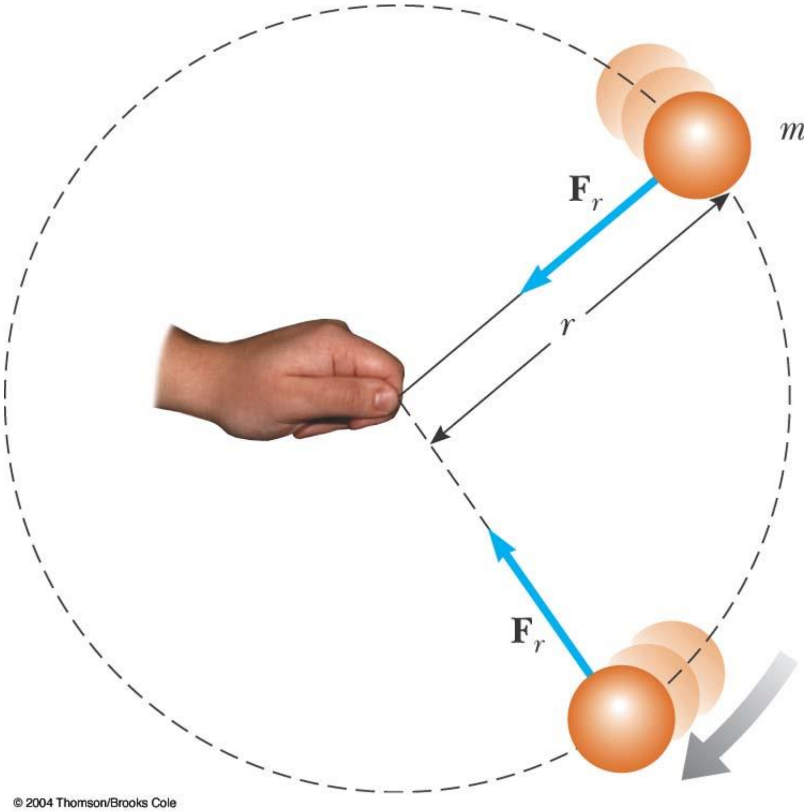
Chapter 6: Circular Motion and Other Applications of Newton's Laws

6.1 Newton's Second Law Applied to Uniform Circular Motion

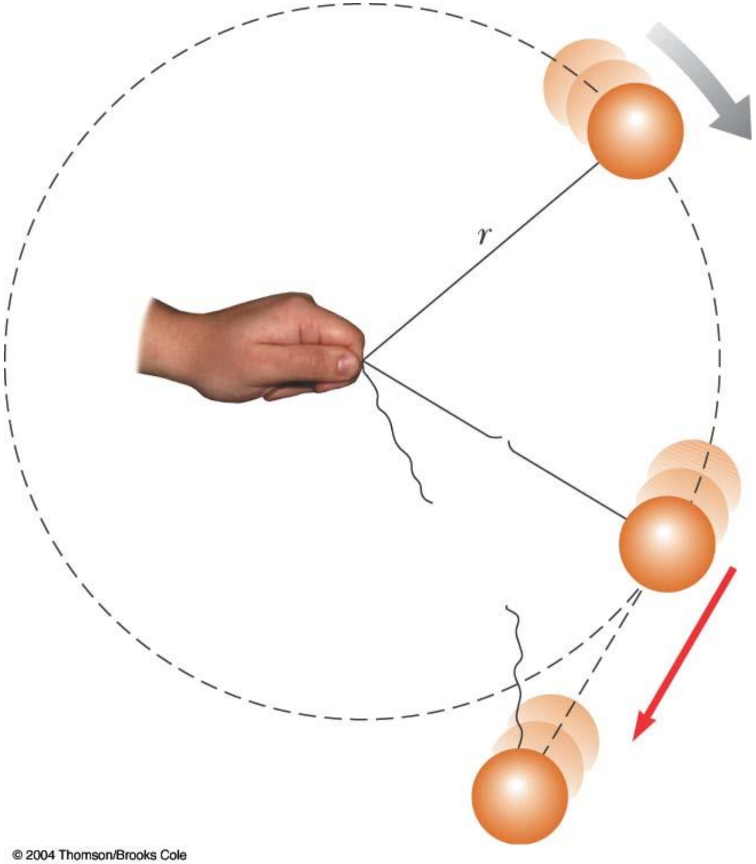
Uniform Circular Motion

- A force, $\mathbf{F}_r$, is directed toward the center of the circle
- This force is associated with an acceleration, $\mathbf{a}_c$
- Applying Newton's Second Law along the radial direction gives



