Phys 103
Chapter 4

Motion in Two Dimensions

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LECTURE OUTLINE

4.1 The Position, Velocity, and Acceleration Vectors

4.2 Two-Dimensional Motion with Constant Acceleration

4.3 Projectile Motion

4.4 Uniform Circular Motion

4.5 Tangential and Radial Acceleration

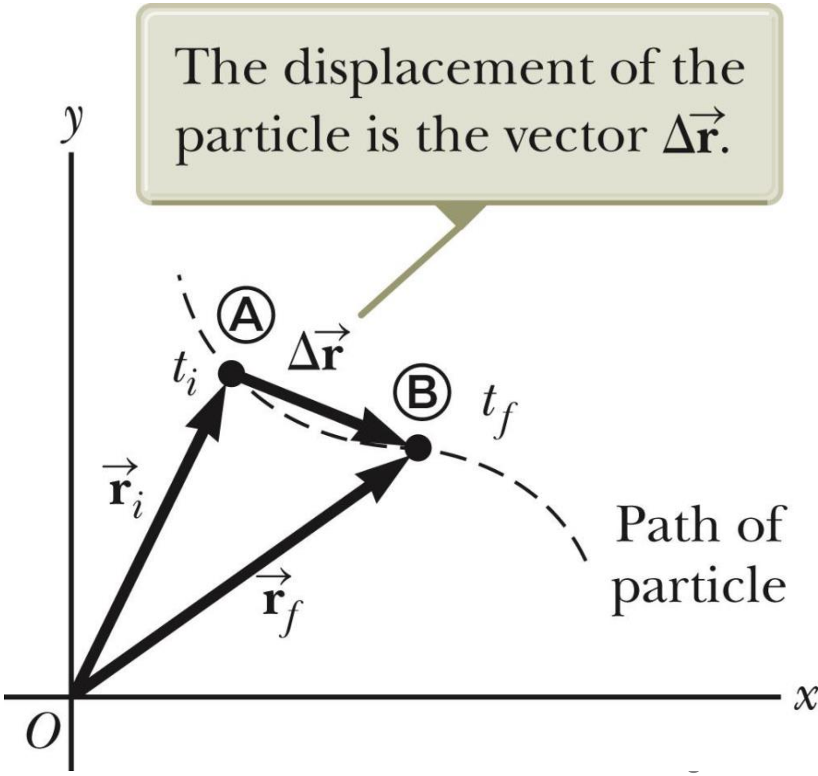
4.1 The Position, Velocity, and Acceleration Vectors

Position and Displacement

The position of an object is described by its position vector, $\vec{r}$.

The **displacement** of the object is defined as the **change in its position.**

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$



4.1 The Position, Velocity, and Acceleration Vectors

Average Velocity

The average velocity is the ratio of the displacement to the time interval for the displacement.

$$\vec{\boldsymbol{v}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}$$

The direction of the average velocity is the direction of the displacement vector.

The average velocity between points is independent of the path taken.

The average velocity between points is independent of the path taken .

4.1 The Position, Velocity, and Acceleration Vectors

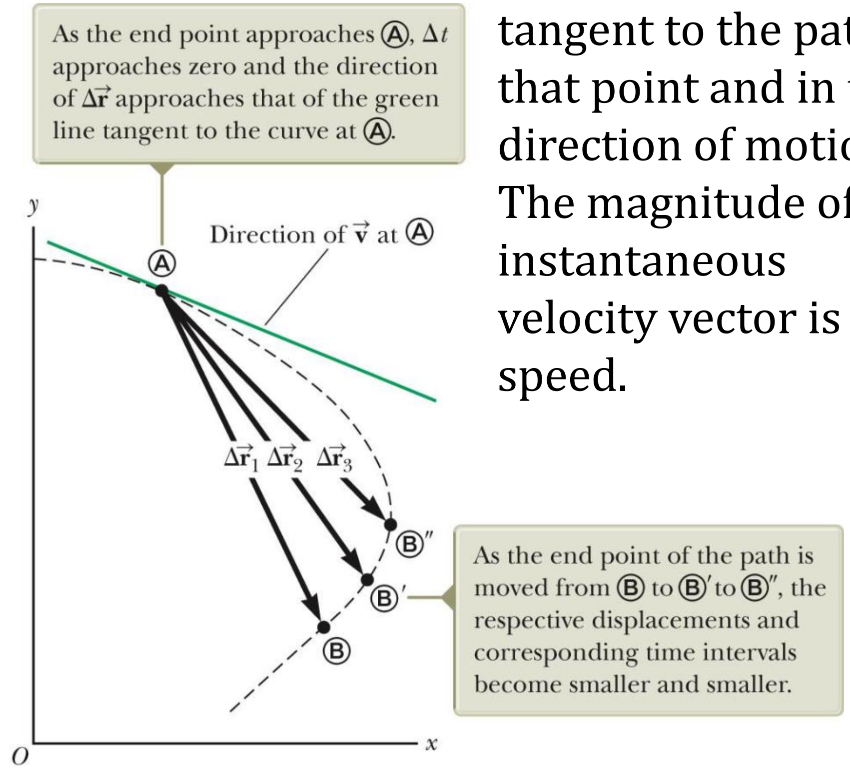
Instantaneous Velocity

The instantaneous velocity is the limit of the average velocity as Δt approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

As the time interval becomes smaller, the direction of the displacement approaches that of the line tangent to the curve.

- The speed is a scalar quantity.

