Potential Energy

8.1 Potential Energy of a System

8.2 The Isolated System–Conservation of Mechanical Energy

8.3 Conservative and Nonconservative Forces

8.4 Changes in Mechanical Energy for Nonconservative Forces

8.5 Relationship Between Conservative Forces and Potential Energy

8.1 Potential Energy of a System

- Let us now consider systems of two or more particles or objects interacting via a force that is *internal* to the system.

- The potential energy of a system can only be associated with specific types of forces acting between members of a system.
- The amount of potential energy in the system is determined by the *configuration* of the system.

- For a system consisting of a book and the Earth, interacting via the gravitational force.
- We do some work on the system by lifting the book slowly from rest through a vertical displacement
- This work done on the system must appear as an increase in energy of the system.

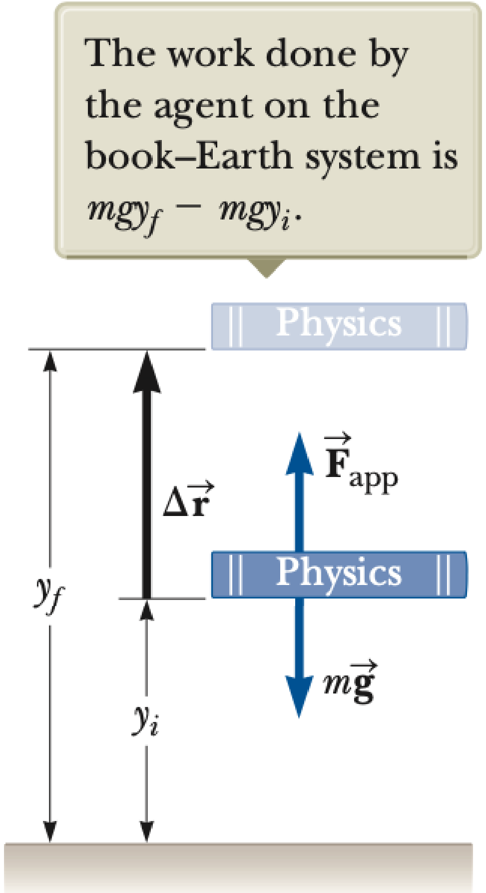


Figure 7.15 An external agent lifts a book slowly from a height y_i to a height y_f.

- The work done by the external agent on the system (object and the Earth)

$$W_{ext} = \overrightarrow{\mathbf{F}}_{\mathbf{app}} \cdot \Delta\vec{\boldsymbol{r}} = mg\,\hat{\mathbf{j}} \cdot [(y_{\boldsymbol{f}} - y_{\boldsymbol{i}})\hat{\mathbf{j}}]$$

$$W_{ext} = mgy_f - mgy_i$$

The net work done on the system because the applied force is the only force on the system from the environment.

The gravitational force is *internal* to the system

The work represents a transfer of energy into the system and the system energy appears in a form, which we have called **potential energy.**

Gravitational potential energy

- The **gravitational potential energy** Ug of the system of an object of mass m and the Earth:

$$U_g = mgy$$

The units of gravitational potential energy are joules

It is valid only for objects near the surface of the Earth, where g is approximately constant.

It depends only on the vertical height of the object

$$W_{ext} = \Delta U_g$$

The net external work done on the system appears as a change in the gravitational potential energy of the system.

Quick Quiz 8.1 Choose the correct answer. The gravitational potential energy of a system (a) is always positive (b) is always negative (c) can be negative or positive.

Quick Quiz 8.2 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the kinetic energy of the system, we (a) must include the kinetic energy of both the object and the Earth (b) can ignore the kinetic energy of the Earth because it is not part of the system (c) can ignore the kinetic energy of the Earth because the Earth is so massive compared to the object.

Quick Quiz 8.3 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the potential energy of the system, we identify the system as (a) both the object and the Earth (b) only the object (c) only the Earth.

Example 8.1 The Bowler and the Sore Toe

A bowling ball held by a careless bowler slips from the bowler's hands and drops on the bowler's toe (foot). Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the ball–Earth system as the ball falls. Repeat the calculation, using the top of the bowler's head as the origin of coordinates.

Conceptualize The bowling ball changes its vertical position with respect to the surface of the Earth. Associated with this change in position is a change in the gravitational potential energy of the ball–Earth system.
Categorize We evaluate a change in gravitational potential energy defined in this section, so we categorize this example as a substitution problem. Because there are no numbers provided in the problem statement, it is also an estimation problem.

We need to estimate a few values. A bowling ball has a mass of approximately 7 kg, and the top of a person's toe is about 0.03 m above the floor. Also, we shall assume the ball falls from a height of 0.5m.

The gravitational potential energy of the ball–Earth system:

Before the ball is released:

$$U_{gi} = mgy_i = (7\text{ kg})(9.80\text{ m}/s^2)(0.5\text{m}) = 34.3\text{ J}$$

When the ball reaches his toe:

$$U_{gf} = mgy_f = (7\text{ kg})(9.80\text{ m}/\text{s}^2)(0.03\text{m}) = 2.06\text{ J}$$

The change in gravitational potential energy of the ball–Earth system is

$$\Delta U_g = U_{gf} - U_{gi} = -32.24\text{ J} \approx -30\text{ J}$$

The second case; using the top of the bowler's head as the origin of coordinates (which we estimate to be 1.50 m above the floor):

The gravitational potential energy of the ball–Earth system:

Before the ball is released:

$$U_{gi} = mgy_i = (7\ \text{kg})(9.80\ \text{m/s}^2)(-1\ \text{m}) = 68.6\ \text{J}$$

When the ball reaches his toe:

$$U_{gf} = mgy_f = (7\ \text{kg})(9.80\ \text{m/s}^2)(-1.47\ \text{m}) = -100.8\ \text{J}$$

The change in gravitational potential energy of the ball–Earth system is

$$\Delta U_g = U_{gf} - U_{gi} = -32.24\ \text{J} \approx -30\ \text{J}$$

This is the same value as before, as it must be.

8.2 The Isolated System–Conservation of Mechanical Energy

A system is chosen such that no energy crosses the system boundary by any method

$$\Delta E_{\mathrm{mech}} = 0$$

conservation of mechanical energy for an isolated system with no nonconservative forces acting.

Mechanical energy of a system is the sum of its the kinetic and potential energies

$$E_{\mathrm{mech}} = K + U$$

$$\Delta K + \Delta U = 0$$

Example: the work done *on the book alone* by the gravitational force.