$$\sum F = ma_c = m\frac{v^2}{r}$$

- A force causing a centripetal acceleration acts toward the center of the circle
- It causes a change in the direction of the velocity vector
- If the force vanishes, the object would move in a straight-line path tangent to the circle

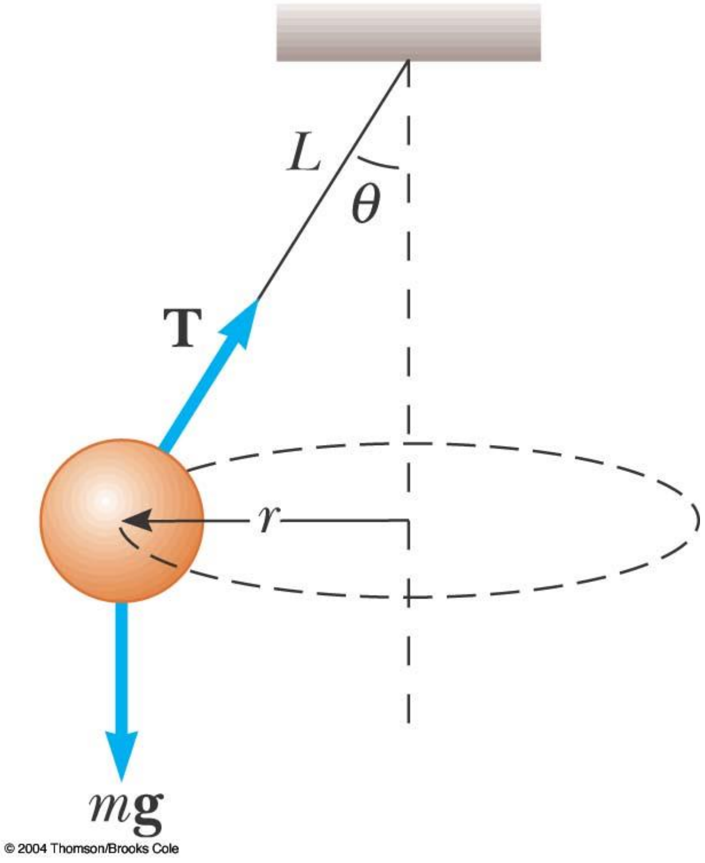


Centripetal Force

- The force causing the centripetal acceleration is sometimes called the ***centripetal force***
- This is not a new force, it is a new *role* for a force
- It is a force *acting in the role of a force that causes a circular motion*

Conical Pendulum

- An object of mass *m* is suspended from a string of length *L*. The object revolves with constant speed in a horizontal circle of radius *r*.



Conical Pendulum

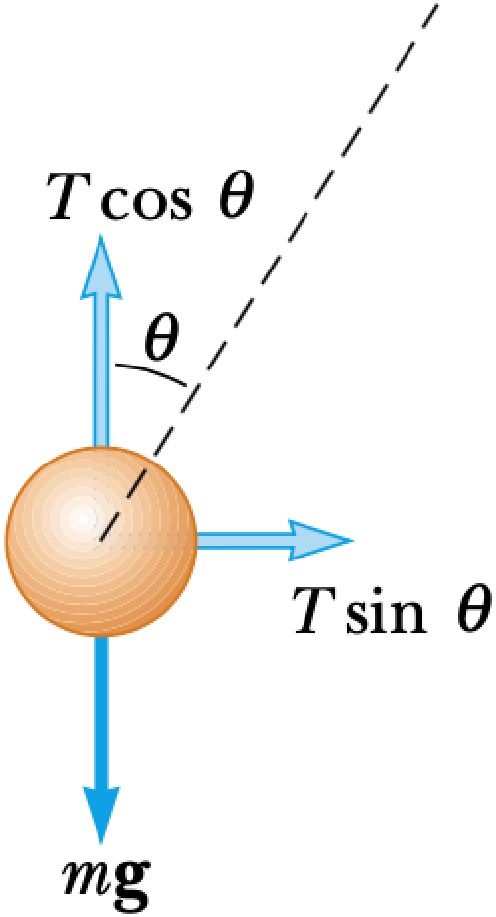
The object is in equilibrium in the vertical direction