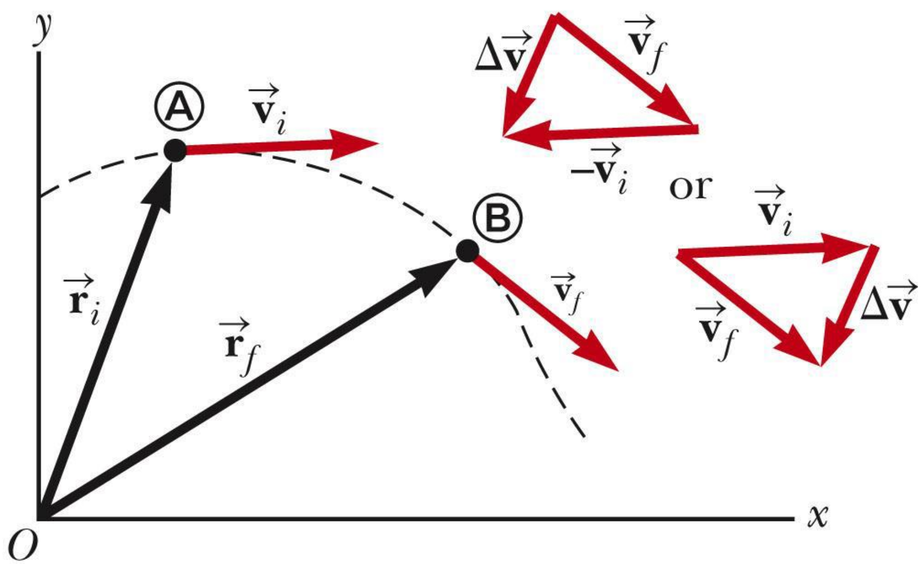


The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion. The magnitude of the instantaneous velocity vector is the speed.

4.1 The Position, Velocity, and Acceleration Vectors

Average Acceleration

The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.



$$\vec{a}_{avg} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

As a particle moves, the direction of the change in velocity is found by vector subtraction.

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

The average acceleration is a vector quantity directed along $\Delta\vec{v}$.

4.1 The Position, Velocity, and Acceleration Vectors

Instantaneous Acceleration

The instantaneous acceleration (acceleration as a function of time) is the limiting value of the ratio as Δt approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{v}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{v}}}{\boldsymbol{d}t}$$

The instantaneous equals the derivative of the velocity vector with respect to time.

Note: the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion.

- the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (2-d) motion.

4.1 The Position, Velocity, and Acceleration Vectors

Producing An Acceleration

- Various changes in a particle's motion may produce an acceleration.
- The magnitude of the velocity vector may change.
- The direction of the velocity vector may change.
  - Even if the magnitude remains constant
- Both may change simultaneously

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

The Let us consider 2-D motion during which the acceleration remains constant in both magnitude and direction.

The position vector for a particle moving in the yy plane can be written:

$$\vec{\boldsymbol{r}} = x\hat{\imath} + y\hat{\jmath}$$

- The velocity vector can be found from the position vector.

$$\vec{v} = \frac{\boldsymbol{d}\vec{\boldsymbol{r}}}{\boldsymbol{d}t} = v_x\hat{\imath} + v_y\hat{\jmath}$$

Since acceleration is constant, we can also find an expression for the velocity as a function of time:

$$\vec{\boldsymbol{v}}_{\boldsymbol{f}} = \overrightarrow{\boldsymbol{v}_{\boldsymbol{i}}} + \vec{a}t$$

Because **a** is assumed constant, its components a_x and a_y also are also constants.

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

For constant acceleration

$a_x = Cons$ **and** $\Delta \boldsymbol{t} = t$

$$\overline{v_x} = \frac{v_{xi} + v_{xf}}{2}$$

For 2-D motion we will have 2 sets of Equations; one for each direction.

- For x-direction; we have:

$$v_{xf} = v_{xi} + a_x t$$
$$\boldsymbol{x_f = x_i + v_{xi} t + \frac{1}{2} a_x \mathbf{t}^2}$$
$$v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$$

For y-direction; we have:

$$v_{yf} = v_{yi} + a_y t$$
$$\boldsymbol{y_f = y_i + v_{yi} t + \frac{1}{2} a_y \mathbf{t}^2}$$
$$v_{yf}{}^2 = v_{yi}{}^2 + 2a_y(y_f - y_i)$$

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

vectors of velocity v and position r are.

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$
$$v_{xf} = v_{xi} + a_x t$$
$$v_{yf} = v_{yi} + a_y t$$
$$\overrightarrow{v_f} = v_{x\mathrm{f}}\hat{\imath} + v_{y\mathrm{f}}\hat{\jmath}$$
$$\overrightarrow{v_f} = (v_{xi} + a_x t)\,\hat{i} + (v_{yi} + a_y t)\hat{\jmath}$$
$$\overrightarrow{v_f} = \left(v_{xi}\hat{\imath} + v_{yi}\hat{\jmath}\right) + (a_{xi}\hat{\imath} + a_{yi}\hat{\jmath})$$

Because **a** is assumed constant, its components a_x and a_y also are also constants.

$$a_{x=}a_x = a$$

$$\overrightarrow{v_f} = \overrightarrow{v_i} + at$$

4.2 Two-D Motion with Cons. Acceleration

Kinematic Equations for Two-Dimensional Motion

vectors of velocity v and position r are.

$$\overrightarrow{v_f} = \overrightarrow{v_i} + at$$

$$\begin{aligned} \boldsymbol{r_f} &= \left(\boldsymbol{v_{xi}t} + \frac{1}{2}\boldsymbol{a_x}\mathbf{t}^2\right)\hat{i} + \left(\boldsymbol{v_{yi}t} + \frac{1}{2}\boldsymbol{a_y}\mathbf{t}^2\right)\hat{j} \\ &= (\boldsymbol{v_{xi}}\,\hat{i} + \boldsymbol{v_{yi}}\,\hat{j})\boldsymbol{t} + \frac{1}{2}(\boldsymbol{a_x}\,\hat{i} + \boldsymbol{a_y}\,\hat{j})\mathbf{t}^2 \\ &= v_i\boldsymbol{t} + \frac{1}{2}\boldsymbol{a}\,\mathbf{t}^2 \end{aligned}$$

Summary

Displacement of a particle in 2-D is:

$$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i$$

The average velocity is defined as:

$$\vec{\boldsymbol{v}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}$$

Instantaneous velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{r}}}{\boldsymbol{d}t}$$

