Linear Momentum and Collisions

9.1 Linear Momentum and Its Conservation

9.2 Impulse and Momentum

9.3 Collisions in One Dimension

9.4 Two-Dimensional Collisions

Consider what happens when a bowling ball strikes a pin.

A moving bowling ball carries momentum, the topic of this chapter. In the collision between the ball and the pins, momentum is transferred to the pins.

One of the main objectives of this chapter is to enable you to understand and analyze such events in a simple way. First, we introduce the concept of *momentum*, which is useful for describing objects in motion.

Momentum Analysis Models

- Force and acceleration are related by Newton's second law.
- When force and acceleration vary by time, the situation can be very complicated.
- The techniques developed in this chapter will enable you to understand and analyze these situations in a simple way.
- Will develop momentum versions of analysis models for isolated and non-isolated systems
- These models are especially useful for treating problems that involve collisions and for analyzing rocket propulsion.

9.1 Linear Momentum and Its Conservation

- An archer stands on frictionless ice and fires an arrow. What is the archer's velocity after firing the arrow?
  - Motion models such as a particle under constant acceleration cannot be used.
    - No information about the acceleration of the arrow
  - Model of a particle under constant force cannot be used.
    - No information about forces involved
  - Energy models cannot be used.
    - No information about the work or the energy (energies) involved
- A new quantity is needed – linear momentum.

The **linear momentum** of a particle or an object that can be modeled as a particle of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:

$$\mathbf{p} \equiv m\mathbf{v}$$

Linear momentum is a vector quantity

Its direction is the same as the direction of the velocity

It has dimensions ML/T, and its SI unit is kg · m/s.

Momentum have three components:

$$\mathrm{p}_x = mv_x \qquad \mathrm{p}_y = mv_y \qquad \mathrm{p}_z = mv_z$$

Newton’s second law and the linear momentum

Using Newton’s second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle.

$$\boldsymbol{\Sigma F} = m\boldsymbol{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt}$$

$$\boldsymbol{\Sigma F} = \frac{d\mathbf{p}}{dt}$$

The mass *m* is assumed to be constant.

The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

This alternative form of Newton’s second law

Let us apply the General Problem-Solving Strategy and *conceptualize* an isolated system of two particles with masses m_1 and m_2 and moving with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ at an instant of time.

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

We can express this condition as

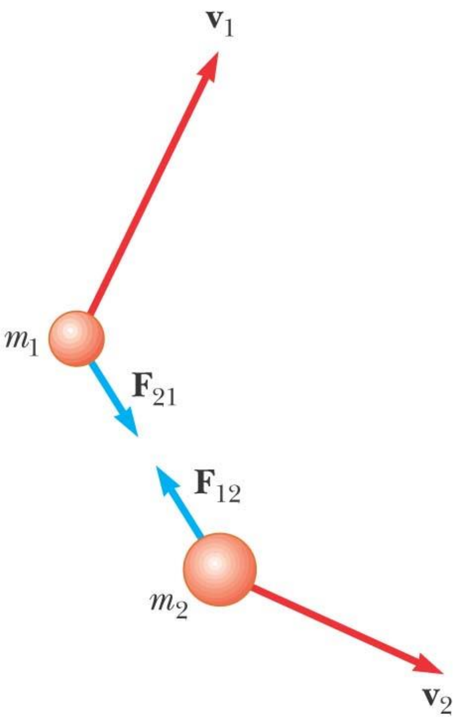
$$\mathbf{F}_{12} + \mathbf{F}_{21} = 0$$

$$m_1\boldsymbol{a}_1 + m_2\boldsymbol{a}_2 = 0$$

$$m_1\frac{d\mathbf{v}_1}{dt} + m_2\frac{d\mathbf{v}_2}{dt} = 0$$

$$\frac{d}{dt}(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = 0$$

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = \text{constant}$$



The law of conservation of momentum

The total momentum of a system for the two particles for the time interval during which the particles interact:

$$\mathbf{p}_{\text{total}} = \mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$$

or,

$$\mathbf{p}_{1\text{i}} + \mathbf{p}_{2\text{i}} = \mathbf{p}_{1\text{f}} + \mathbf{p}_{2\text{f}}$$

In component form:

$$\mathrm{p}_{ix} = \mathrm{p}_{fx} \qquad \mathrm{p}_{iy} = \mathrm{p}_{fy} \qquad \mathrm{p}_{iz} = \mathrm{p}_{fz}$$

This result, known as the **law of conservation of linear momentum,** can be extended to any number of particles in an isolated system.

Conservation of momentum

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

The total momentum of an isolated system at all times equals its initial momentum.

$$\mathbf{p}_{1\mathrm{i}} + \mathbf{p}_{2\mathrm{i}} = \mathbf{p}_{1\mathrm{f}} + \mathbf{p}_{2\mathrm{f}}$$

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

Example 9.1 The Archer

A 60-kg archer stands at rest on frictionless ice and fires a 0.50-kg arrow horizontally at 50 m/s (Fig. 9.2). With what velocity does the archer move across the ice after firing the arrow?

Let us take the system to consist of the archer (including the bow) and the arrow.

The archer is particle 1 and the arrow is particle 2.

We choose the direction of firing of the arrow as the positive *x* direction.

There are no external forces in the horizontal direction, and we can consider the system to be isolated in terms of momentum components in this direction.

The total horizontal momentum of the system before the arrow is fired is zero ($m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}$=0).

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$
$$0 = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$m_1 = 60\text{kg}, \qquad m_2 = 0.50\text{ km}$$
$$\mathbf{v}_{1f} = ?, \qquad \mathbf{v}_{2f} = 50\,\hat{\mathbf{i}}\text{ m/s}$$

$$\mathbf{v}_{1f} = -\frac{m_2}{m_1}\mathbf{v}_{2f}$$
$$= -\frac{0.50\text{ km}}{60\text{ km}}(50\,\hat{\mathbf{i}}\text{ m/s}) = -0.42\,\hat{\mathbf{i}}\text{ m/s}$$

9.2 Impulse and Momentum

The momentum of a particle changes if a net force acts on the particle.

Let us assume that a single force **F** acts on a particle and that this force may vary with time. According to Newton's second law,

$$\mathrm{d}\mathbf{p} = \mathbf{F}\mathrm{d}t$$

We can integrate this expression to find the change in the momentum of a particle when the force acts over some time interval:

$$\Delta\mathbf{p} = \mathbf{p}_{\mathrm{f}} - \mathbf{p}_{\mathrm{i}} = \int_{t_i}^{t_f} \mathbf{F}\,\mathrm{d}t$$

The **impulse** of the force **F** acting on a particle over the time interval Δt

$$\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F}\, \mathrm{dt}$$

The impulse of the force **F** acting on a particle equals the change in the momentum of the particle.

$$\Delta \mathbf{p} = \mathbf{I}$$

This is the most general statement of the principle of conservation of momentum and is called **the conservation of momentum** equation.

This form applies to **non-isolated systems**.