$$\Sigma F_y = ma_y = 0$$

$$T\cos\theta = mg$$

and undergoes uniform circular motion in the horizontal direction

$$\sum F = T\sin\theta = ma_c = \frac{mv^2}{r}$$



Conical Pendulum

Dividing the two equations, we eliminate T and find that

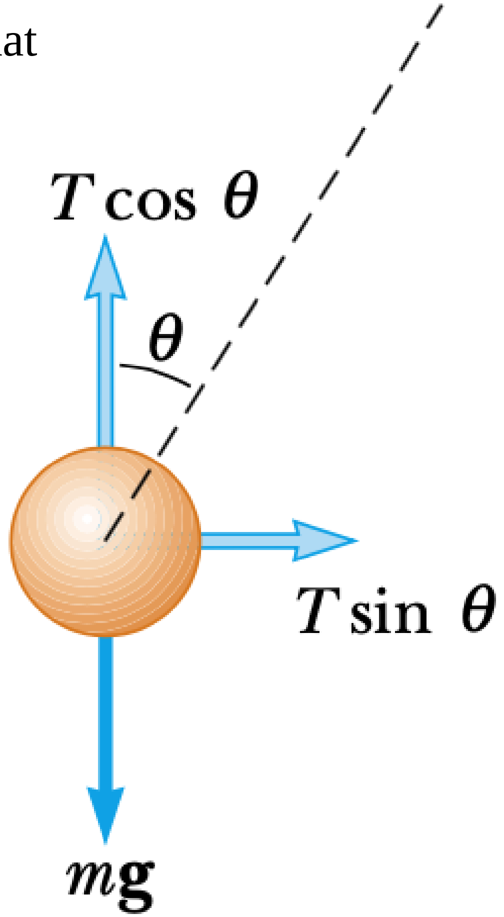
$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

From the geometry, $r = L\sin\theta$

$$v = \sqrt{Lg\sin\theta\tan\theta}$$

- v is independent of m



Example 6.2 The Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in Figure 6.4. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for v.

Solution Conceptualize the problem with the help of Figure 6.4. We categorize this as a problem that combines equilibrium for the ball in the vertical direction with uniform circular motion in the horizontal direction. To analyze the problem, begin by letting θ represent the angle between the string and the vertical. In the free-body diagram shown, the force $\mathbf{T}$ exerted by the string is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of revolution. Because the object does not accelerate in the vertical direction, $\sum F_y = ma_y = 0$ and the upward vertical component of $\mathbf{T}$ must balance the downward gravitational force. Therefore,

$$(1) \qquad T\cos\theta = mg$$

Because the force providing the centripetal acceleration in this example is the component $T\sin\theta$, we can use Equation 6.1 to obtain

$$(2) \qquad \sum F = T\sin\theta = ma_c = \frac{mv^2}{r}$$

Dividing (2) by (1) and using $\sin\theta/\cos\theta = \tan\theta$, we eliminate T and find that

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

From the geometry in Figure 6.4, we see that $r = L\sin\theta$; therefore,

$$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$

Note that the speed is independent of the mass of the object.

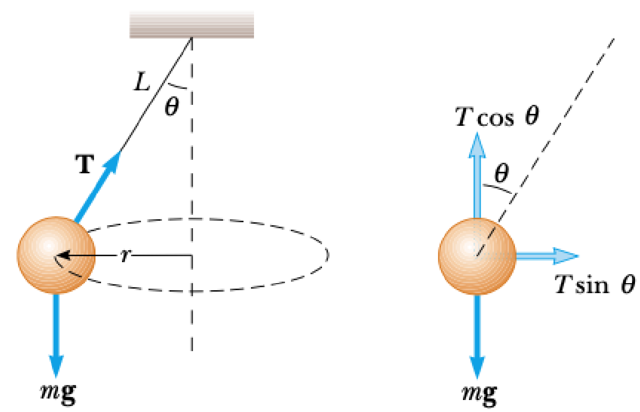


Figure 6.4 (Example 6.2) The conical pendulum and its free-body diagram.