The average acceleration is defined as:

$$\vec{\boldsymbol{a}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{v}}}{\Delta \boldsymbol{t}} = \frac{\vec{\boldsymbol{v}}_f - \vec{\boldsymbol{v}}_i}{t_f - t_i}$$

The instantaneous acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{v}}}{\boldsymbol{d}t}$$

Summary

Constant Acceleration motion of a particle in 2-D:

$v_{xf} = v_{xi} + a_x t$ $x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$ $v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$	$v_{yf} = v_{yi} + a_y t$ $y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$ $v_{yf}{}^2 = v_{yi}{}^2 + 2a_y(y_f - y_i)$

Velocity and position in Vector form in 2-D motion:

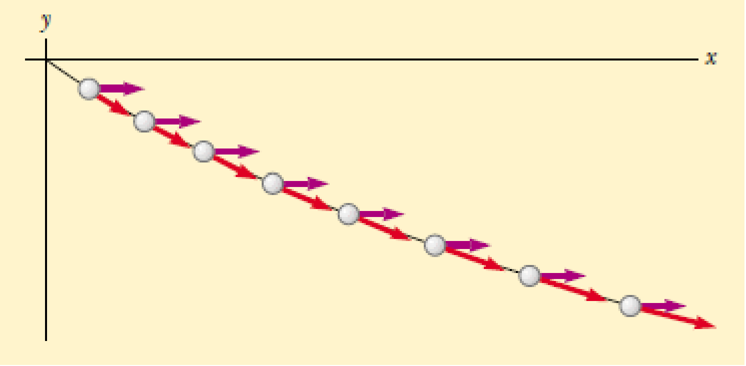
$$\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath}$$
$$\overrightarrow{v_f} = (v_{xi} + a_x t)\hat{\imath} + (v_{yi} + a_y t)\hat{\jmath}$$
$$\overrightarrow{v_f} = \left(v_{xi}\hat{\imath} + v_{yi}\hat{\jmath}\right) + (a_{xi}\hat{\imath} + a_{yi}\hat{\jmath})$$
$$\overrightarrow{v_f} = \overrightarrow{v_i} + at$$

$$r_f = \left(v_{xi} t + \frac{1}{2} a_x t^2\right)\hat{\imath} + \left(v_{yi} t + \frac{1}{2} a_y t^2\right)\hat{\jmath}$$
$$= (v_{xi}\,\hat{\imath} + v_{yi}\,\hat{\jmath})t + \frac{1}{2}(a_x\,\hat{\imath} + a_y\,\hat{\jmath})t^2$$
$$= v_i t + \frac{1}{2} a\, t^2$$

Example 4.1 Motion in a Plane

A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0\ \text{m/s}^2$.

(A) Determine the components of the velocity vector at any time and the total velocity vector at any time.



Equations 4.8a give

(1) $$v_{xf} = v_{xi} + a_x t = (20 + 4.0t)\ \text{m/s}$$

(2) $$v_{yf} = v_{yi} + a_y t = -15\ \text{m/s} + 0 = -15\ \text{m/s}$$

Therefore

$$\mathbf{v}_f = v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}} = \boxed{[(20 + 4.0t)\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]\ \text{m/s}}$$

(B) Calculate the velocity and speed of the particle at $t = 5.0$ s.

$$\mathbf{v}_f = [(20 + 4.0(5.0))\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]\ \text{m/s} = \boxed{(40\hat{\mathbf{i}} - 15\hat{\mathbf{j}})\ \text{m/s}}$$

$$(3) \qquad \theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15\ \text{m/s}}{40\ \text{m/s}}\right) = \boxed{-21^\circ}$$

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^{\,2} + v_{yf}^{\,2}} = \sqrt{(40)^2 + (-15)^2}\ \text{m/s} = \boxed{43\ \text{m/s}}$$

(C) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

$$x_i = y_i = 0 \text{ at } t = 0,$$

$$x_f = v_{xi}t + \tfrac{1}{2}a_x t^2 = \boxed{(20t + 2.0t^2)\text{ m}}$$

$$y_f = v_{yi}t = \boxed{(-15t)\text{ m}}$$

Therefore, the position vector at any time t is

$$(4) \qquad \mathbf{r}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} = \boxed{[(20t + 2.0t^2)\hat{\mathbf{i}} - 15t\hat{\mathbf{j}}]\text{ m}}$$

$$r_f = |\mathbf{r}_f| = \sqrt{(150)^2 + (-75)^2}\text{ m} = 170\text{ m}$$

4.3 Projectile Motion

Projectile Motion

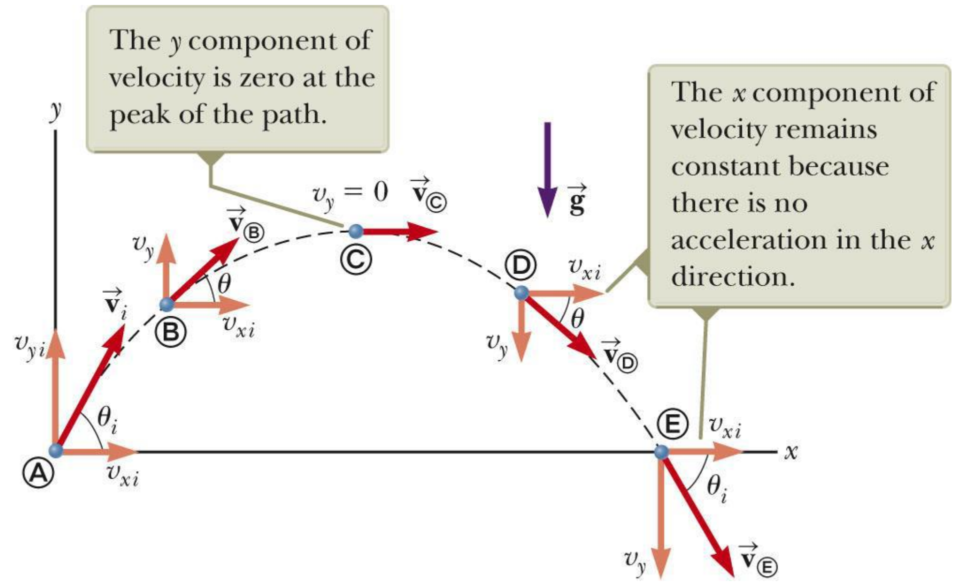
An object may move in both the x and y directions simultaneously. The form of two-dimensional motion we will deal with is called **projectile motion.**

4.3 Projectile Motion

Projectile Motion Diagram

Anyone who has observed a baseball in motion has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze if we make two assumptions: (1) the free-fall acceleration g is constant over the range of motion and is directed downward, and (2) the effect of air resistance is negligible.