A time-averaged force

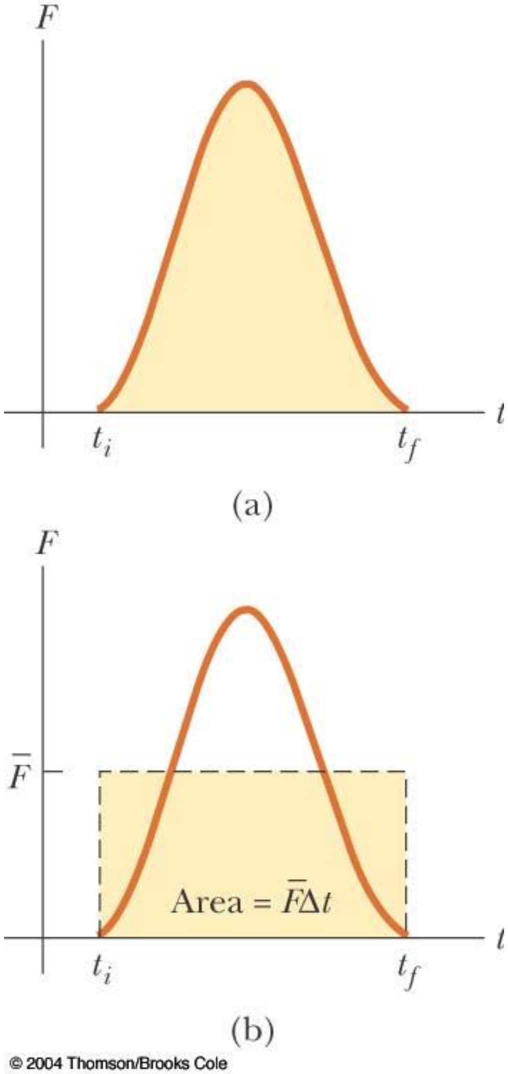
$$\bar{\mathbf{F}} = \frac{\mathbf{1}}{\Delta \mathrm{t}} \int_{t_i}^{t_f} \mathbf{F}\, dt$$

Therefore,

$$\boxed{\mathbf{I} = \bar{\mathbf{F}} \Delta \mathrm{t}}$$

If the force acting on the particle is constant.

$$\boxed{\mathbf{I} = \mathbf{F} \Delta \mathrm{t}}$$



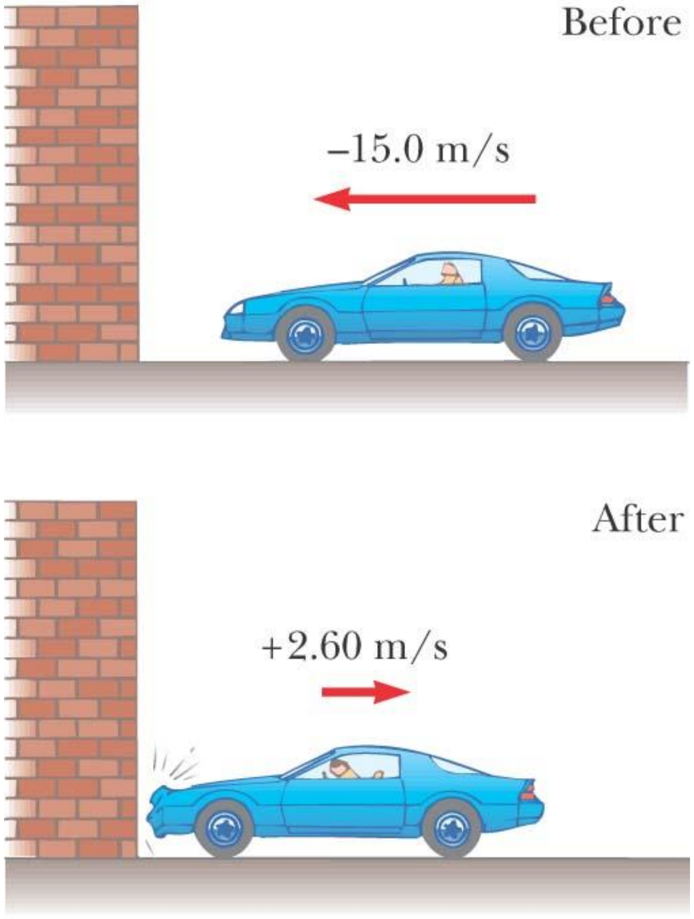
Impulse approximation

In many physical situations, we shall use the **impulse approximation,** in which we assume that one of the forces exerted on a particle acts for a short time but is much greater than any other force present.

Example: when a baseball is struck with a bat, the time of the collision is about 0.01 s and the average force that the bat exerts on the ball in this time is typically several thousand newtons.

Example 9.4 How Good Are the Bumpers?

In a particular crash test, a car of mass 1 500 kg collides with a wall, as shown in Figure 9.6. The initial and final velocities of the car are $\mathbf{v}_i$ = -15.0**i** m/s and $\mathbf{v}_f$ = 2.60**i** m/s, respectively. If the collision lasts for 0.150 s, find the impulse caused by the collision and the average force exerted on the car.



The initial and final momenta of the car are

$$\mathbf{p}_i = m\mathbf{v}_i = (1500 \text{ kg})(-15.0\hat{\mathbf{i}} \text{ m/s})$$
$$= -2.25 \times 10^4 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s}$$

$$\mathbf{p}_f = m\mathbf{v}_f = (1500 \text{ kg})(2.60\hat{\mathbf{i}} \text{ m/s})$$
$$= 0.39 \times 10^4 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s}$$

Hence, the impulse is equal to

$$\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = 0.39 \times 10^4 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s} - (-2.25 \times 10^4 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s})$$

$$\mathbf{I} = \boxed{2.64 \times 10^4 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s}}$$

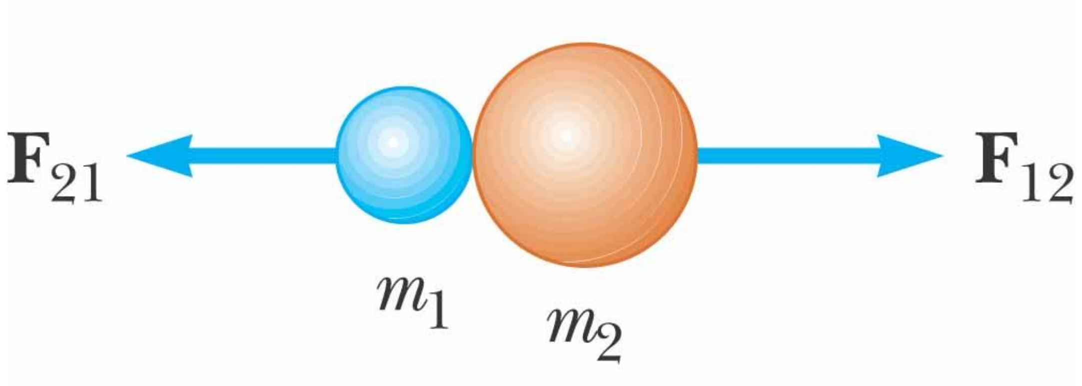
The average force exerted by the wall on the car is

$$\overline{\mathbf{F}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{2.64 \times 10^4 \hat{\mathbf{i}} \text{ kg} \cdot \text{m/s}}{0.150 \text{ s}} = \boxed{1.76 \times 10^5 \hat{\mathbf{i}} \text{ N}}$$

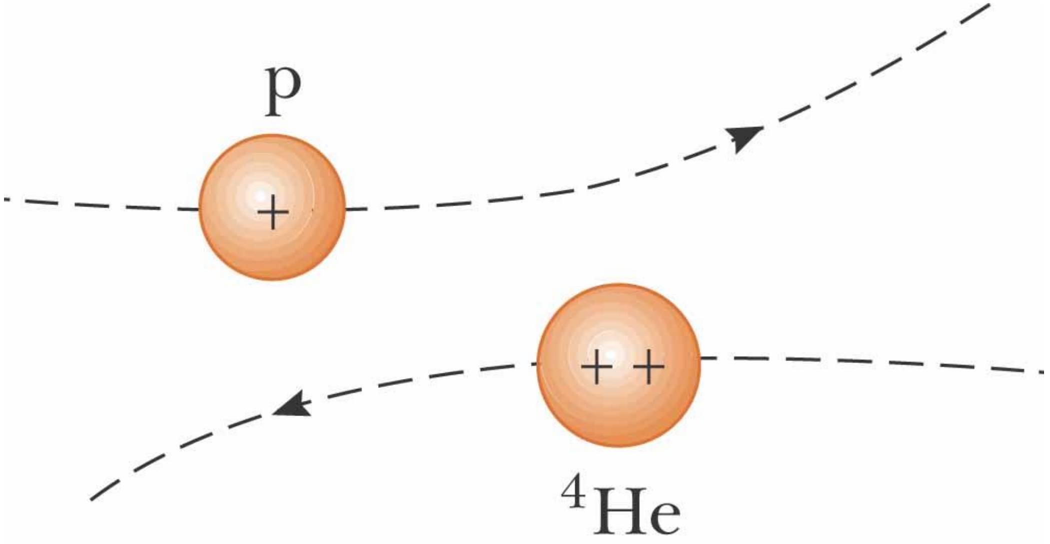
9.3 Collisions in One Dimension

- The law of conservation of linear momentum will be used to describe what happens when two particles collide.
- The term **collision** to represent an event during which two particles come close to each other and interact by means of forces.
- The time interval during which the velocities of the particles change from initial to final values is assumed to be short.
- The interaction forces are assumed to be much greater than any external forces present, so we can use **the impulse approximation**.
- A collision may involve physical contact between two macroscopic objects, but the notion of what we mean by collision must be generalized.