The work done by the gravitational force on the book is

$$W_{on\ book} = -\Delta U_{\mathrm{g}}$$

the work done on the book is equal to the change in the kinetic energy of the book

$$W_{on\ book} = \Delta K_{\mathrm{book}}$$

Equate these two expressions for the work done on the book:

$$\Delta K_{\mathrm{book}} = -\Delta U_{\mathrm{g}}$$

$$\Delta K + \Delta U = 0$$

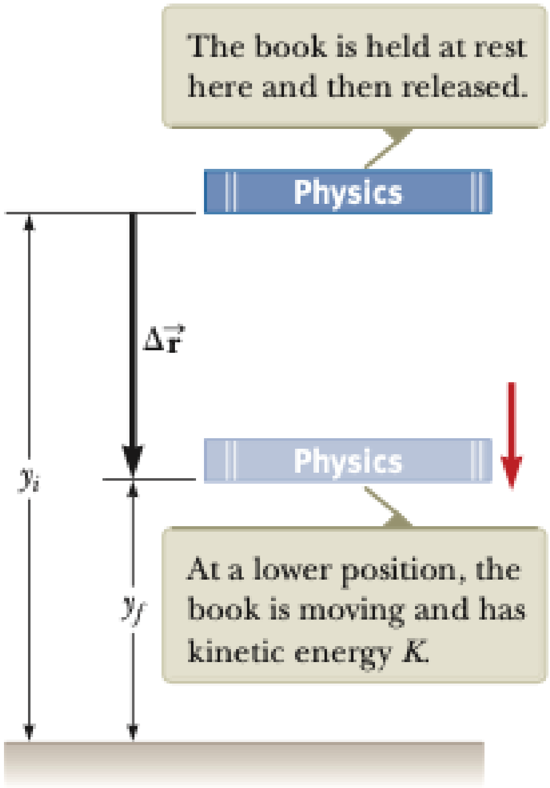


Figure 8.2 A book is released from rest and falls due to work done by the gravitational force on the book.

Let us write the changes in energy in Equation explicitly:

$$\Delta K + \Delta U = 0$$

$$(K_f - K_i) + (U_f - U_i) = 0$$

$$K_f + U_f = K_i + U_i$$

For the gravitational situation of the falling book:

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

$$E_{\text{total},f} = E_{\text{total},i}$$

If there are nonconservative forces acting within the system, mechanical energy is transformed to internal energy.

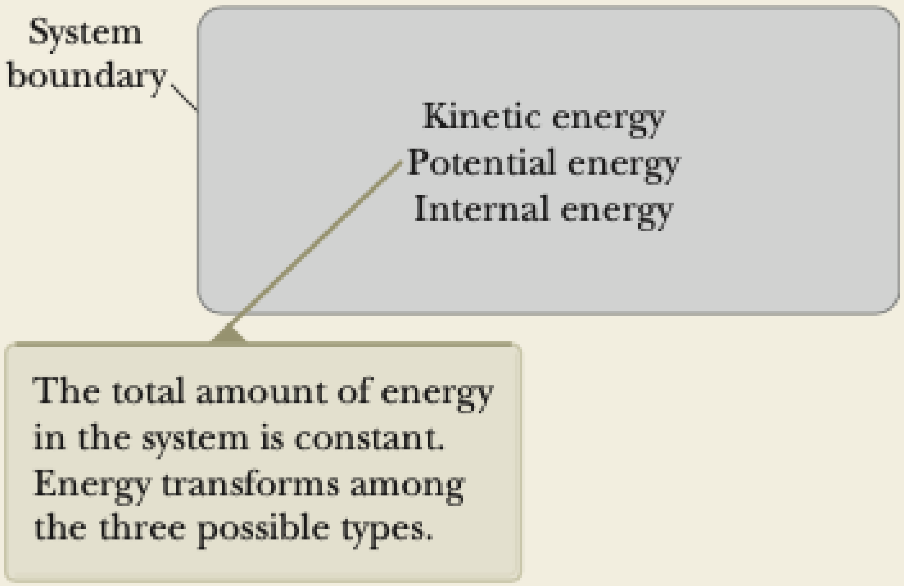
If nonconservative forces act in an isolated system, the total energy of the system is conserved although the mechanical energy is not.

In that case, we can express the conservation of energy of the system as

$$\Delta E_{\text{system}} = 0$$

Analysis Model Isolated System (Energy)

Imagine you have identified a system to be analyzed and have defined a system boundary. Energy can exist in the system in three forms: kinetic, potential, and internal. Imagine also a situation in which no energy crosses the boundary of the system by any method. Then, the system is isolated; energy transforms from one form to another and Equation 8.2 becomes



$$\Delta E_{\text{system}} = 0 \qquad \textbf{(8.10)}$$

If no nonconservative forces act within the isolated system, the mechanical energy of the system is conserved, so

$$\Delta E_{\text{mech}} = 0 \qquad \textbf{(8.8)}$$

Examples:

- an object is in free-fall; gravitational potential energy transforms to kinetic energy: $\Delta K + \Delta U = 0$
- a basketball rolling across a gym floor comes to rest; kinetic energy transforms to internal energy: $\Delta K + \Delta E_{int} = 0$
- a pendulum is raised and released with an initial speed; its motion eventually stops due to air resistance; gravitational potential energy and kinetic energy transform to internal energy, $\Delta K + \Delta U + \Delta E_{int} = 0$ (Chapter 15)

Elastic Potential Energy

- Consider a system consisting of a block and a spring
- The spring force is the interaction between the two members of the system.
- The external work done by an applied force on the block–spring system is given by

$$W_{ext} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

The **elastic potential energy** is

$$U_s = \frac{1}{2}kx^2$$

$$W_{ext} = \Delta U_s$$

External work is done on a system and a form of energy storage in the system changes as a result.