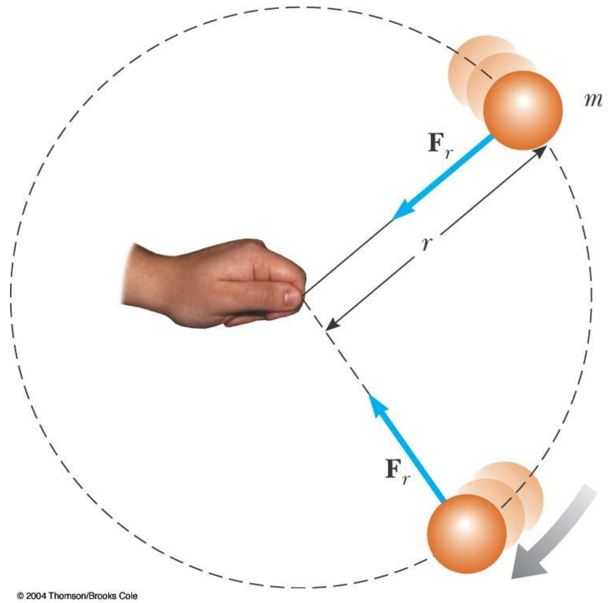
Motion in a Horizontal Circle

- The speed at which the object moves depends on the mass of the object and the tension in the cord
- The centripetal force is supplied by the tension

$$T = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{Tr}{m}}$$

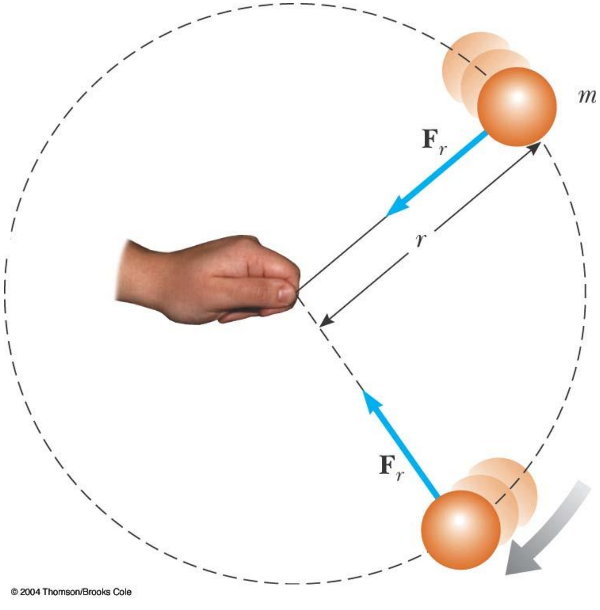
This shows that v increases with T and decreases with larger m, as we expect to see—for a given v, a large mass requires a large tension and a small mass needs only a small tension.



Example 6.3 How Fast Can It Spin?

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as shown in Figure 6.1. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the ball can be whirled before the cord breaks? Assume that the string remains horizontal during the motion.

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\text{ N})\ (1.50\text{ m})}{0.500\text{ kg}}} = 12.2\text{ m/s}$$



What If? **Suppose that the ball is whirled in a circle of larger radius at the same speed *v*. Is the cord more likely to break or less likely?**

$$T_1 = \frac{mv^2}{r_1} \qquad T_2 = \frac{mv^2}{r_2}$$

Dividing the two equations gives us,

$$\frac{T_2}{T_1} = \frac{\left(\dfrac{mv^2}{r_2}\right)}{\left(\dfrac{mv^2}{r_1}\right)} = \frac{r_1}{r_2}$$