- When two particles of masses m_1 and m_2 collide, the impulsive forces may vary in time in complicated ways. Regardless of the complexity of the time behavior of the force of interaction,
  - this force is internal to the system of two particles.
  - the two particles form an isolated system,
  - and the momentum of the system must be conserved.
    - Therefore, the total momentum of an isolated system just before a collision equals the total momentum of the system just after the collision.



- Example of of a collision without physical contact between the objects: two charged particles.
- It can be analyzed in the same way as those that include physical contact.



Type of collisions

1) An **inelastic collision:**

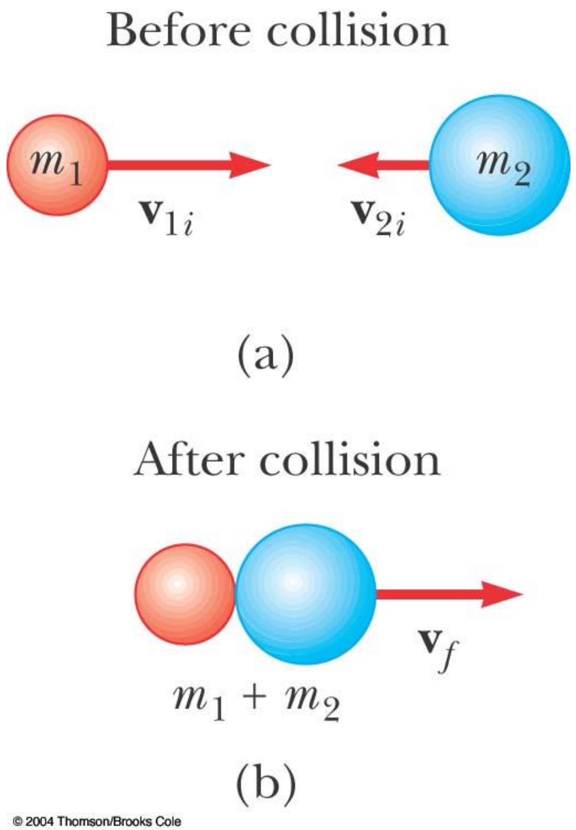
- the total kinetic energy of the system is not the same before and after the collision.
- The momentum of the system is conserved.
- If the objects stick together after the collision, it is a **perfectly inelastic** collision (a meteorite collides with the Earth).
- When the colliding objects do not stick together, but some kinetic energy is lost, as in the case of a rubber ball colliding with a hard surface, the collision is called inelastic.

Elastic and perfectly inelastic collisions are limiting cases; most collisions fall somewhere between them.

Perfectly Inelastic Collisions

The total momentum before the collision equals the total momentum of the composite system after the collision:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$



Solving for the final velocity gives

$$\boxed{\mathbf{v}_f = \frac{m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}}{m_1 + m_2}}$$

Type of collisions

2) An **elastic collision** between two objects:

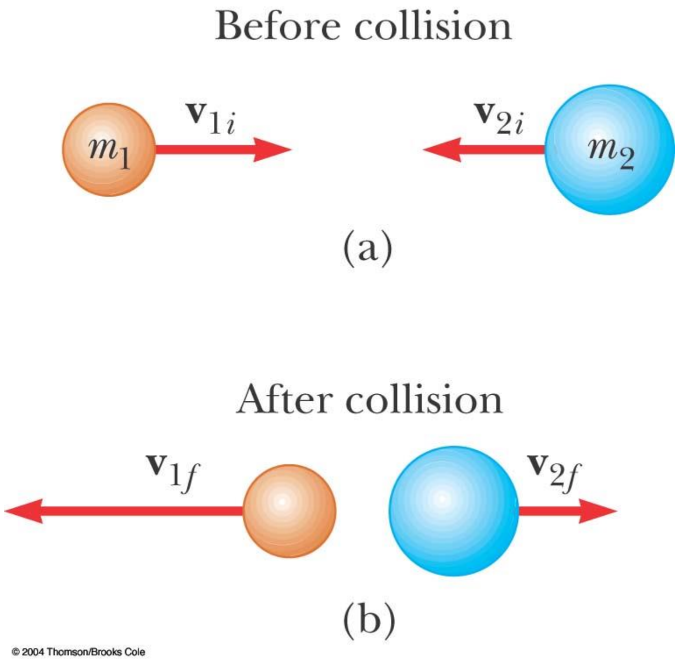
- The total kinetic energy (as well as total momentum) of the system is the same before and after the collision.
- Collisions between certain objects in the macroscopic world, are only *approximately* elastic because some deformation and loss of kinetic energy take place.
- For example, you can hear a billiard ball collision, so you know that some of the energy is being transferred away from the system by sound.
- An elastic collision must be perfectly silent! *Truly* elastic collisions occur between atomic and subatomic particles.

Elastic Collisions

- Considering velocities along the horizontal direction:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$\frac{1}{2}m_1{v_{1i}}^2 + \frac{1}{2}m_2{v_{2i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$



Typically, there are two unknowns to solve for and so you need two equations.
Remember to use the appropriate signs for all velocities

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

Special cases:

Let us consider some special cases.

i) If $m_1 = m_2$, then, $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$.

ii) If $v_{2i} = 0$,

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i}$$

iii) If m_1 is much greater than m_2 and $v_{2i} = 0$, then $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$.

iv) If m_2 is much greater than m_1 and $v_{2i} = 0$, then $v_{1f} \approx -v_{1i}$ and $v_{2f} \approx 0$.

Example 9.5 The Executive Stress Reliever

An ingenious device that illustrates conservation of momentum and kinetic energy is shown in Figure 9.10. It consists of five identical hard balls supported by strings of equal lengths. When ball 1 is pulled out and released, after the almost-elastic collision between it and ball 2, ball 5 moves out, as shown in Figure 9.10b. If balls 1 and 2 are pulled out and released, balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1, as in Figure 9.10c?