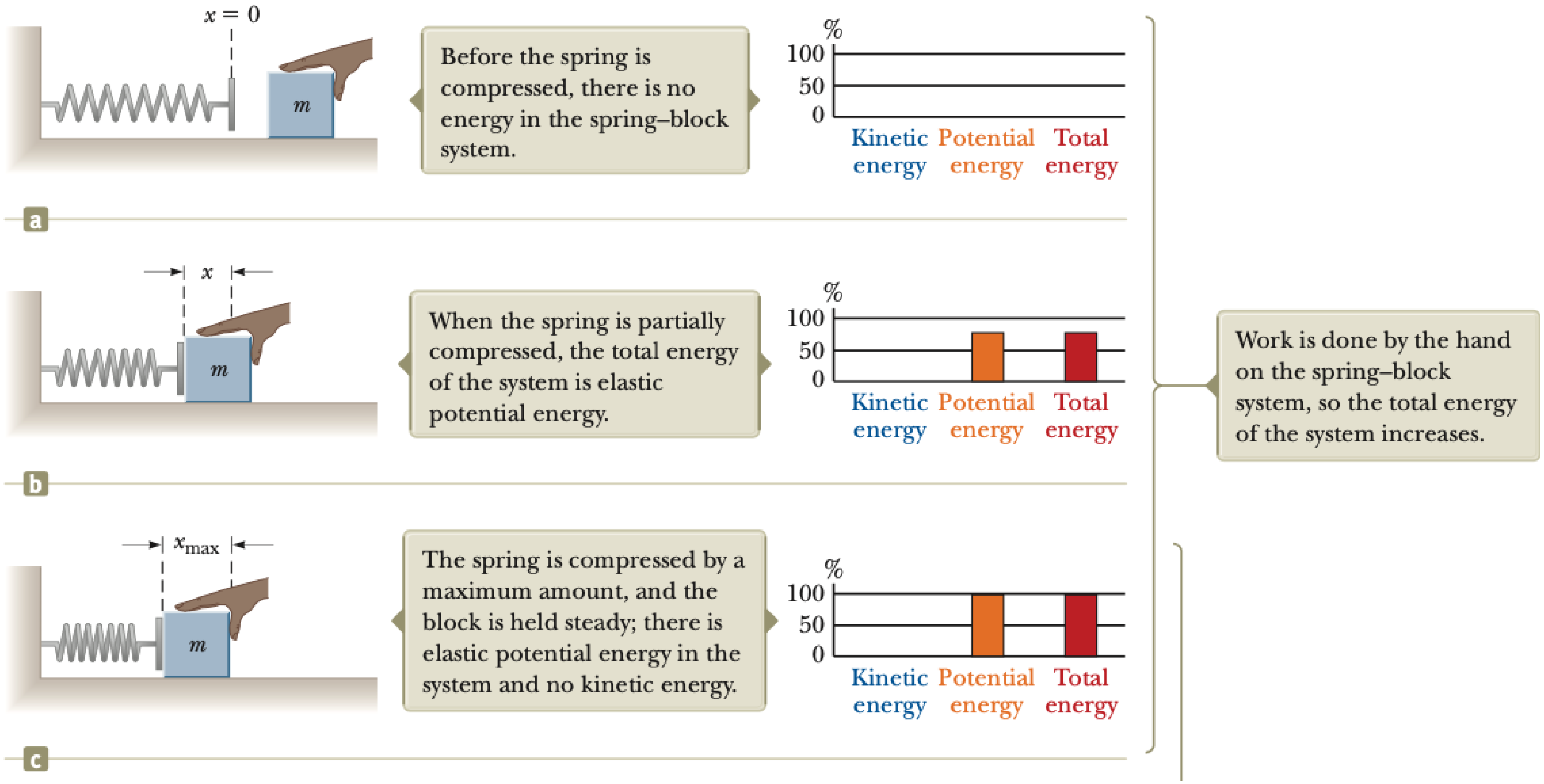


Figure 7.16 A spring on a frictionless, horizontal surface is compressed a distance x_{max} when a block of mass m is pushed against it. The block is then released and the spring pushes it to the right, where the block eventually loses contact with the spring. Parts (a) through (e) show various instants in the process. Energy bar charts on the right of each part of the figure help keep track of the energy in the system.

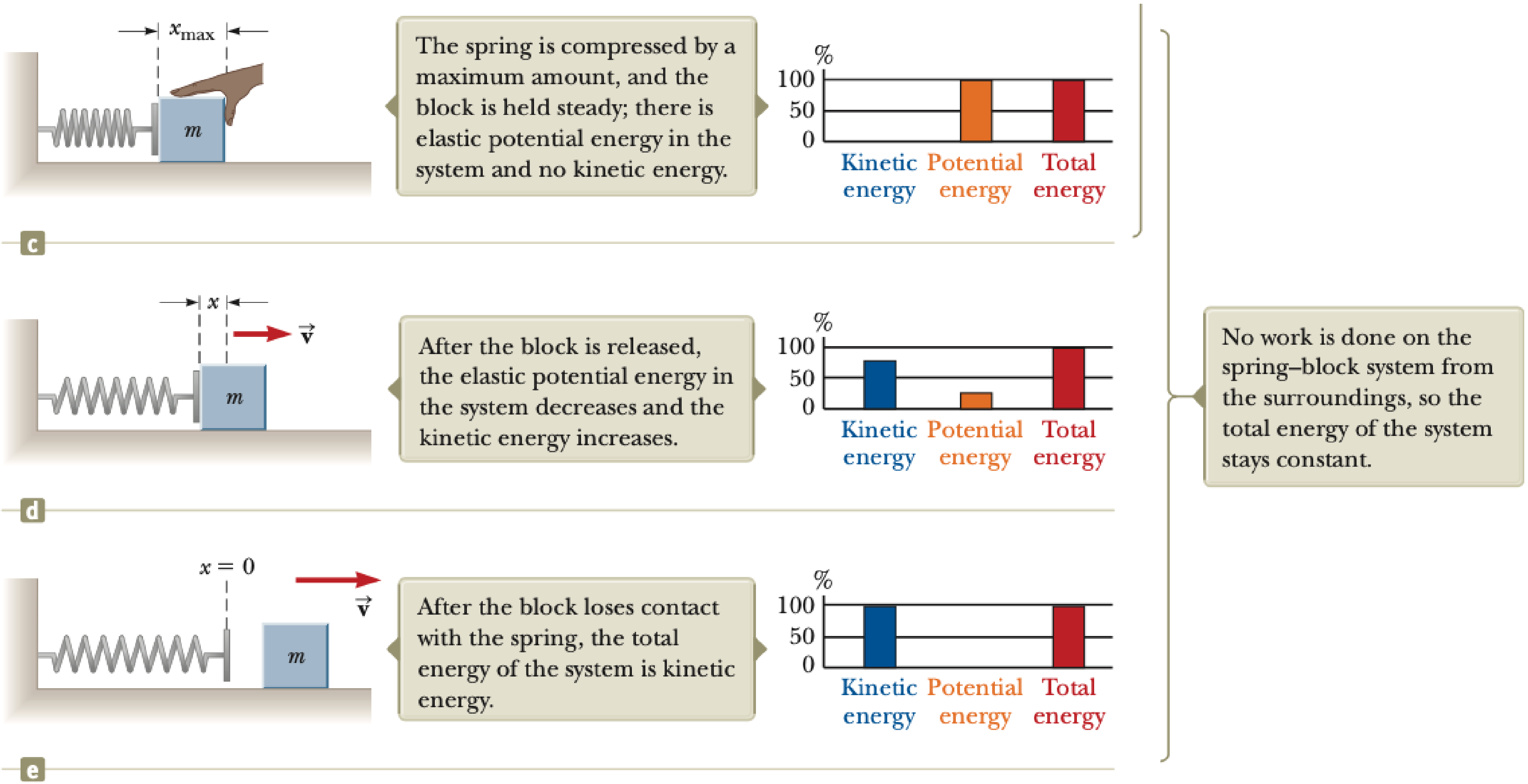
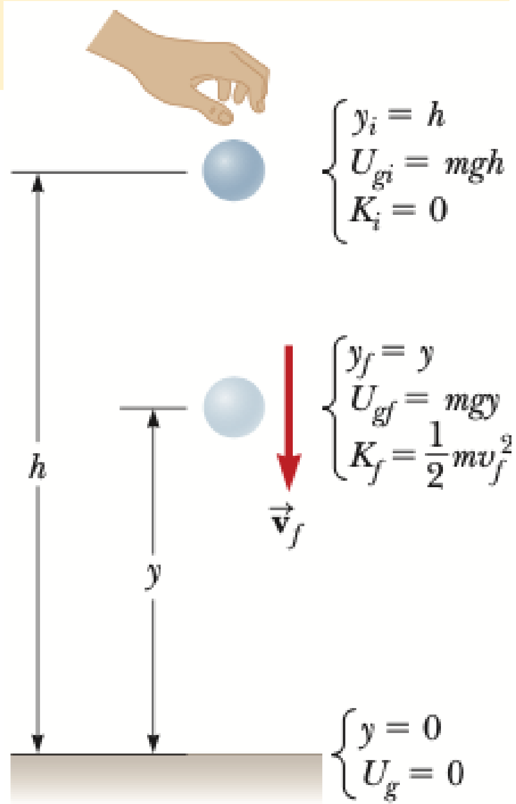


Figure 7.16 A spring on a frictionless, horizontal surface is compressed a distance x_{max} when a block of mass m is pushed against it. The block is then released and the spring pushes it to the right, where the block eventually loses contact with the spring. Parts (a) through (e) show various instants in the process. Energy bar charts on the right of each part of the figure help keep track of the energy in the system.