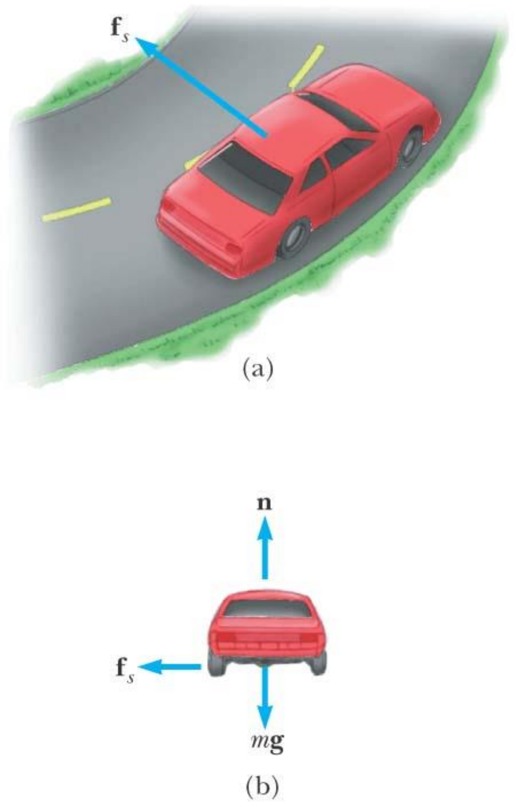
If we choose $r_2 > r_1$, we see that $T_2 < T_1$. Thus, less tension is required to whirl the ball in the larger circle and the string is less likely to break.

Horizontal (Flat) Curve

- The force of static friction supplies the centripetal force
- The maximum speed at which the car can negotiate the curve is

$$v = \sqrt{\mu_s g r}$$

- Note, this does not depend on the mass of the car

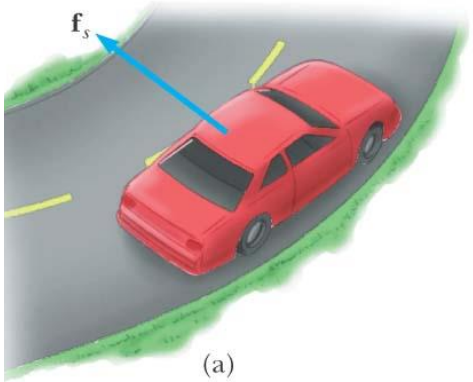


Example 6.4 What Is the Maximum Speed of the Car?

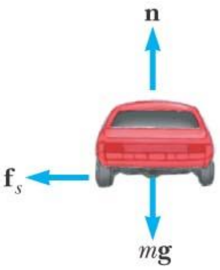
A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure 6.5. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully.

Solution In this case, the force that enables the car to remain in its circular path is the force of static friction. (*Static* because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero—for example, if the car were on an icy road—the car would continue in a straight line and slide off the road.) Hence, from Equation 6.1 we have

$$(1) \qquad f_s = m\frac{v^2}{r}$$



(a)



(b)

The maximum speed the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,\,\max} = \mu_s n$. Because the car shown in Figure 6.5b is in equilibrium in the vertical direction, the magnitude of the normal force equals the weight ($n = mg$) and thus $f_{s,\,\max} = \mu_s mg$. Substituting this value for f_s into (1), we find that the maximum speed is

$$(2) \qquad v_{\max} = \sqrt{\frac{f_{s,\,\max}\, r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

$$= \sqrt{(0.500)(9.80\ \text{m/s}^2)(35.0\ \text{m})}$$

$$= \boxed{13.1\ \text{m/s}}$$

Note that the maximum speed does not depend on the mass of the car. That is why curved highways do not need multiple speed limit signs to cover the various masses of vehicles using the road.

What If? **Suppose that a car travels this curve on a wet day and begins to skid on the curve when its speed reaches only 8.00 m/s. What can we say about the coefficient of static friction in this case?**

Answer The coefficient of friction between tires and a wet road should be smaller than that between tires and a dry road. This expectation is consistent with experience with driving, because a skid is more likely on a wet road than a dry road.