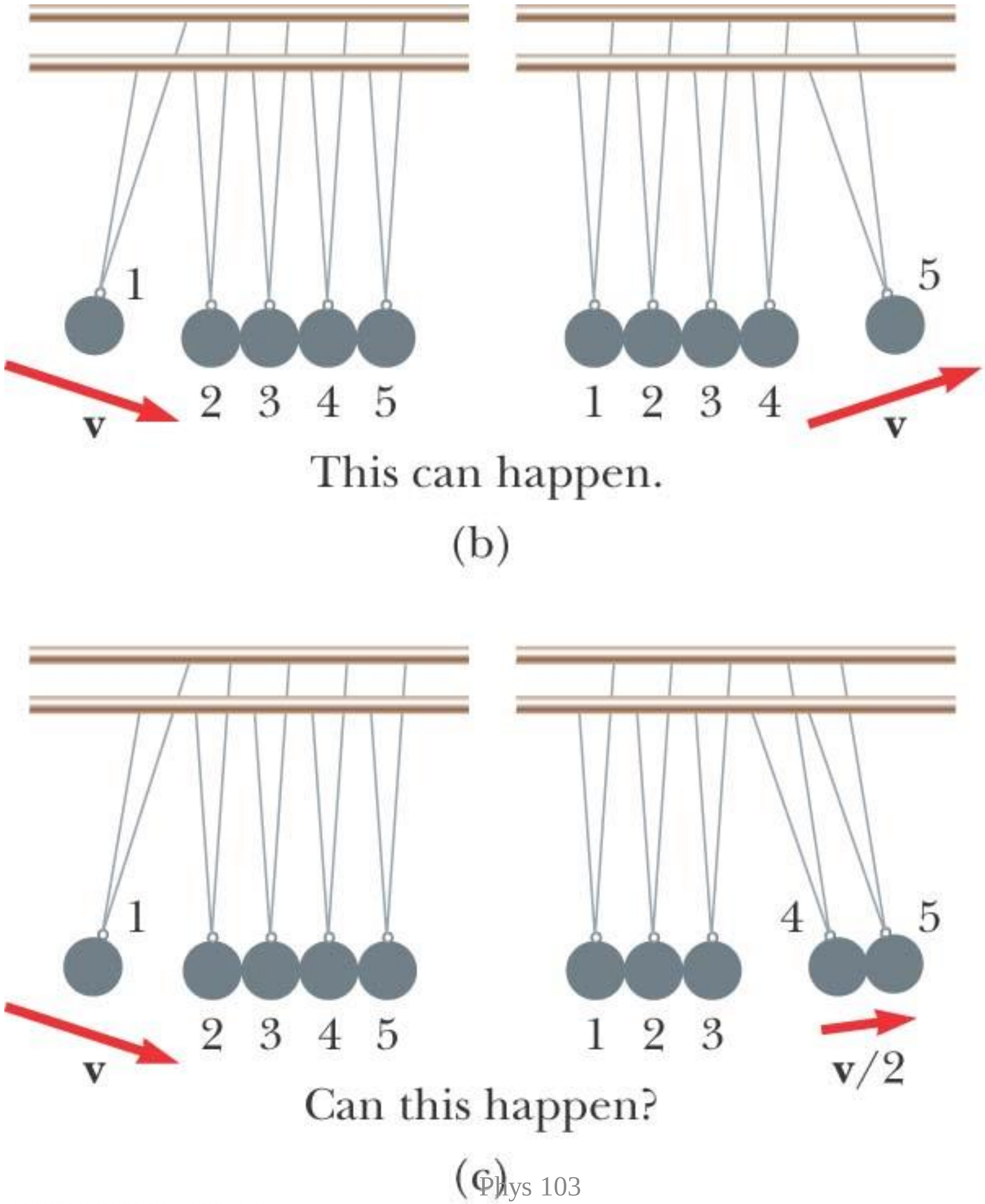

1
2 3 4 5
v
1 2 3 4
5
v
This can happen.
(b)
1
2 3 4 5
v
1 2 3
4 5
v/2
Can this happen?
(c)

Fig. 9.10, p.263

Solution:

No, such movement can never occur if we assume the collisions are elastic.

The momentum of the system before the collision $m\mathbf{v}_{1i}$

After the collision (if two balls moving with $\mathbf{v}_{1i}/2$), $m\frac{\mathbf{v}_{1i}}{2} + m\frac{\mathbf{v}_{1i}}{2}$

The total momentum of the system after the collision would be

$$m\mathbf{v}_{1i} = m\mathbf{v}_{1i}$$

Thus, momentum of the system is **conserved**.

However, the kinetic energy just before the collision is $\frac{1}{2}m{v_{1i}}^2$

and that after the collision is $\frac{1}{8}m{v_{1i}}^2 + \frac{1}{8}m{v_{1i}}^2 = \frac{1}{4}m{v_{1i}}^2$

Thus, kinetic energy of the system is ***not* conserved**.

The only way to have both momentum and kinetic energy conserved is for one ball to move out when one ball is released, two balls to move out when two are released, and so on.

What If? **Consider what would happen if balls 4 and 5 are glued together so that they must move together. Now what happens when ball 1 is pulled out and released?**

Answer We are now forcing balls 4 and 5 to come out together. We have argued that we cannot conserve both momentum and energy in this case. However, we assumed that ball 1 stopped after striking ball 2. What if we do not make this assumption? Consider the conservation equations with the assumption that ball 1 moves after the collision. For conservation of momentum,

$$p_i = p_f$$

$$mv_{1i} = mv_{1f} + 2mv_{4,5f}$$

where $v_{4,5f}$ refers to the final speed of the ball 4–ball 5 combination. Conservation of kinetic energy gives us

$$K_i = K_f$$

$$\frac{1}{2}m{v_{1i}}^2 = \frac{1}{2}m{v_{1f}}^2 + \frac{1}{2}(2m)\,v_{4,5f}^2$$

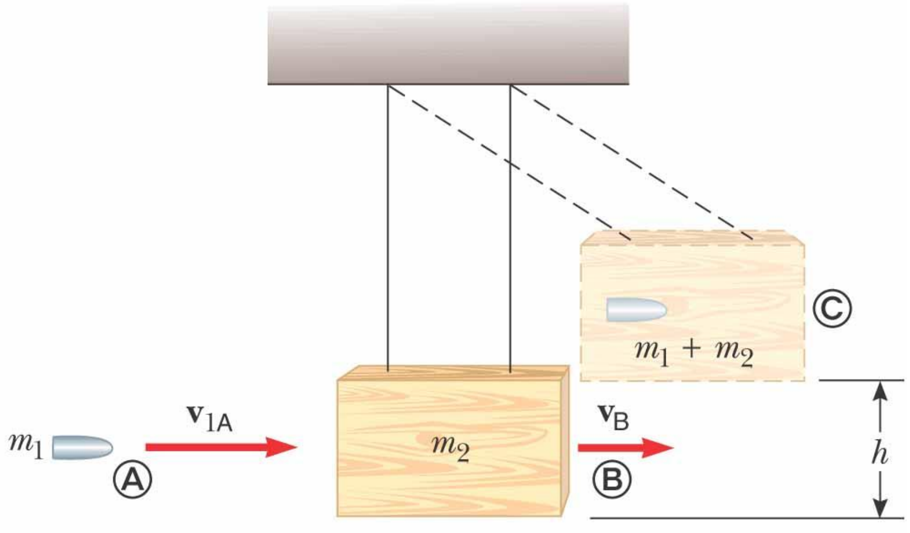
Combining these equations, we find

$$v_{4,5f} = \frac{2}{3}v_{1i} \qquad v_{1f} = -\frac{1}{3}v_{1i}$$

Thus, balls 4 and 5 come out together and ball 1 bounces back from the collision with one third of its original speed.

Example 9.7 The Ballistic Pendulum

The ballistic pendulum (Fig. 9.11) is an apparatus used to measure the speed of a fast-moving projectile, such as a bullet. A bullet of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height h. How can we determine the speed of the bullet from a measurement of h?



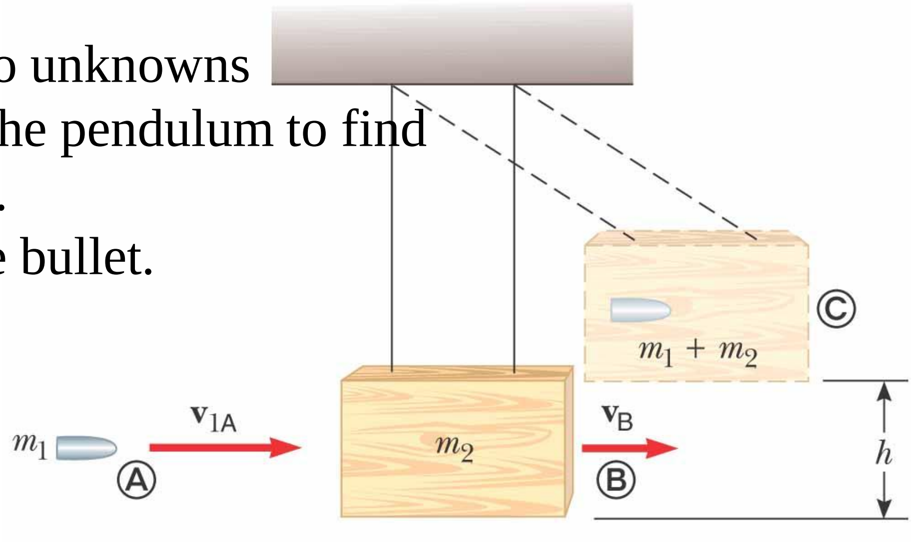
(a)

Conceptualize

- Observe diagram
- The projectile enters the pendulum, which swings up to some height where it momentarily stops.