Example 8.2 Ball in Free Fall

A ball of mass *m* is dropped from a height *h* above the ground, as shown in Figure 8.6.
(A) Neglecting air resistance, determine the speed of the ball when it is at a height *y* above the ground.



Conceptualize Figure allows us to conceptualize the situation, and let us practice an energy approach.

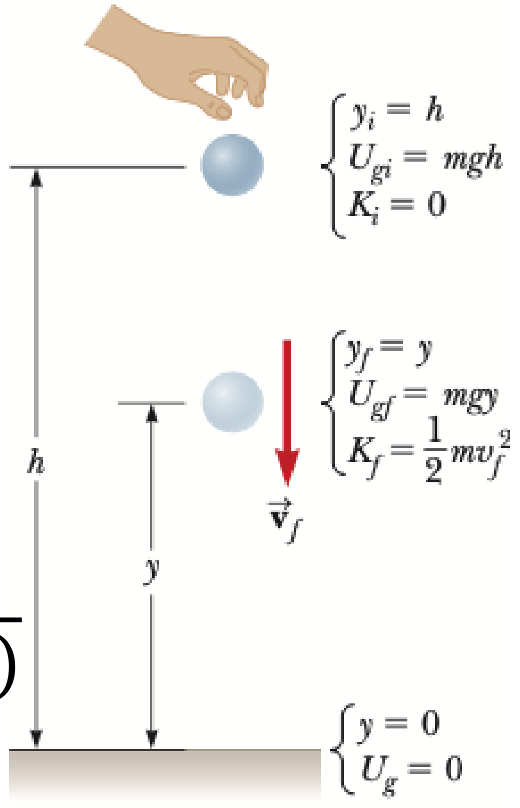
Categorize we identify the system as the ball and the Earth. The system is isolated and we use the *isolated system* model. The only force between members of the system is the gravitational force, which is conservative.

Analyze Because the system is isolated and there are no nonconservative forces acting within the system, we apply the principle of conservation of mechanical energy to the ball–Earth system.

At the instant the ball is released, its kinetic energy is $K = 0$ and the gravitational potential energy of the system is $U = mgh$.

When the ball is at a position y above the ground, its kinetic energy is $K_f = 1/2mv_f^2$ and the potential energy relative to the ground is $U_{gf} = mgy$.

The only types of energy in the system that change are kinetic energy and gravitational potential energy:



$$\Delta K + \Delta U_g = 0$$

$$(\frac{1}{2}mv_f^2 - 0) + (mgy - mgh) = 0$$

$$v_f^2 = 2g(h-y) \rightarrow v_f = \sqrt{2g(h-y)}$$

(B) Determine the speed of the ball at *y* if at the instant of release it already has an initial upward speed v_i at the initial altitude *h*.

$$\Delta K + \Delta U_g = 0$$

$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (mgy - mgh) = 0$$

$$v_f^2 = v_i^2 + 2g(h - y) \rightarrow v_f = \sqrt{v_i^2 + 2g(h - y)}$$

Example 8.3 The Pendulum

A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure 8.7. The sphere is released from rest at point A when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless.

(A) Find the speed of the sphere when it is at the lowest point B.

The only force that does work on the sphere is the gravitational force.

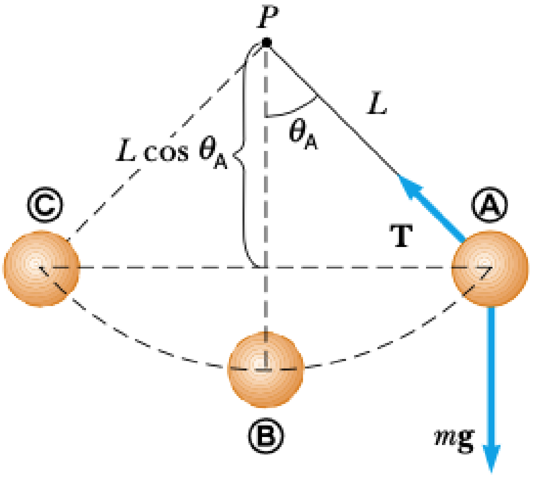
If we measure the y coordinates of the sphere from the center of rotation, then

$y_A = -L\, cos\theta_A$

$y_B = -L.$

Therefore,

$$U_A = -mgL\, cos\theta_A$$
$$U_B = -mgL$$



Applying the principle of conservation of mechanical energy to the system gives

$$\Delta K + \Delta U_g = 0$$

$$K_B + U_B = K_A + U_A$$

$$\frac{1}{2} m v_B^2 - mgL = 0 - mgL\, cos\theta_A$$

$$v_B = \sqrt{2gL(1 - cos\theta_A)}$$

(B) What is the tension in the cord at B?

Solution Because the tension force does no work, it does not enter into an energy equation, and we cannot determine the tension using the energy method. To find T_B, we can apply Newton's second law to the radial direction. First, recall that the centripetal acceleration of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Because $r = L$ in this example, Newton's second law gives

$$(2) \qquad \sum F_r = mg - T_B = ma_r = -m\frac{{v_B}^2}{L}$$

Substituting Equation (1) into Equation (2) gives the tension at point Ⓑ as a function of θ_A:

$$(3) \qquad T_B = mg + 2mg(1 - \cos\theta_A) = mg(3 - 2\cos\theta_A)$$

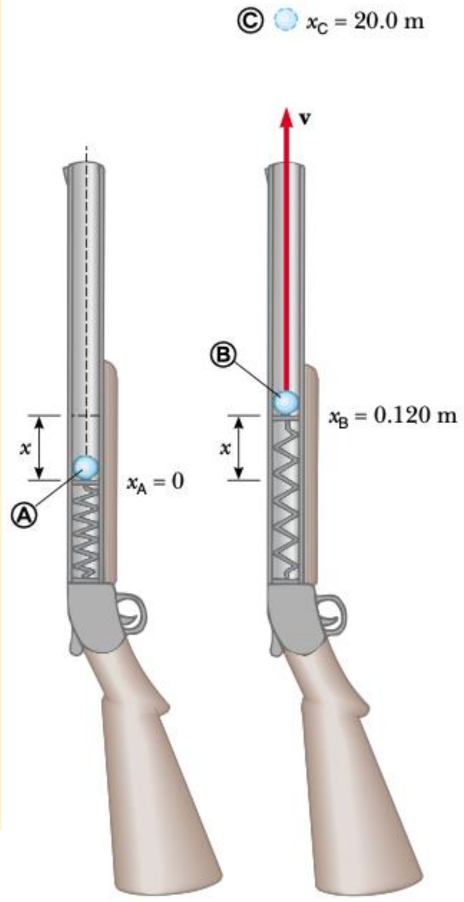
From Equation (2) we see that the tension at Ⓑ is greater than the weight of the sphere. Furthermore, Equation (3) gives the expected result that $T_B = mg$ when the initial angle $\theta_A = 0$. Note also that part (A) of this example is categorized as an energy problem while part (B) is categorized as a Newton's second law problem.