- We find that the path of a projectile, which we call its trajectory, is always a ***parabola***



The parabolic path of a projectile that leaves the origin with a velocity *vi*. The x component of v remains constant in time. The y component of velocity is zero at the peak of the path.

4.3 Projectile Motion

Acceleration at the Highest Point

The vertical velocity is zero at the top. The acceleration is not zero anywhere along the trajectory.

- The horizontal and vertical components of a projectile's motion are completely independent of each other and can be handled separately, with time t as the common variable for both components.

Assumptions of projectile motion :

1. The velocity in the x-direction is always constant, therefore the acceleration in x-direction is always zero over the range of motion,
2. The velocity in the y -direction varies throughout the projectile motion. The acceleration in the y-direction is the free-fall acceleration (-g) which is constant throughout the whole motion and is directed downward. .
3. The time t as the common variable for the motion in the x & y-directions
4. The effect of air resistance is negligible.

4.3 Projectile Motion

Analyzing Projectile Motion

Consider the motion as the superposition of the motions in the x- and y-directions. The actual position at any time is given by:

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}\,t^2$$

The initial velocity can be expressed in terms of its components.

$$v_{xi} = v_i \cos\theta_i \; and \; v_{yi} = v_i \sin\theta_i$$

$$x_f = v_{xi}t \; and \;\; y_f = v_{yi}t + \frac{1}{2}a_y t^2$$

The x-direction has constant velocity.

$a_x = 0$

The y-direction is free fall.

- $a_y = -g$

4.3 Projectile Motion

Analyzing Projectile Motion

We will be having 2 sets of equations: 1 for x and 1 for y directions*:*

$$x_f = v_{xi}t$$

$$t = \frac{x_f}{v_i \cos \theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = v_i\, sin\, \theta_i \frac{x_f}{v_i \cos \theta_i} - \frac{1}{2} g \left(\frac{x_f}{v_i \cos \theta_i}\right)^2$$

$$y_f = tan\, \theta_i\, x_f - \frac{g}{2{v_i}^2 \cos {\theta_i}^2} {x_f}^2$$

Or

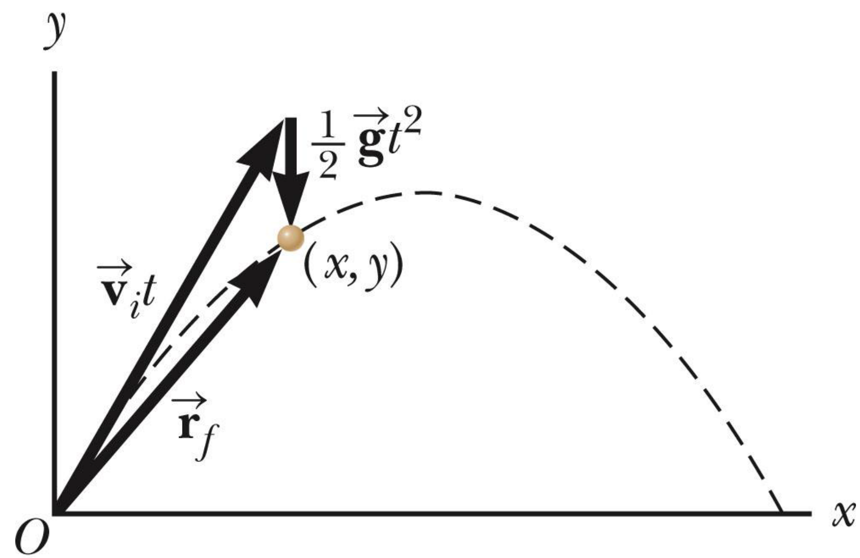
$$y = ax - bx^2$$

4.3 Projectile Motion

Projectile Motion Vectors

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}\,\mathbf{t^2}$$

- The final position is the vector sum of the initial position, the position resulting from the initial velocity and the position resulting from the acceleration.

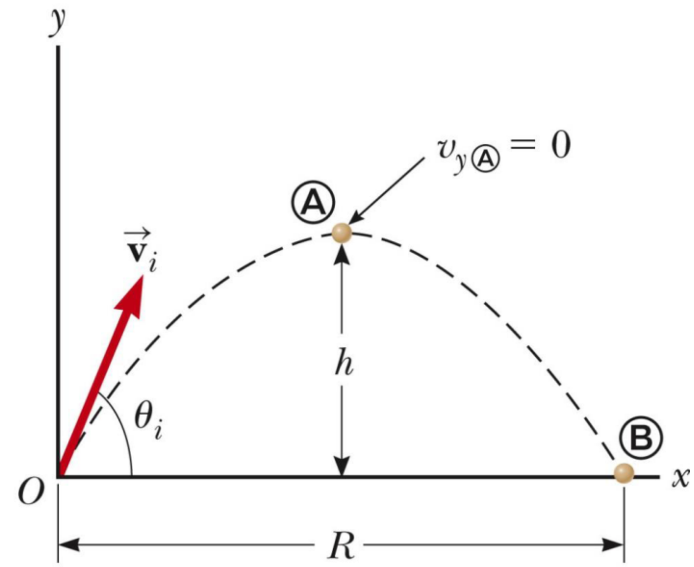


4.3 Projectile Motion

Time of Flight of a Projectile

We will consider the maximum height reached by a projectile:

1st: time of flight: at maxi. height $v_{yf} = 0$



$$v_{yf} = v_{yi} + a_y t = 0$$

$$\therefore\ 0 = v_i \sin \boldsymbol{\theta}_i - g t_{max}$$

$$t_{max} = \frac{v_i \sin \boldsymbol{\theta}_i}{g}$$

$$t_{flight} = \frac{2\, v_i \sin \boldsymbol{\theta}_i}{g}$$

Time of flight is twice the time required to reach to the maximum point. We call this Time-of –flight and is true only if the projecile final destination is on the same level as its starting point.