To check our suspicion, we can solve (2) for the coefficient of friction:

$$\mu_s = \frac{v_{\text{max}}^2}{gr}$$

Substituting the numerical values,

$$\mu_s = \frac{v_{\text{max}}^2}{gr} = \frac{(8.00\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(35.0\ \text{m})} = 0.187$$

This is indeed smaller than the coefficient of 0.500 for the dry road.

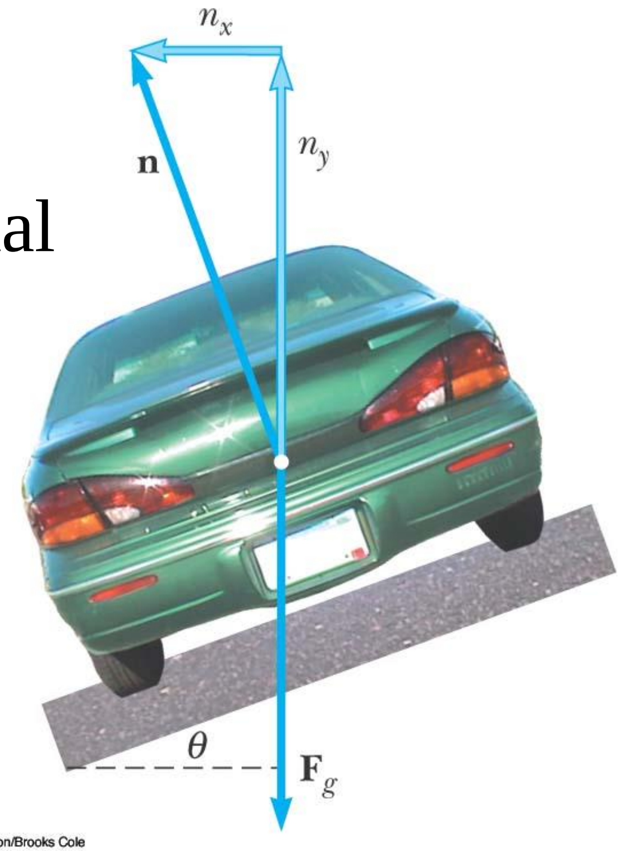
Banked Curve

- These are designed with friction equaling zero
- There is a component of the normal force that supplies the centripetal force

$$\sum F_r = n \sin\theta = \frac{mv^2}{r}$$

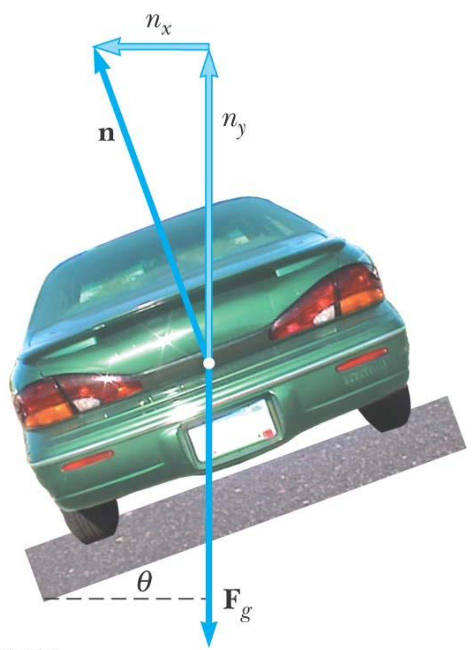
$$n \cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$



Example 6.5 The Banked Exit Ramp

A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually *banked*; this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s (30.0 mi/h) and the radius of the curve is 50.0 m. At what angle should the curve be banked?



$$\tan\theta = \frac{v^2}{rg}$$

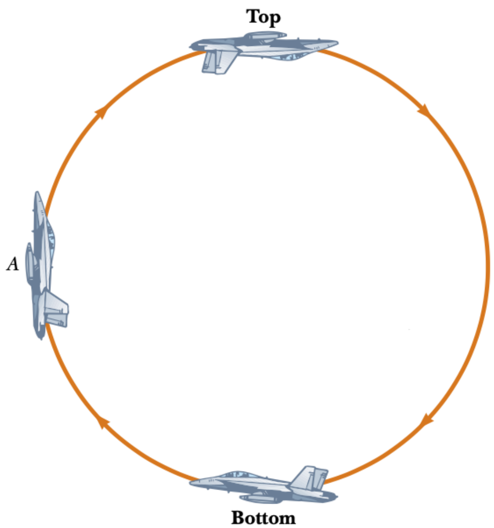
$$\theta = \tan^{-1}\left(\frac{(13.4\ \text{m/s})^2}{(50.0\ \text{m})(9.80\ \text{m/s}^2)}\right) = 20.1^\circ$$