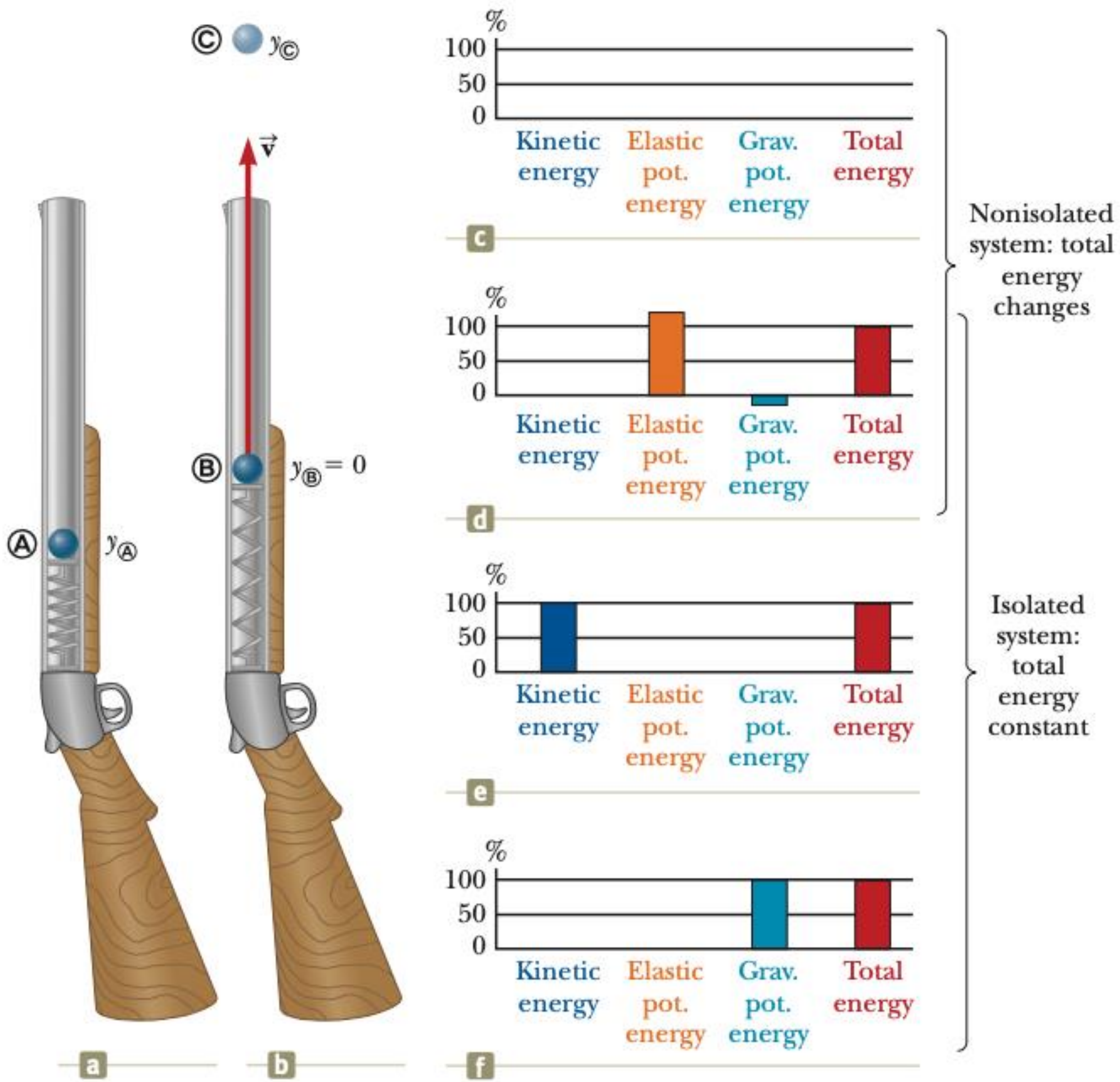
Example 8.5 The Spring-Loaded Popgun

The launching mechanism of a toy gun consists of a spring of unknown spring constant (Fig. 8.9a). When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing.

(A) Neglecting all resistive forces, determine the spring constant.

(B) Find the speed of the projectile as it moves through the equilibrium position of the spring (where x_B = 0.120m) as shown in Figure8.9b.





Ⓒ $y_Ⓒ$
$\vec{v}$
Ⓑ $y_Ⓑ = 0$
Ⓐ $y_Ⓐ$
a
b
%
100
50
0
Kinetic energy
Elastic pot. energy
Grav. pot. energy
Total energy
c
d
e
f
Nonisolated system: total energy changes
Isolated system: total energy constant

(A) Neglecting all resistive forces, determine the spring constant.

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

$$(0 - 0) + (mgy_{©} - mgy_{Ⓐ}) + (0 - \tfrac{1}{2}kx^2) = 0$$

$$k = \frac{2mg(y_{©} - y_{Ⓐ})}{x^2}$$

$$k = \frac{2(0.035\,0\text{ kg})(9.80\text{ m/s}^2)[20.0\text{ m} - (-0.120\text{ m})]}{(0.120\text{ m})^2} = \boxed{958\text{ N/m}}$$

(B) Find the speed of the projectile as it moves through the equilibrium position of the spring (where x_B = 0.120m) as showninFigure8.9b.

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

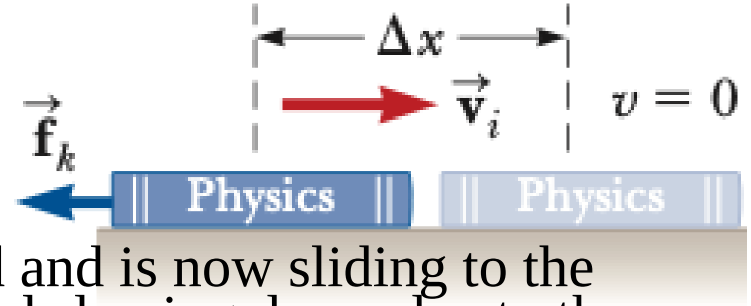
$$\left(\tfrac{1}{2}mv_{Ⓑ}^2 - 0\right) + \left(0 - mgy_{Ⓐ}\right) + \left(0 - \tfrac{1}{2}kx^2\right) = 0$$

$$v_{Ⓑ} = \sqrt{\frac{kx^2}{m} + 2gy_{Ⓐ}}$$

$$v_{Ⓑ} = \sqrt{\frac{(958\ \text{N/m})(0.120\ \text{m})^2}{(0.035\,0\ \text{kg})} + 2(9.80\ \text{m/s}^2)(-0.120\ \text{m})} = \boxed{19.8\ \text{m/s}}$$

