y are two independent poisson r.v.s

$\therefore$ The prob. mass fn of Z is

$$pr(Z = n) = \sum_{k=0}^{n} pr\{X = k\}\, pr\{Y = n-k\}$$

$$pr(Z = n) = \sum_{k=0}^{n} \frac{\mu^k e^{-\mu}}{k!} \cdot \frac{\upsilon^{n-k} e^{-\upsilon}}{(n-k)!} = \frac{1}{n!} e^{-(\mu+\upsilon)} \sum_{k=0}^{n} \frac{n!\, \mu^k \upsilon^{n-k}}{k!\,(n-k)!}$$

$$\therefore pr(Z = n) = \frac{e^{-(\mu+\upsilon)}}{n!}(\mu + \upsilon)^n$$ By using Binomial formula

$\therefore Z \sim \text{poisson}(\mu + \upsilon)$. \#

5

* pb 1.3.13 p. 26 Textbook

Suppose that a sample of 10 is taken from a day's output of a machine that produces parts of which 5% are normally defective. If 100% of a day's production is inspected whenever the sample of 10 gives 2 or more defective parts, then what is the probability that 100% of a day's production will be inspected? What assumptions did you make?

Ans:

Assume that the inspected items are independently defective or good.

Let X be # of defects in a sample

$$pr\{X=0\} = (0.95)^{10} = 0.5987$$

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$$pr\{X=1\} = \binom{10}{1}(0.95)^9(0.05)^1 \quad \text{Binomial dist}^{\underline{n}}$$

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$$= 10\,(0.95)^9(0.05)$$

$$= 0.3151$$

$$pr\{X \geq 2\} = 1-(0.5987+0.3151)$$

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$= 0.0862 = 8.62\%$ which is the probability that 100% of a day's production will be inspected.

* pb 1.3.15 p. 26 Textbook

Suppose that X is a poisson distributed random variable with mean $\lambda=2$. Determine $pr\{X\leq\lambda\}$

Ans:

$$pr\{X\leq 2\} = \sum_{x=0}^{2} \frac{e^{-2}2^x}{x!}$$

$X \sim poisson(\lambda)$

$\Rightarrow P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, $x=0,1,2,\ldots$

$$= e^{-2}\left[1+\frac{2}{1!}+\frac{2^2}{2!}\right]$$

$$\therefore pr\{X\leq 2\} = 5e^{-2} = 0.6767$$

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