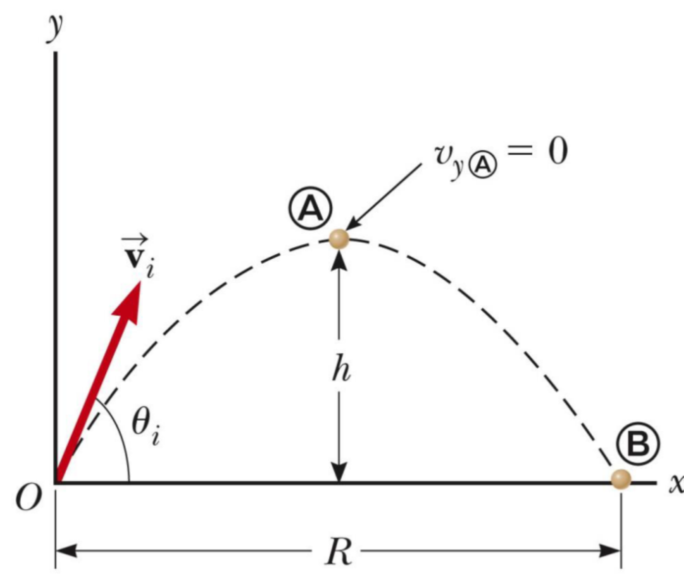
4.3 Projectile Motion

Range and Maximum Height of a Projectile

When analyzing projectile motion, two characteristics are of special interes.t The range, R, is the horizontal distance of the projectile. The maximum height the projectile reaches is h.

The maximum height of the projectile can be found in terms of the initial velocity vector:

$$h = \frac{v^2{}_i \, sin^2\theta_i}{2g}$$



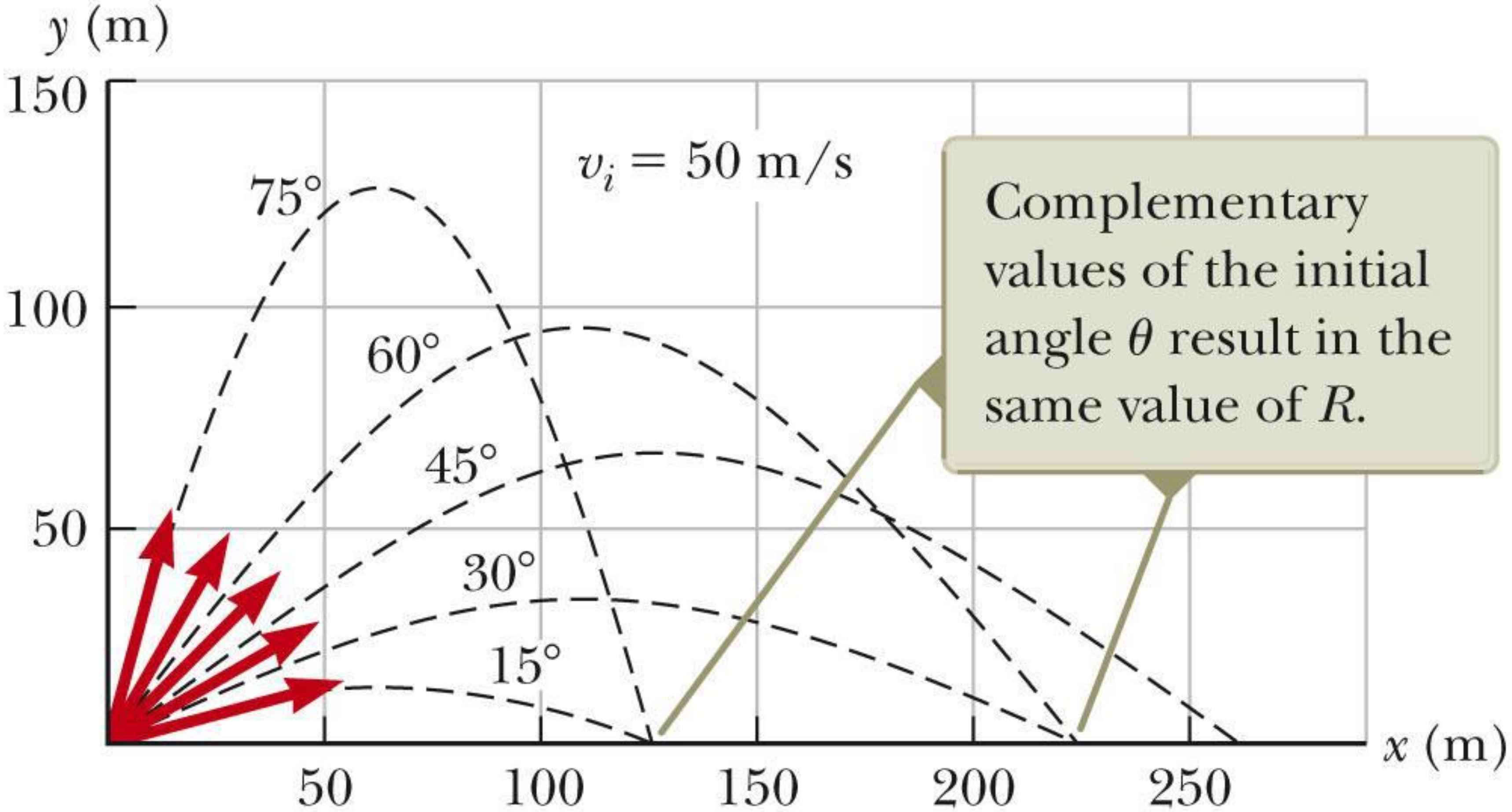
The range of a projectile can be expressed in terms of the initial velocity vector:

$$R = \frac{v^2{}_i \, sin^2 2\theta_i}{g}$$

This equation is valid only for symmetric

4.3 Projectile Motion

More About the Range of a Projectile



4.3 Projectile Motion

More About the Range of a Projectile

Range of a Projectile, final The maximum range occurs at $\theta_i = 45^o$. Complementary angles will produce the same range.