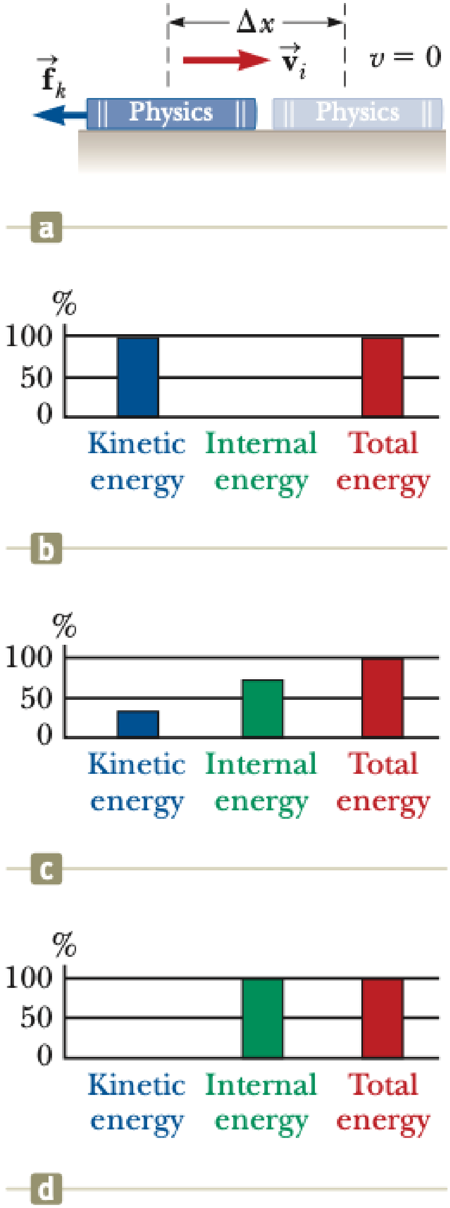
8.3 Conservative and Nonconservative Forces

Internal Energy



A book has been accelerated by a hand and is now sliding to the right on the surface of a heavy table and slowing down due to the friction force.

- The surface is the system.
- The friction force from the sliding book does work on the surface.
- Positive work has been done on the surface, yet there is no increase in the surface's kinetic energy or the potential energy of any system.
- The work that was done on the surface has gone into warming the surface.
- The energy associated with an object's temperature is called its **internal energy**, E_{int}.



A book sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left.

At the initial instant of time, the energy of the system is all kinetic energy.

While the book is sliding,
the kinetic energy of the system decreases as it is transformed to internal energy.

After the book has stopped, the energy of the system is all internal energy.

- Now consider in more detail an object moving downward near the surface of the Earth. The work done by the gravitational force on the object does not depend on whether it falls vertically or slides down a sloping incline with friction. All that matters is the change in the object’s elevation.
- The energy transformation to internal energy due to friction on that incline, however, depends very much on the distance the object slides.
- In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider the energy transformation due to friction forces.
- We can use this varying dependence on path to classify forces as either *conservative* or *nonconservative*.

Conservative Forces have two equivalent properties:

1) The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

2) The work done by a conservative force on a particle moving through any closed path is zero.

A closed path is one for which the beginning point and the endpoint are identical.

Examples:
-The gravitational force
-A spring force exerts on any object attached to the spring.

We can associate a potential energy for a system with a force acting between members of the system.

-we can do so only if the force is conservative.

the work W_{int} done by a conservative force on an object is

$$W_{int} = U_i - U_f = -\Delta U$$

W_{int} (internal to the system)

Positive work done on a component of a system by a conservative force *internal to the system* causes a decrease in the potential energy of the system.

Nonconservative Forces

The work done by a nonconservative force is path-dependent.

Nonconservative forces acting within a system cause a *change* in the **mechanical energy** of the system.

$$E_{mech} = K + U$$

K includes the kinetic energy of all moving members of the system.
U includes all types of potential energy in the system.

For example, for a book sent sliding on a horizontal surface that is not frictionless the mechanical energy of the book–surface system is transformed to internal energy.

- As an example of the path dependence of the work for a nonconservative force,

  Suppose you displace a book between two points on a table and along two different paths (blue and brown).

  - You do a certain amount of work against the kinetic friction force to keep the book moving at a constant speed.
  - You perform more work against friction along the curved path (brown) than along the straight path (blue) because the curved path is longer.
  - The work done on the book depends on the path, so the friction force *cannot* be conservative.

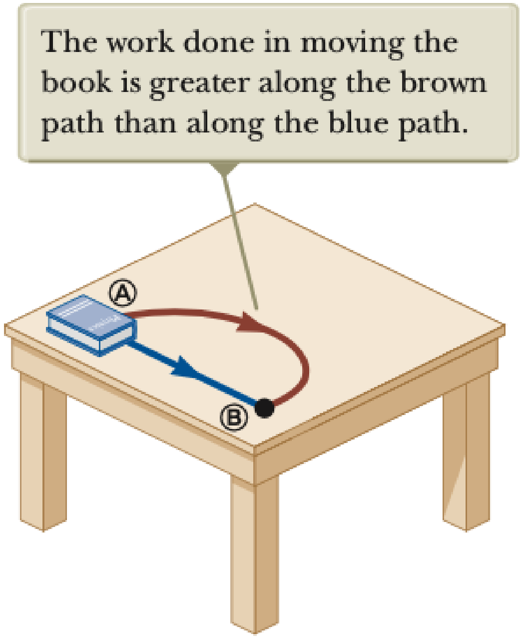


Figure 7.19 The work done against the force of kinetic friction depends on the path taken as the book is moved from Ⓐ to Ⓑ.

8.4 Changes in Mechanical Energy for Nonconservative Forces

- Consider the book moves on an incline that is not frictionless, there is a change in both the kinetic energy and the gravitational potential energy of the book–Earth system.
- If a nonconservative force acts within an isolated system,

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

- If a nonconservative force acts in a nonisolated system,

$$\Sigma W_{\text{other forces}} = W = \Delta K + \Delta U + \Delta E_{\text{int}}$$

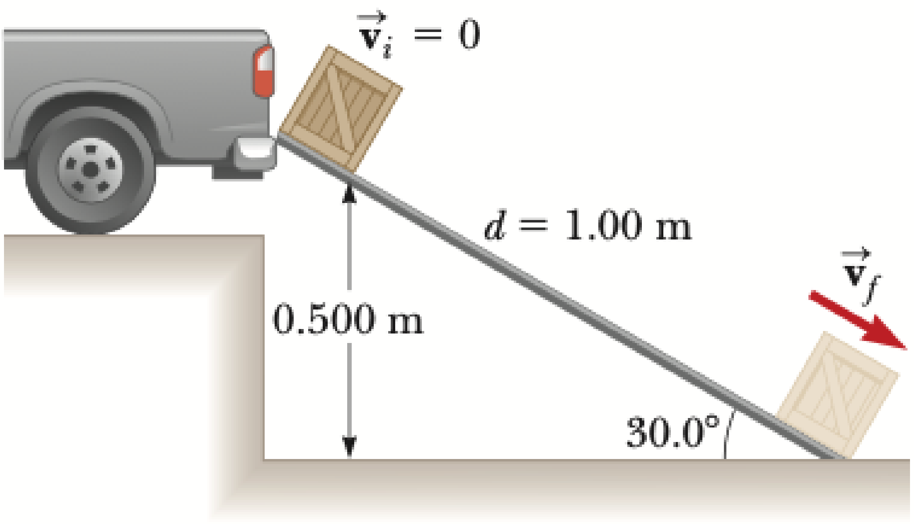
Example 8.6 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00m in length and inclined at an angle of 30.0°, as shown in Figure 8.11. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00N, and continues to move a short distance on the horizontal floor after it leaves the ramp.