Categorize

- Isolated system in terms of momentum for the projectile and block.
- Perfectly inelastic collision – the bullet is embedded in the block of wood.
- Momentum equation will have two unknowns
- Use conservation of energy from the pendulum to find the velocity just after the collision.
- Then you can find the speed of the bullet.



(a)

Before the collision:

$\mathbf{v}_{2i} = 0$

The system right after the collision:

$$\mathrm{v}_B = \frac{m_1 \mathrm{v}_{1i} + m_2 \mathrm{v}_{2i}}{m_1 + m_2}$$

$$v_B = \frac{m_1 v_{1A}}{m_1 + m_2}$$

The total kinetic energy

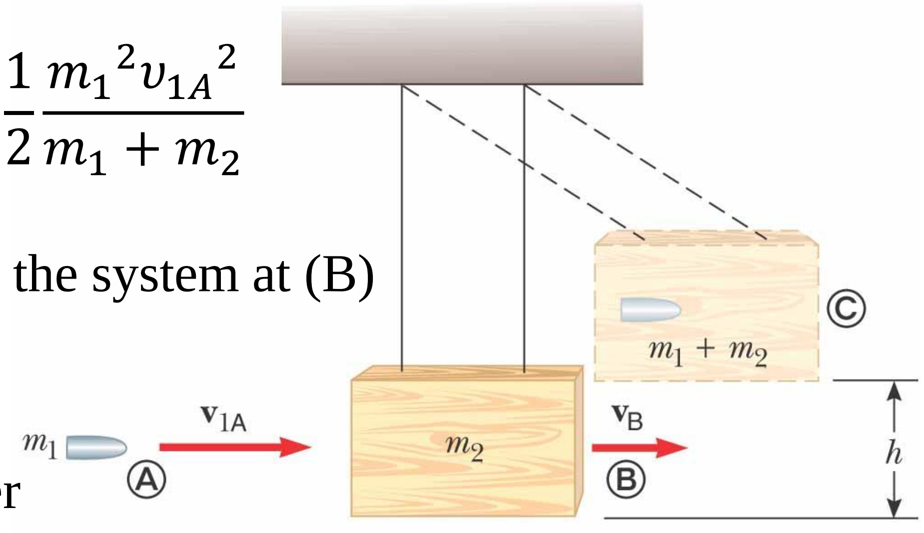
$$K_B = \frac{1}{2}(m_1 + m_2)\, v_B{}^2$$

$$K_B = \frac{1}{2}\frac{m_1{}^2 v_{1A}{}^2}{m_1 + m_2}$$

The gravitational potential energy of the system at (B)

$$U_{\mathrm{gB}} = 0$$

This kinetic energy immediately after the collision is *less* than the initial kinetic energy of the bullet



(a)

At the configuration ©:

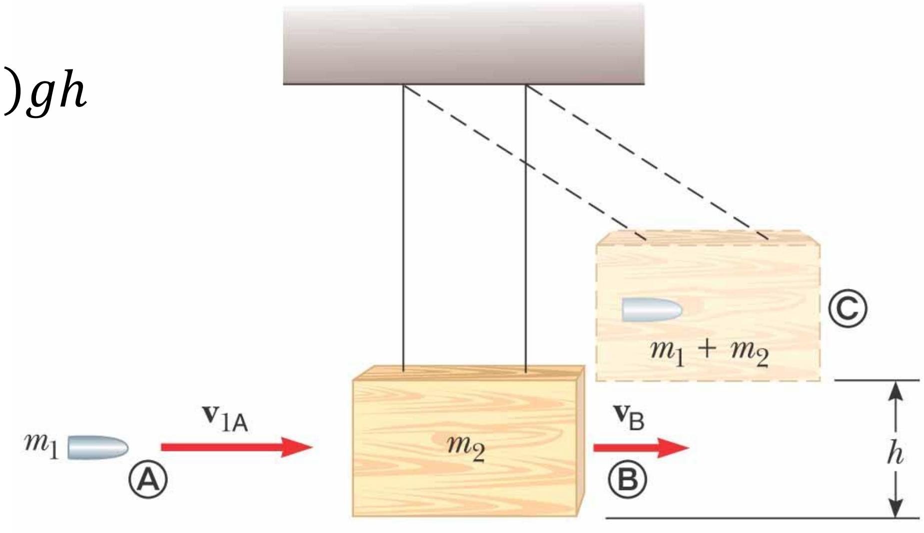
The gravitational potential energy of the system
$U_{gC} = (m_1 + m_2)gh$

Conservation of energy now leads to

$$K_{\mathrm{B}} + U_{\mathrm{g}B} = K_{\mathrm{C}} + U_{\mathrm{gC}}$$

$$\frac{{m_1}^2\, {v_{1A}}^2}{2(m_1 + m_2)} + 0 = 0 + (m_1 + m_2)gh$$

$$v_{1A} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$$



(a)

To finalize this problem, note that we had to solve this problem in two steps.

Each step involved a different system and a different conservation principle.

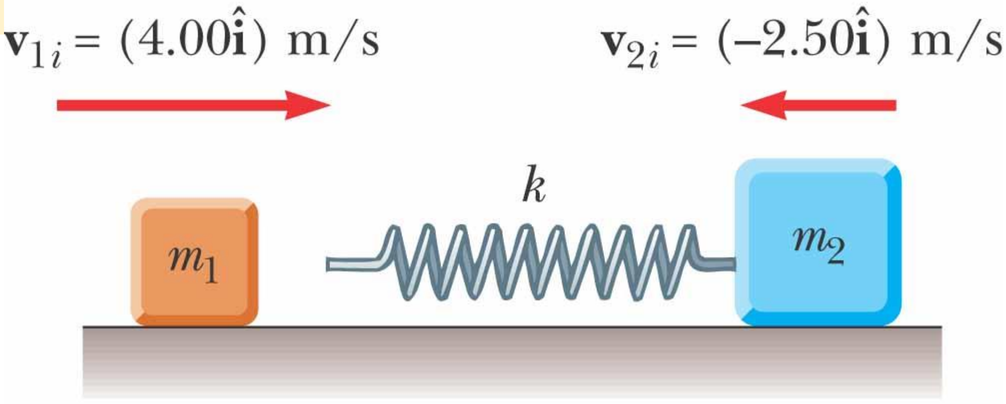
Because the collision was assumed to be perfectly inelastic, some mechanical energy was converted to internal energy.

It would have been *incorrect* to equate the initial kinetic energy of the incoming bullet to the final gravitational potential energy of the bullet–block–Earth combination.

Example 9.8 A Two-Body Collision with a Spring

A block of mass m_1 = 1.60 kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass m_2 = 2.10 kg initially moving to the left with a speed of 2.50 m/s, as shown in Figure 9.12a. The spring constant is 600 N/m.

(A) Find the velocities of the two blocks after the collision.

