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| **Second Semester 1442** | **Second Exam** | **King Saud University** |
| **(without calculators)** | **Time: 8 - 9:30 am** | **College of Science** |
| **Wednesday 11-8-1442** | **240 Math** | **Math. Department** |

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Q1: Let $W=\{(2x+3,0):x\in\mathbb{R}\}$ be a subset of the Euclidean vector space $\mathbb{R}^2$. Show that W is **a subspace** of $\mathbb{R}^2$. (4 marks)

Q2: Let $S=\{(1,1,0),(1,2,0),(3,4,5)\}$.

(i) Show that the set S forms a basis for the vector space $\mathbb{R}^3$. (3 marks)

(ii) Find the vector $v\in \mathbb{R}^3$, where $(v)_S=(2,2,1)$. (1 mark)

Q3: Let $B=\{(1,2,1),(3,0,3),(5,4,5)\}$ be a subset of the vector space $\mathbb{R}^3$. Find a subset of B that forms a basis for span(B). (3 marks)

Q4: Use the Wronskian to show that the set {x, sin(x), cos(x)} is linearly independent. (2 marks)

Q5: Let $S=\{(1,2),(3,7)\}$ and $B=\{(2,1),(1,3)\}$ be two bases for the vector space $\mathbb{R}^2$. Find the transition matrix from B to S. (3 marks)

Q6: Let

$$A=\begin{bmatrix}1 & 1 & 1 & 3\\ 2 & 3 & 4 & 5\\ 1 & 0 & 0 & 1\\ 0 & 1 & 1 & 2\end{bmatrix}$$

(i) Find bases for col(A) and row(A). (4 marks)

(ii) Deduce nullity($A^T$) **without solving any equations**. (2 marks)

Q7:(i) If $S = \{v_1, v_2, \ldots, v_n\}$ is a basis for a vector space V, then prove that every vector v in V can be expressed in the form $v = c_1v_1 + c_2v_2 + \cdots + c_nv_n$ in exactly one way. (1 mark)

(ii) For any matrix A, prove that rank(A)=rank($A^T$). (1 mark)

(iii) Let $S=\{p_1,p_2,p_3\}$ be a linearly independent set in the vector space $P_3$. Is S a basis for $P_3$? Why? (1 mark)