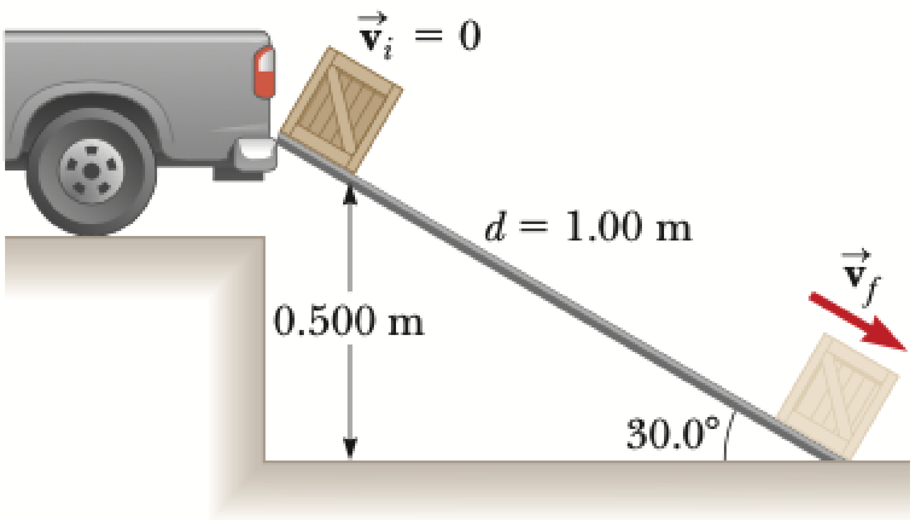
A) Use energy methods to determine the speed of the crate at the bottom of the ramp.

Conceptualize The larger the friction force, the more slowly the crate will slide.

Categorize We identify the crate, the surface, and the Earth as an *isolated system* with a nonconservative force acting.



Analyze Because $v_i = 0$, the initial kinetic energy of the system when the crate is at the top of the ramp is zero.



$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

$$\left(\frac{1}{2}mv_f^2 - 0\right) + (mgy_f - mgy_i) + f_k d = 0$$

$$v_f = \sqrt{\frac{2}{m}(mgy_{\text{i}} - f_{\text{k}}\text{d})}$$

$$v_f = \sqrt{\frac{2}{3.00\text{ kg}}[(3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) - (5.00\text{ N})(1.00\text{ m})]}$$

$$v_f = 2.54\text{ m/s}$$

(B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N?

$$W = \Delta K + \Delta U + \Delta E_{\text{int}}$$

$$\Delta K + \Delta E_{\text{int}} = 0$$

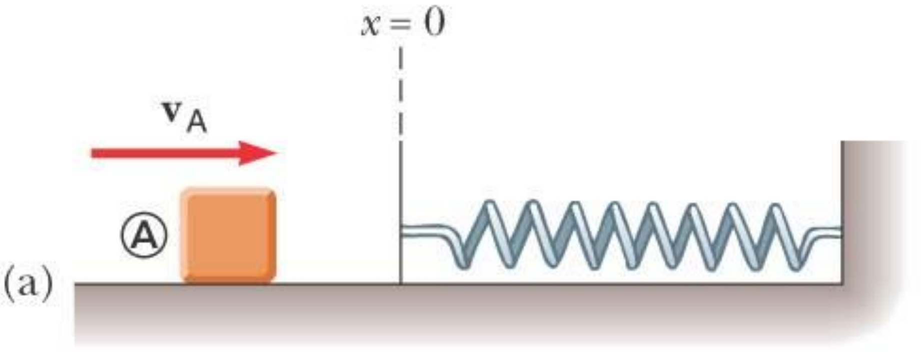
$$\left(0 - \frac{1}{2} m v_i^2\right) + f_{\text{k}} d = 0$$

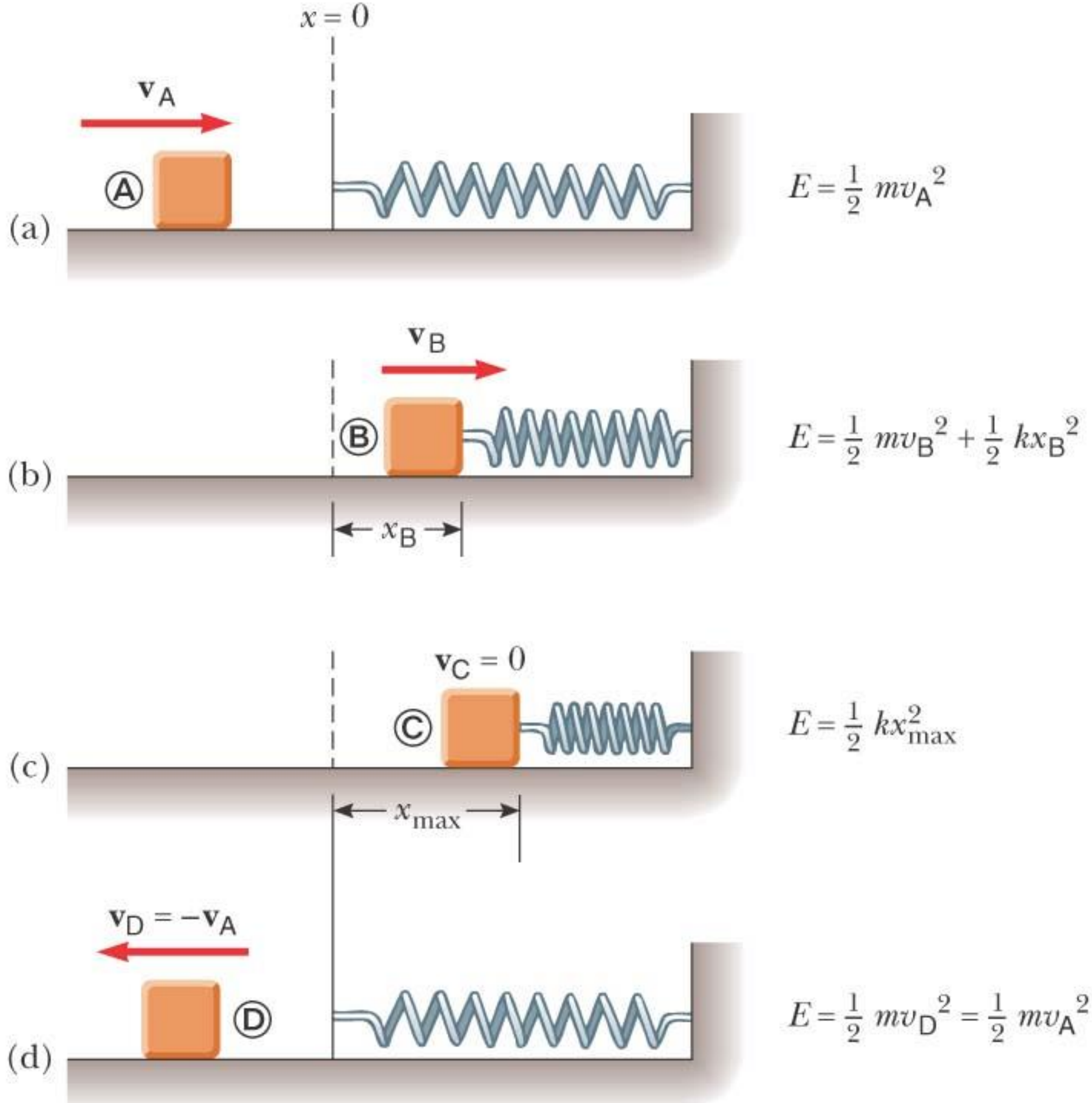
$$d = \frac{m v_i^2}{2 f_{\text{k}}} = \frac{(3.00\ \text{kg})(2.54\ \text{m/s})^2}{2(5.00\ \text{N})} = 1.94\ \text{m}$$