- The maximum height will be different for the two angles.
- The times of the flight will be different for the two angles.

4.3 Projectile Motion

Projectile Motion – Problem Solving Hints

Conceptualize

- Establish the mental representation of the projectile moving along its trajectory. Categorize
- Confirm air resistance is neglected.
- Select a coordinate system with x in the horizontal and y in the vertical direction. Analyze
- If the initial velocity is given, resolve it into x and y components.
- Treat the horizontal and vertical motions independently.

4.3 Projectile Motion

Projectile Motion – Problem Solving Hints, cont.

Analysis, cont.

- Analyze the horizontal motion with the particle-under-constant-velocity model.
- Analyze the vertical motion with the particle-under-constant-acceleration model.
- Remember that both directions share the same time.

Finalize

- Check to see if your answers are consistent with the mental and pictorial representations.
- Check to see if your results are realistic.

Example 4.2 the long jump page 87

A long jumper leaves the ground at an angle of 20° above the horizontal and at a speed of 11 m/s.

(a) How far does he jump in the horizontal direction?

(b) What is the maximum height reached?

$v_i = 11\ \text{m/s}$
$\theta_i = 20°$

(a) How far does he jump in the horizontal direction?

$$\text{Horizontal range}: R = \frac{v_i^2 \sin 2\theta_i}{g} = \frac{(11)^2 \sin 40}{9.8} = 7.94\ \text{m}$$

(b) What is the maximum height reached?

$$\text{Maximum height}: h = \frac{v_i^2 \sin^2 \theta_i}{2g} = \frac{(11)^2 (\sin 20)^2}{(2)(9.8)} = 0.722\ \text{m}$$

Example 4.5 The End of the Ski Jump

A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25 m/s as shown in Figure 4.14. The landing incline below him falls off with a slope of 35°. Where does he land on the incline?

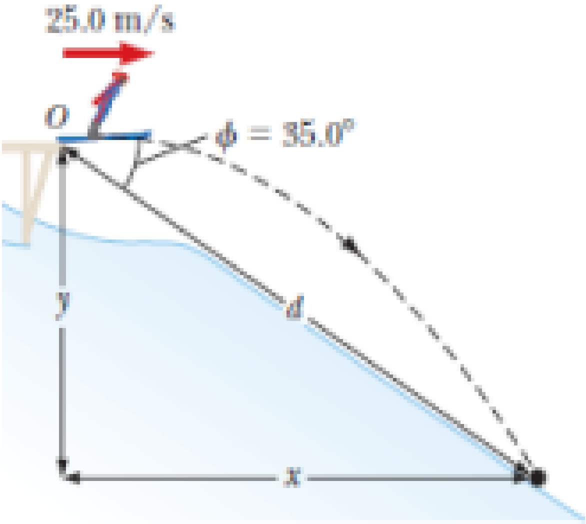
$v_{yi} = 0\ \text{m/s} \quad \& \quad v_{xi} = 25\ \text{m/s} \qquad \varphi_i = 35°$

$\Delta x = v_{xi} t$ .... (1)

$v_{yf} = v_{yi} - gt$ .... (2)

$\Delta y = v_{yi} t - \frac{1}{2} g\, t^2$ .... (3)

${v_{yf}}^2 = {v_{yi}}^2 - 2g(\Delta y)$ ..... (4)



$$x_f = v_{xi} t = d\cos\varphi \qquad \Rightarrow \qquad t = \frac{d\cos\varphi}{v_{xi}}$$

$$y_f = -\tfrac{1}{2} g\, t^2 = -d\sin\varphi$$

$$-d\sin\varphi = -\frac{1}{2} g\, t^2 = -\frac{1}{2} g \left(\frac{d\cos\varphi}{v_{xi}}\right)^2$$

$$d\sin\varphi = \frac{1}{2} g \left(\frac{d\cos\varphi}{v_{xi}}\right)^2 \qquad \Rightarrow \qquad d\sin\varphi = \frac{1}{2} g\, \frac{d^2(\cos\varphi)^2}{{v_{xi}}^2}$$

$$\Rightarrow \qquad d = \frac{2\,{v_{xi}}^2 \sin\varphi}{g\,(\cos\varphi)^2} = \frac{2(25)^2\,(\sin 35)}{(9.8)\,(\cos 35)^2} = 109\ \text{m}$$

$$x_f = d\cos\varphi = (109)(\cos 35^\circ) = 89.3\ \text{m}$$

$$y_f = -\tfrac{1}{2} g\, t^2 = -d\sin\varphi = -(109)\,(\sin 35^\circ) = -62.5\ \text{m}$$

