First Semester 1442	**Second Exam**	**King Saud University**
(without calculators)	**Time: 8 - 9:30 am**	**College of Science**
Wednesday 11-3-1442	**240 Math**	**Math. Department**

Q1: Let $W=\{(x+1,0):x\in\mathbb{R}\}$ be a subset of the vector space $\mathbb{R}^2$. Show that W is **<u>a subspace</u>** of $\mathbb{R}^2$. (3 marks)

Q2: Let $S=\{(1,0,0),(1,1,0),(3,3,3)\}$. Show that the set S forms a basis for the vector space $\mathbb{R}^3$. (3 marks)

Q3: Let $B=\{(1,2,1),(2,0,2),(4,4,4)\}$ be a subset of the vector space $\mathbb{R}^3$. Find a subset of B that forms a basis of span(B). Also, find dim(span(B)). (3 marks)

Q4: Use the Wronskian to show that the set $\{1, \sin(x), e^x\}$ is linearly independent. (2 marks)

Q5: Let $S=\{(1,1),(2,5)\}$ and $B=\{(2,-1),(1,4)\}$ be two bases of the vector space $\mathbb{R}^2$. Find the transition matrix from B to S. (3 marks)

Q6: Let

$$A=\begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 6 & 2 & 4 \\ 1 & 2 & 3 & 6 \end{bmatrix}$$

(i) Find a basis for the column space of A. (3 marks)

(ii) Deduce nullity(A^T) **<u>without solving any equations</u>**. (2 marks)

Q7: If $S = \{v_1, v_2, \ldots, v_n\}$ is a basis for a vector space V, then prove that every vector v in V can be expressed in the form $v = c_1v_1 + c_2v_2 + \cdots + c_nv_n$ in exactly one way. (2 marks)

Q8: Show that the operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by
$T(x_1, x_2)=(w_1, w_2)= (2x_1 + 2x_2, 3x_1 + 4x_2)$
is one-to-one, and find $T^{-1}(w_1,w_2)$. (4 marks)