Example 8.9 Block–Spring Collision

A block having a mass of 0.80 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring of negligible mass and force constant k = 50 N/m, as shown in Figure 8.14.

A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.




x = 0
$\mathbf{v}_A$
A
(a)
$E = \frac{1}{2} m v_A^2$
$\mathbf{v}_B$
B
(b)
x_B
$E = \frac{1}{2} m v_B^2 + \frac{1}{2} k x_B^2$
$\mathbf{v}_C = 0$
C
(c)
x_{max}
$E = \frac{1}{2} k x_{max}^2$
$\mathbf{v}_D = -\mathbf{v}_A$
D
(d)
$E = \frac{1}{2} m v_D^2 = \frac{1}{2} m v_A^2$

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

$$\Delta K + \Delta Us = 0$$

$$(0 - \frac{1}{2} m v_A^2) + (\frac{1}{2} k x_f^2 + 0) = 0$$

$$x_f = \sqrt{\frac{m}{k}} v_A = \sqrt{\frac{0.8 \text{ kg}}{50 \text{ N/m}}} (1.2 \text{ m/s}) = 0.15 \text{ m}$$

(B) Suppose a constant force of kinetic friction acts between the block and the surface, with μ_k= 0.50. If the speed of the block at the moment it collides with the spring is v = 1.2 m/s, what is the maximum compression x_C in the spring?

$$f_k = \mu_k n = \mu_k mg$$

$$\Delta E_{mech} + \Delta E_{int} = 0$$

$$\Delta K + \Delta Us = -\Delta E_{int}$$

$$\Delta K + \Delta Us = -f_k x_C$$

$$(0 - \frac{1}{2} m v_A^2) + (\frac{1}{2} k x_f^2 + 0) = -f_k x_C$$

$$\frac{1}{2}(50) x_f^2 - \frac{1}{2}(0.80)(1.2)^2 = -3.92\, x_C$$

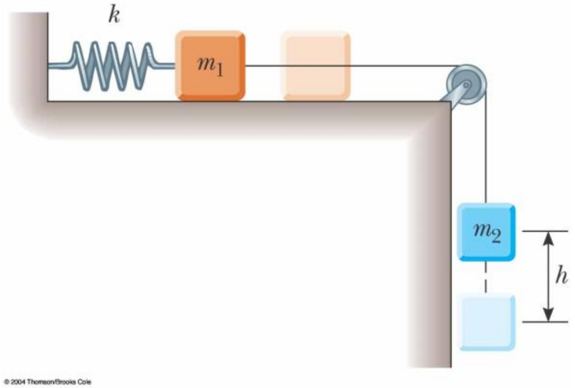
$$x_C = 0.092 \text{ m}$$

Example 8.10 Connected Blocks in Motion

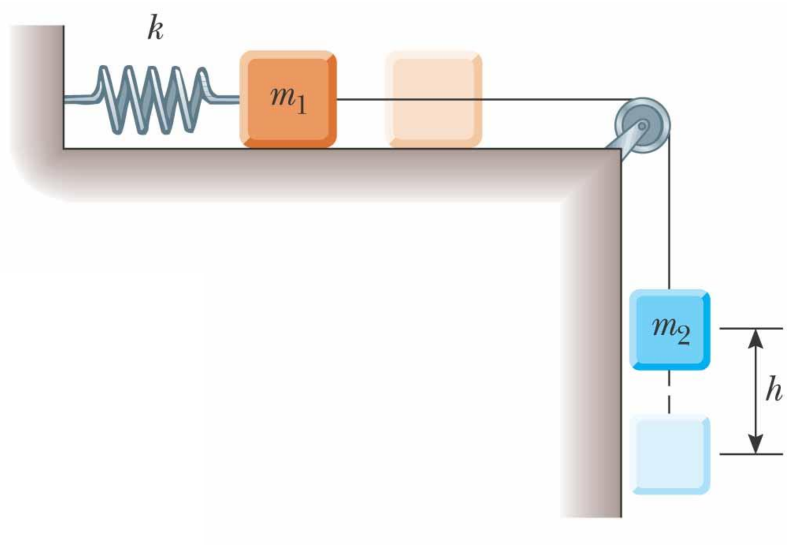
Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.15. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

Conceptualize The key word *rest* appears twice in the problem statement. This word suggests that the configurations of the system associated with rest are good candidates for the initial and final configurations because the kinetic energy of the system is zero for these configurations.

Categorize In this situation, the system consists of the two blocks, the spring, the surface, and the Earth. This is an *isolated system* with a nonconservative force acting. We also model the sliding block as a *particle in equilibrium* in the vertical direction, leading to $n = m_1 g$.



Analyze We need to consider two forms of potential energy for the system, gravitational and elastic: $\Delta U_g = U_{gf} - U_{gi}$ is the change in the system's gravitational potential energy, and $\Delta U_s = U_{sf} - U_{si}$ is the change in the system's elastic potential energy. The change in the gravitational potential energy of the system is associated with only the falling block because the vertical coordinate of the horizontally sliding block does not change. The initial and final kinetic energies of the system are zero, so $\Delta K = 0$.



$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

$\Delta K = 0$,

$$\Delta U_g + \Delta U_s + \Delta E_{\text{int}} = 0$$

$$(0 - m_2 g h) + \left(\frac{1}{2} k h^2 - 0\right) + f_k h = 0$$

$$f_k = \mu_k n = \mu_k m g$$

$$-m_2 g h + \frac{1}{2} k h^2 + \mu_k m_1 g h = 0$$

$$\mu_k = \frac{m_2 g h - \frac{1}{2} k h^2}{m_1 g h}$$

$$\mu_k = \frac{m_2 g - \frac{1}{2} k h}{m_1 g}$$

8.5 Relationship Between Conservative Forces and Potential Energy

The conservative force is related to the potential energy function through

$$F_x = -\frac{dU}{dx}$$

where the *x* component of a conservative force acting on a member within a system equals the negative derivative of the potential energy of the system with respect to *x*.

Selected Problems - Chapter 8:

Problems: 2, 5, 6, 11, 13, 17, 31, 33, 36, 38, 42, 55, 57, 59, 60