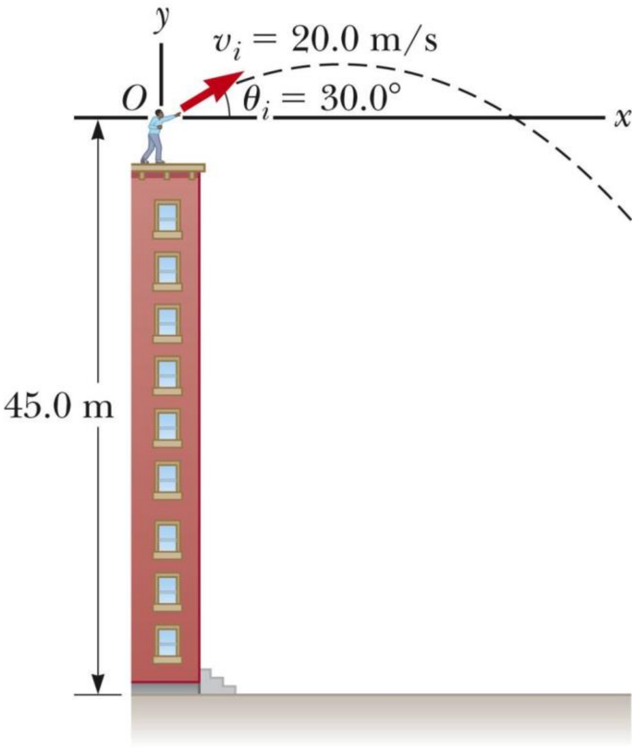
4.3 Projectile Motion

Non-Symmetric Projectile Motion

Follow the general rules for projectile motion.

- Break the y-direction into parts.
- up and down or
- symmetrical back to initial height

and then the rest of the height Apply the problem solving process to determine and solve the necessary equations. May be non-symmetric in other ways



Example 4.4 That's Quite an Arm! AM

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s as shown in Figure 4.13. The height from which the stone is thrown is 45.0 m above the ground.

(A) How long does it take the stone to reach the ground?

$$x_i = y_i = 0,\ y_f = -45.0\ \mathrm{m}, \qquad a_y = -g, \text{ and } v_i = 20.0\ \mathrm{m/s}$$

$$v_{xi} = v_i \cos\theta_i = (20.0\ \mathrm{m/s})\cos 30.0° = 17.3\ \mathrm{m/s}$$

$$v_{yi} = v_i \sin\theta_i = (20.0\ \mathrm{m/s})\sin 30.0° = 10.0\ \mathrm{m/s}$$

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$

$$-45.0\ \mathrm{m} = 0 + (10.0\ \mathrm{m/s})t + \tfrac{1}{2}(-9.80\ \mathrm{m/s^2})t^2$$

$$-45.0\ \mathrm{m} = 0 + (10.0\ \mathrm{m/s})t + \tfrac{1}{2}(-9.80\ \mathrm{m/s^2})t^2$$

$$t = \boxed{4.22\ \mathrm{s}}$$

(B) What is the speed of the stone just before it strikes the ground?

$$v_{yf} = v_{yi} - gt$$

$$v_{yf} = 10.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.3 \text{ m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3 \text{ m/s})^2 + (-31.3 \text{ m/s})^2} = \boxed{35.8 \text{ m/s}}$$

4.4 Uniform Circular Motion

Uniform circular motion occurs when an object moves in a circular path with a constant speed. The associated **analysis model** is a particle in uniform circular motion. An acceleration exists since the direction of the motion is changing .

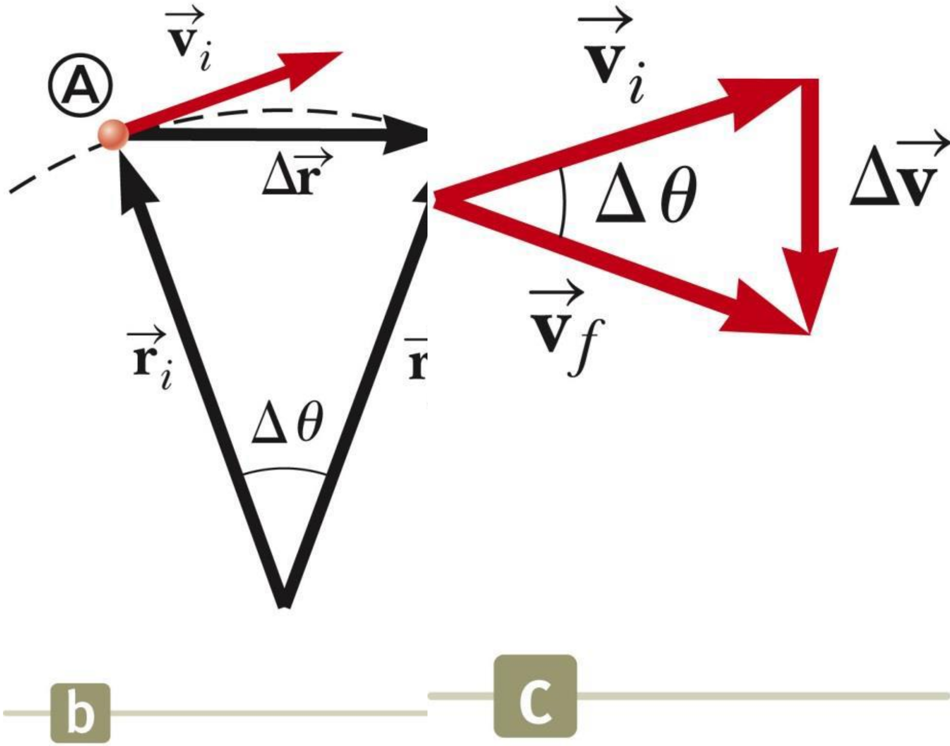
- This change in velocity is related to an acceleration. The constant-magnitude velocity vector is always tangent to the path of the object.

4.4 Uniform Circular Motion

Changing Velocity in Uniform Circular Motion

The change in the velocity vector is due to the change in direction.

- The direction of the change in velocity is toward the center of the circle.
- The vector diagram shows $\vec{v}_f = \vec{v}_i + \Delta v_i$



4.4 Uniform Circular Motion

Centripetal Acceleration

The acceleration is always perpendicular to the path of the motion. The acceleration always points toward the center of the circle of motion. This acceleration is called the **centripetal acceleration.**

The magnitude of the centripetal acceleration vector is given by

$$a_c = \frac{V^2}{r}$$

The direction of the centripetal acceleration vector is always changing, to stay directed toward the center of the circle of motion.