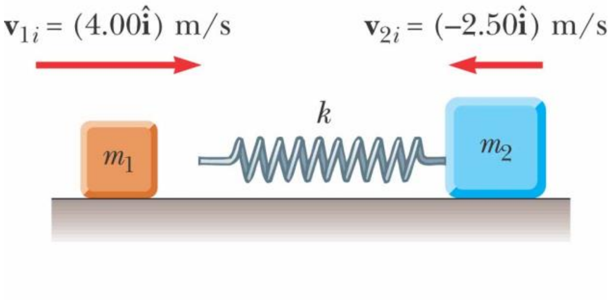


(a)

- The spring force is conservative, no kinetic energy is converted to internal energy during the compression of the spring.
- Ignoring any sound made when the block hits the spring, we can model the collision as being elastic.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg})(-2.50\text{ m/s})$$
$$= (1.60\text{ kg})v_{1f} + (2.10\text{ kg})v_{2f}$$
$$(1) \qquad 1.15\text{ kg}\cdot\text{m/s} = (1.60\text{ kg})v_{1f} + (2.10\text{ kg})v_{2f}$$



(a)

Equation 9.19 gives us

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$4.00\text{ m/s} - (-2.50\text{ m/s}) = -v_{1f} + v_{2f}$$

$$(2) \qquad 6.50\text{ m/s} = -v_{1f} + v_{2f}$$

Multiplying Equation (2) by 1.60 kg gives us

$$(3) \qquad 10.4\ \mathrm{kg\cdot m/s} = -(1.60\ \mathrm{kg})v_{1f} + (1.60\ \mathrm{kg})v_{2f}$$

Adding Equations (1) and (3) allows us to find v_{2f}:

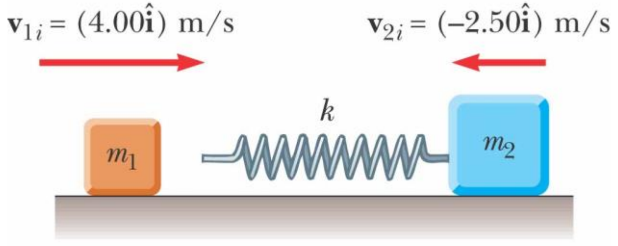
$$11.55\ \mathrm{kg\cdot m/s} = (3.70\ \mathrm{kg})v_{2f}$$

$$v_{2f} = \frac{11.55\ \mathrm{kg\cdot m/s}}{3.70\ \mathrm{kg}} = \boxed{3.12\ \mathrm{m/s}}$$

Now, Equation (2) allows us to find v_{1f}:

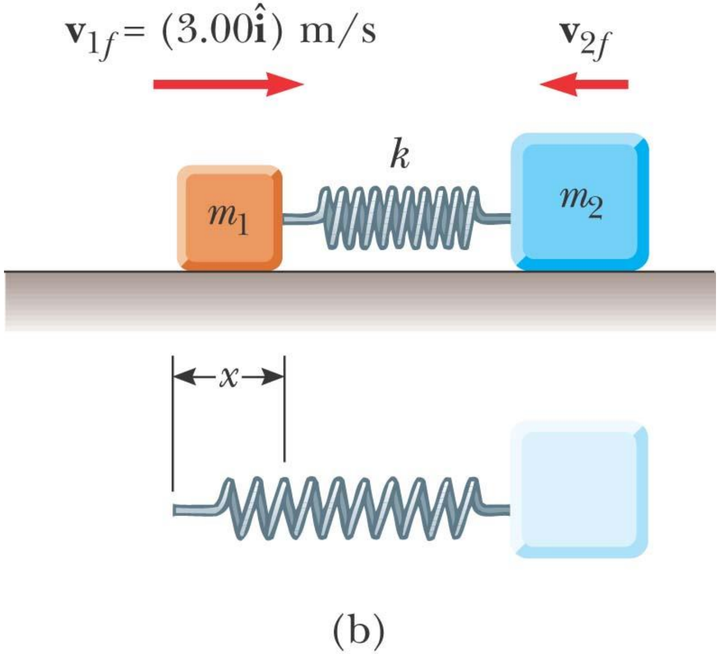
$$6.50\ \mathrm{m/s} = -v_{1f} + 3.12\ \mathrm{m/s}$$

$$v_{1f} = \boxed{-3.38\ \mathrm{m/s}}$$



(a)

(B) During the collision, at the instant block 1 is moving to the right with a velocity of +3.00 m/s, as in Figure 9.12b, determine the velocity of block 2.



Solution Because the momentum of the system of two blocks is conserved *throughout* the collision for the system of two blocks, we have, for *any* instant during the collision,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

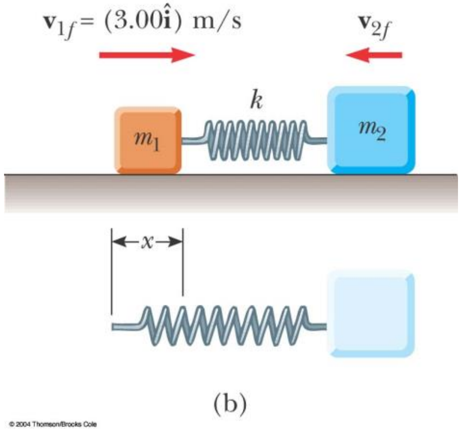
We choose the final instant to be that at which block 1 is moving with a velocity of $+3.00$ m/s:

$$(1.60 \text{ kg})(4.00 \text{ m/s}) + (2.10 \text{ kg})(-2.50 \text{ m/s})$$
$$= (1.60 \text{ kg})(3.00 \text{ m/s}) + (2.10 \text{ kg}) v_{2f}$$
$$v_{2f} = \boxed{-1.74 \text{ m/s}}$$

The negative value for v_{2f} means that block 2 is still moving to the left at the instant we are considering.

(C) Determine the distance the spring is compressed at that instant.

Solution To determine the distance that the spring is compressed, shown as x in Figure 9.12b, we can use the principle of conservation of mechanical energy for the system of the spring and two blocks because no friction or other nonconservative forces are acting within the system. We choose the initial configuration of the system to be that existing just before block 1 strikes the spring and the final configuration to be that when block 1 is moving to the right at 3.00 m/s. Thus, we have



(b)

$$K_i + U_i = K_f + U_f$$

$$\tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_2 v_{2i}^2 + 0 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2 + \tfrac{1}{2}kx^2$$

Substituting the given values and the result to part (B) into this expression gives

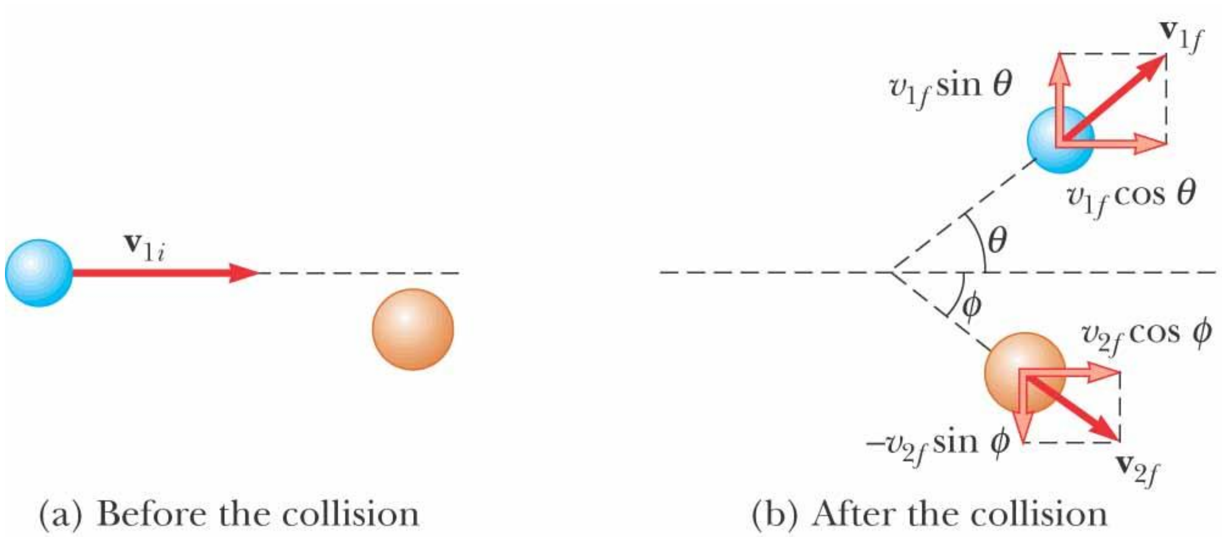
$$x = \boxed{0.173 \text{ m}}$$

9.4 Two-Dimensional Collisions

Let us consider a two-dimensional problem in which particle 1 of mass m_1 collides with particle 2 of mass m_2, we obtain two component equations for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