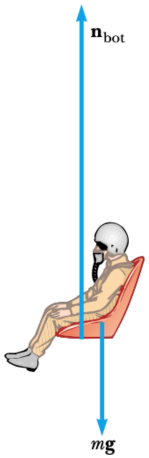
Loop-the-Loop

- This is an example of a vertical circle
- At the bottom of the loop, the upward force experienced by the object is greater than its weight

$$\sum F = n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$





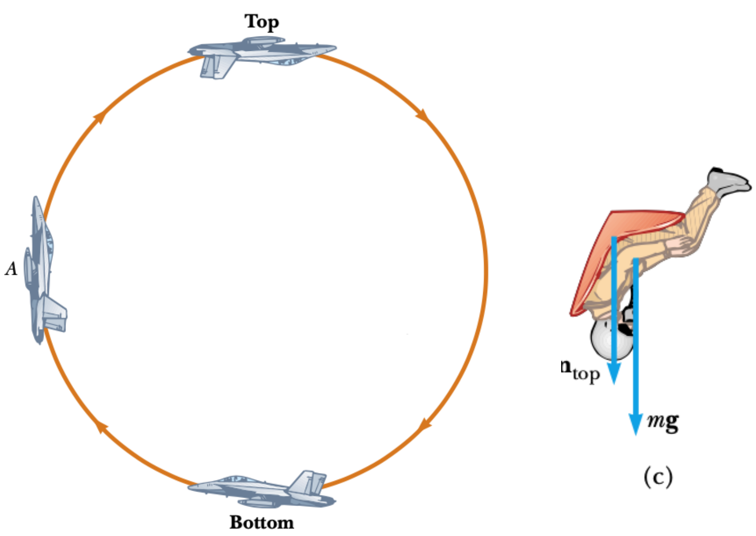
(b)

Loop-the-Loop

- At the top of the circle, the force exerted on the object is less than its weight

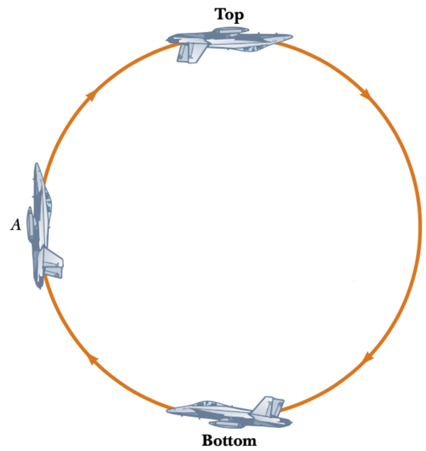
$$\sum F = n_{\text{top}} + mg = m\,\frac{v^2}{r}$$

$$n_{\text{top}} = m\frac{v^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$



Example 6.6 Let's Go Loop-the-Loop!

A pilot of mass m in a jet aircraft executes a loop-the-loop, as shown in Figure 6.7a. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot **(A)** at the bottom of the loop and **(B)** at the top of the loop. Express your answers in terms of the weight of the pilot mg.



$$n_{\text{bot}} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

$$n_{\text{bot}} = mg\left(1 + \frac{(225\text{ m/s})^2}{(2.70 \times 10^3\text{ m})(9.80\text{ m/s}^2)}\right) = 2.91\,mg$$

$$n_{\text{top}} = m\frac{v^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$

$$n_{\text{top}} = mg\left(\frac{(225\text{ m/s})^2}{(2.70 \times 10^3\text{ m})(9.80\text{ m/s}^2)} - 1\right) = 0.913\,mg$$

Suggested Problems from Chapter 6

Problems: 1, 2, 5, 7, 59