King Saud University / College of Sciences / Department of Mathematics
Semester 441 / MATH-244 / Quiz-2

Max. Marks: 10 **Max. Time: 35 Min.**

Name: ______________________ **ID:** ____________ **Signature:** __________

Note: Choose the correct answers to all the 6 questions. Calculators are not allowed!

Question 1 [Marks: 1.5]**:** Let P_2 denote the vector space of polynomials with degree ≤ 2. Given the ordered basis S = $\{2+3t\ ,\ 1-t+t^2\ ,\ 1+t+3t^2\}$ of P_2. If $p \in P_2$ with coordinate vector $[p]_S = [3 \quad 2 \quad -2]^T$, then the polynomial p is equal to:

(a) $p = 6 + 5t - 4t^2$ (b) $p = 4 - 5t - 4t^2$
(c) $p = 3 + 2t - 4t^2$ (d) $p = 3 + 3t - 4t^2$

Question 2 [Marks: 2]**: Let** $\boldsymbol{E} = \{u_1,\ u_2,\ u_3\}$ and $\boldsymbol{F} = \{(1,0,0,1), (-1,1,0,1), (0,0,1,1)\}$ be two ordered bases for a vector subspace for the Euclidean space $\mathbb{R}^4$. **If** ${}_{\boldsymbol{F}}\boldsymbol{P}_{\boldsymbol{E}} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is the transition matrix from $\boldsymbol{E}$ to $\boldsymbol{F}$, **then** the vector u_3 is equal to:

(a) $(4, 1, 1, -1)$ (b) $(4, 1, 2, -1)$ (c) $(-1, 2, 1, 4)$ (d) $(-1, 1, 1, 4)$.

Question 3 [Marks: 1.5]**: For** the matrix $\boldsymbol{A} = \begin{bmatrix} 1 & -2 & 2 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & -2 & 3 \end{bmatrix}$, **which** of the following statements is true?

(a) $nullity(\boldsymbol{A}) = 3$ (b) $nullity(\boldsymbol{A}) = 2$ (c) $rank(\boldsymbol{A}) = 0$ (d) $rank(\boldsymbol{A}) = 3$.

Question 4 [Marks: 1.5]**: If** u and v are linearly independent vectors in a real inner product space $(V,\ <,>)$ with $||u|| = 3$ and $||v|| = 2$, **then** which of the following statements is true?

(a) The number $< u , v >$ is less than 5.
(b) The number $| < u , v > |$ less than 6.
(c) The number $| < u , v > |$ is equal to 6.
(d) The number $| < u , v > |$ is equal to 5.

Question 5 [Marks: 2]**: If** $\boldsymbol{G} = \{u, v, w\}$ is an orthogonal set of non-zero vectors in the Euclidean space $\mathbb{R}^3$, **then** which of the following statements is true?

(a) $span(\boldsymbol{G})$ is a proper subset of $\mathbb{R}^3$.
(b) $\boldsymbol{G}$ is an orthogonal basis for $\mathbb{R}^3$.
(c) The set $\boldsymbol{G}$ is normal in the space $\mathbb{R}^3$.
(d) The set $\boldsymbol{G}$ is linearly dependent in the space $\mathbb{R}^3$.

Question 6 [Marks: 1.5]**: If** the Gram-Schmidt orthogonalization algorithm is applied on the set $\{(0, 1, 1, 0), (1, 0, 0, 1)\}$ of vectors in the Euclidean space $\mathbb{R}^4$, **then** which of the following orthogonal sets is obtained?

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(a) $\{(1, 0, 0, 0), (0, 1, 0, 0)\}$ (b) $\{(0, 1, 1, 0), (1, 0, 0, 0)\}$
(c) $\{(1, 0, 0, 1), (0, 1, 1, 0)\}$ (d) $\{(1, 0, 0, 1), (0, 0, 1, 0)\}$.

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