If particle 2 is initially at rest,

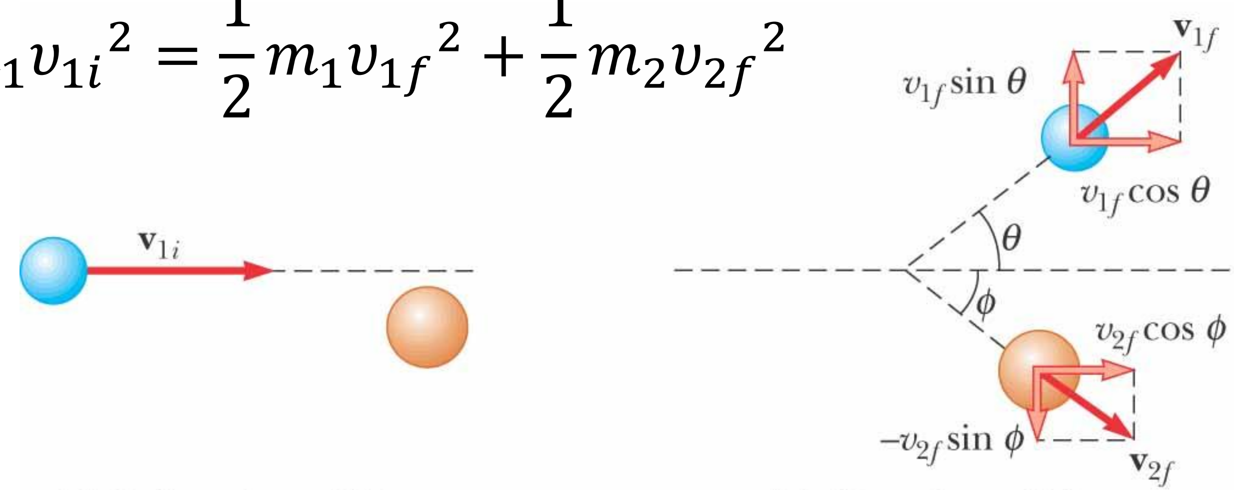
- After the collision, particle 1 moves at an angle θ with respect to the horizontal and particle 2 moves at an angle ϕ with respect to the horizontal. This is called a *glancing* collision.
- Applying the law of conservation of momentum in component form

$$m_1 v_{1i} + 0 = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

If the collision is elastic, from conservation of kinetic energy:

$$\frac{1}{2}m_1 {v_{1i}}^2 = \frac{1}{2}m_1 {v_{1f}}^2 + \frac{1}{2}m_2 {v_{2f}}^2$$



(a) Before the collision (b) After the collision

If the collision is an inelastic collision, the two component equations of the conservation of momentum are:

$$m_1 v_{1ix} + m_2 v_{2ix} = (m_1 + m_2) v_{fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = (m_1 + m_2) v_{fy}$$

$$v_f = \sqrt{{v_{fx}}^2 + {v_{fy}}^2}$$

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

PROBLEM-SOLVING HINTS

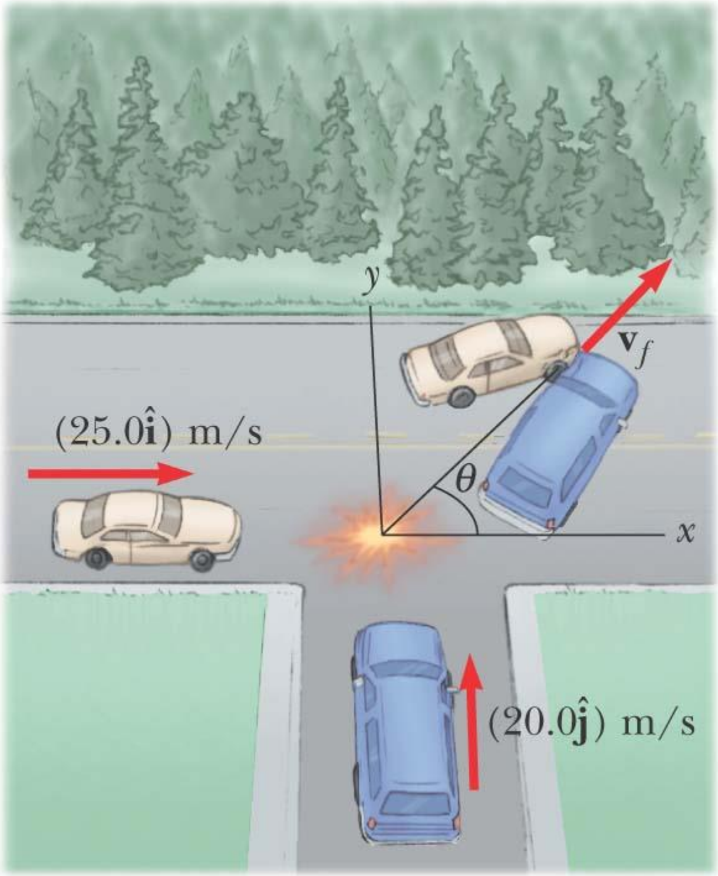
Two-Dimensional Collisions

The following procedure is recommended when dealing with problems involving two-dimensional collisions between two objects:

- Set up a coordinate system and define your velocities with respect to that system. It is usually convenient to have the x axis coincide with one of the initial velocities.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the x and y components of the momentum of each object before and after the collision. Remember to include the appropriate signs for the components of the velocity vectors.
- Write expressions for the total momentum of the system in the x direction before and after the collision and equate the two. Repeat this procedure for the total momentum of the system in the y direction.
- If the collision is inelastic, kinetic energy of the system is *not* conserved, and additional information is probably required. If the collision is *perfectly* inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is *elastic,* kinetic energy of the system is conserved, and you can equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain an additional relationship between the velocities.

Example 9.10 Collision at an Intersection

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.14. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).



Solution

Let us choose east to be along the positive *x* direction and north to be along the positive *y* direction.

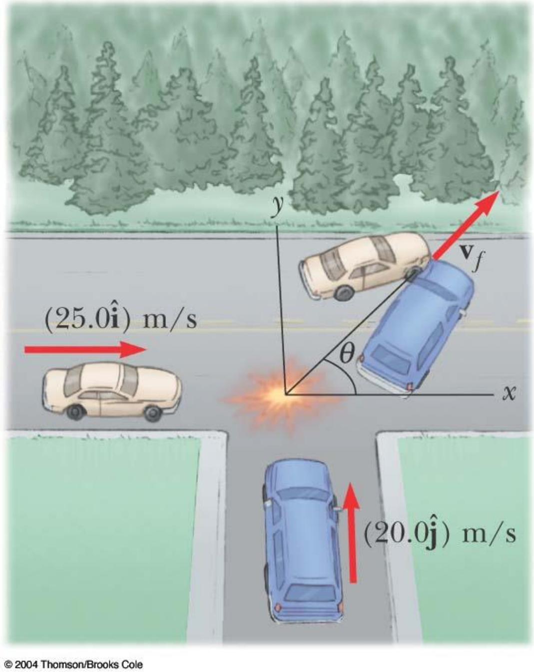
Applying conservation of momentum to **the *x* direction**:

$m_1\, v_{1ix} + m_2 v_{2ix} = (m_{\text{can}} + m_{\text{van}}) v_f \cos\theta$

$m_{\text{car}}\, v_{\text{car}\, i} = (m_{\text{can}} + m_{\text{van}}) v_f \cos\theta$

$(1\,500\ \text{kg})(25.0\ \text{m/s}) = (4\,000\ \text{kg})\, v_f \cos\theta$

$3.75 \times 10^4\ \text{kg.m/s} = (4\,000\ \text{kg})\, v_f \cos\theta \quad (1)$



Solution

Applying conservation of momentum to **the *y* direction**:

$m_{\text{van}}\ v_{\text{van}\ i} = (m_1+m_2)v_f \sin\theta$

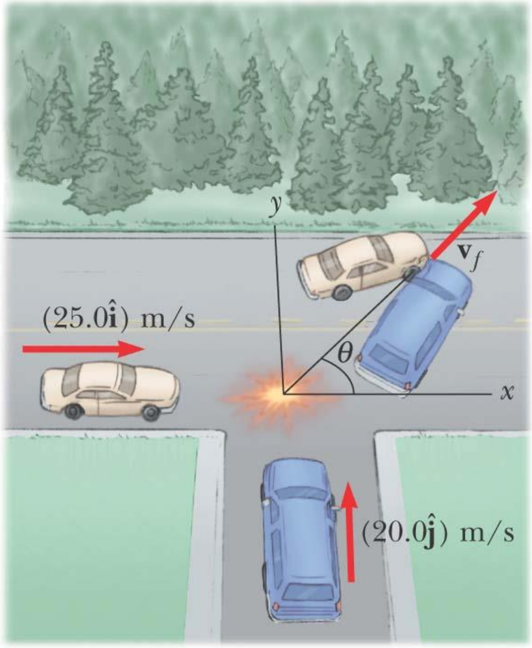
$(2\,500\ \text{kg})(20.0\ \text{m/s}) = (4\,000\ \text{kg})\ v_f \sin\theta$

$5.00 \times 10^4\ \text{kg. m/s} = (4\,000\ \text{kg})\ v_f \sin\theta \qquad (2)$

Divide Equation (2) by (1), one can get,

$3.75 \times 10^4\ \text{kg. m/s} = (4\,000\ \text{kg})\ v_f \cos\theta$

$5.00 \times 10^4\ \text{kg. m/s} = (4\,000\ \text{kg}) v_f \sin\theta$



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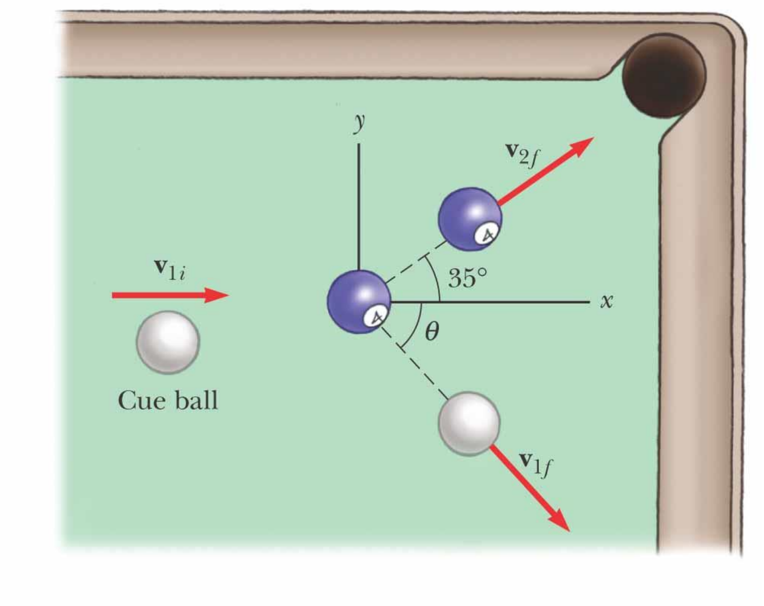
$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{5.00 \times 10^4\ \text{kg. m/s}}{3.75 \times 10^4\ \text{kg. m/s}} = 1.33$$

$\theta = 53.1^{\text{o}}$

$$v_f = \frac{5.00 \times 10^4\ \text{kg. m/s}}{(4\,000\ \text{kg})\sin(53.1^{\text{o}})} = 15.6\ \text{m/s}$$

Example 9.12 Billiard Ball Collision

In a game of billiards, a player wishes to sink a target ball in the corner pocket, as shown in Figure 9.15. If the angle to the corner pocket is 35°, at what angle θ is the cue ball deflected? Assume that friction and rotational motion are unimportant and that the collision is elastic. Also assume that all billiard balls have the same mass m.



Solution Let ball 1 be the cue ball and ball 2 be the target ball. Because the target ball is initially at rest, conservation of kinetic energy (Eq. 9.16) for the two-ball system gives

$$\frac{1}{2}m_1{v_{1i}}^2 = \frac{1}{2}m_1{v_{1f}}^2 + \frac{1}{2}m_2{v_{2f}}^2$$

But $m_1 = m_2 = m$, so that

$${v_{1i}}^2 = {v_{1f}}^2 + {v_{2f}}^2 \qquad (1)$$

Applying conservation of momentum to the two-dimensional collision gives ($m_1 = m_2 = m$)

$$m_1\mathbf{v}_{1i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} \qquad (2)$$

$$\mathbf{v}_{1i} = \mathbf{v}_{1f} + \mathbf{v}_{2f}$$

If we square both sides of Equation (2)

$${v_{1i}}^2 = (\mathbf{v}_{1f}+\mathbf{v}_{2f})\cdot(\mathbf{v}_{1f}+\mathbf{v}_{2f}) = {v_{1f}}^2 + {v_{2f}}^2 + 2\,\mathbf{v}_{1f}\cdot\mathbf{v}_{2f}$$

Because the angle between $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$ is $\theta + 35°$,

$$\mathbf{v}_{1f} \cdot \mathbf{v}_{2f} = v_{1f} v_{2f} \cos(\theta + 35°)$$

Then,

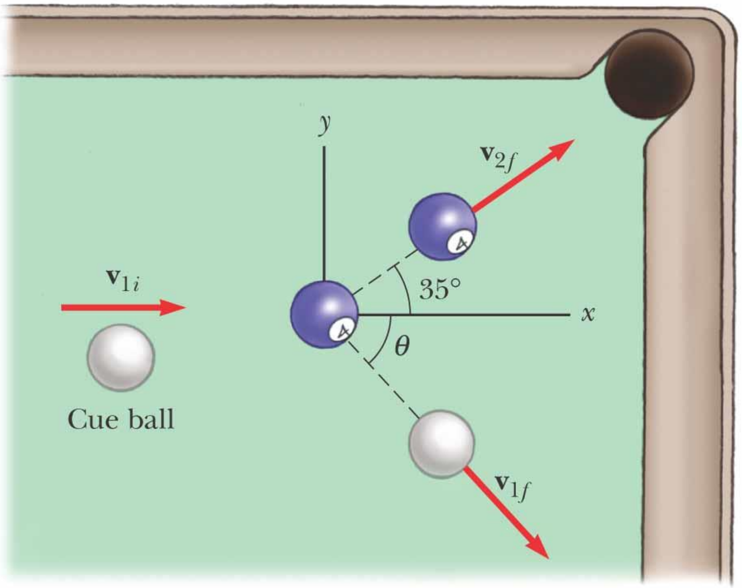
$$v_{1i}^{\,2} = v_{1f}^{\,2} + v_{2f}^{\,2} + 2 v_{1f} v_{2f} \cos(\theta + 35°) \qquad (3)$$

Subtracting Eq. (1) from (3) gives

$$0 = 2 v_{1f} v_{2f} \cos(\theta + 35°)$$

$$0 = \cos(\theta + 35°)$$

$$\theta + 35° = 90° \quad \text{or} \quad \theta = 55°$$



This result shows that whenever two equal masses undergo a glancing elastic collision and one of them is initially at rest, they move in perpendicular directions after the collision.

Selected Problems - Chapter 9:

Problems: 1, 2, 4, 5, 7, 8, 9, 10, 13, 15, 16, 17, 18, 21, 25, 27, 32, 33, 35