4.4 Uniform Circular Motion

Period

The **period**, T, is the time required for one complete revolution. The speed of the particle would be the circumference of the circle of motion divided by the period. Therefore, the period is defined as

$$T = \frac{2\pi r}{V}$$

angular speed ω measured in radians/s or s^{-1}:

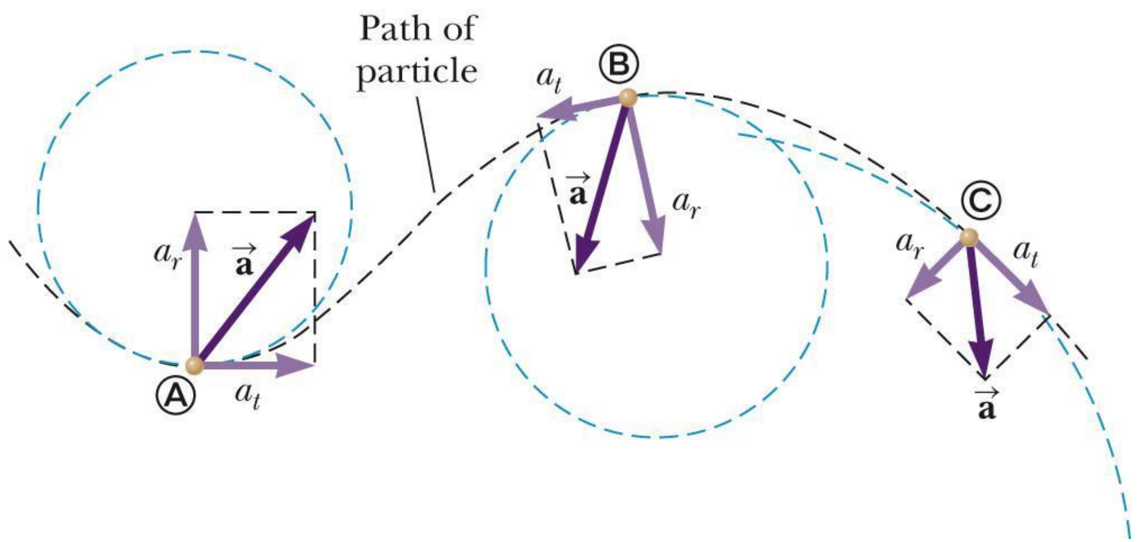
$$\omega = \frac{2\pi}{T}$$

4.5 Tangential and Radial Acceleration

Tangential Acceleration

The magnitude of the velocity could also be changing. In this case, there would be a **tangential acceleration.** The motion would be under the influence of both tangential and centripetal accelerations.

- Note the changing acceleration vectors



Analysis Model Particle in Uniform Circular Motion

Imagine a moving object that can be modeled as a particle. If it moves in a circular path of radius r at a constant speed v, the magnitude of its centripetal acceleration is

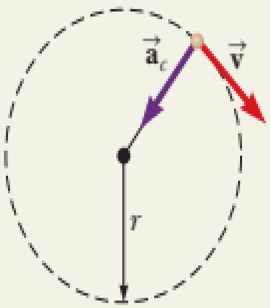
$$a_c = \frac{v^2}{r} \quad (4.14)$$

and the **period** of the particle's motion is given by

$$T = \frac{2\pi r}{v} \quad (4.15)$$

The **angular speed** of the particle is

$$\omega = \frac{2\pi}{T} \quad (4.16)$$



Examples:

- a rock twirled in a circle on a string of constant length
- a planet traveling around a perfectly circular orbit (Chapter 13)
- a charged particle moving in a uniform magnetic field (Chapter 29)
- an electron in orbit around a nucleus in the Bohr model of the hydrogen atom (Chapter 42)

Example 4.6 The Centripetal Acceleration of the Earth AM

(A) What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$a_c = \frac{4\pi^2(1.496 \times 10^{11}\ \text{m})}{(1\ \text{yr})^2}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right)^2 = \boxed{5.93 \times 10^{-3}\ \text{m/s}^2}$$

(B) What is the angular speed of the Earth in its orbit around the Sun?

$$\omega = \frac{2\pi}{1\ \text{yr}}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right) = \boxed{1.99 \times 10^{-7}\ \text{s}^{-1}}$$

4.5 Tangential and Radial Acceleration

Total Acceleration

The tangential acceleration causes the change in the speed of the particle. The radial acceleration comes from a change in the direction of the velocity vector.

4.5 Tangential and Radial Acceleration

Total Acceleration, equations

The tangential acceleration:

$$a_t = |\frac{dv}{dt}|$$

The radial acceleration:

$$a_r = -a_c = -\frac{v^2}{r}$$

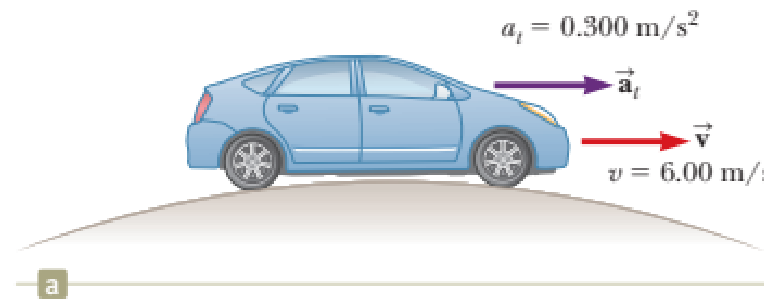
The total acceleration:

- Magnitude

$$a = \sqrt{a^2{}_r + a^2{}_t}$$

Example 4.7 Over the Rise

A car leaves a stop sign and exhibits a constant acceleration of $0.300\ \text{m/s}^2$ parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like an arc of a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What are the magnitude and direction of the total acceleration vector for the car at this instant?



$$a_r = -\frac{v^2}{r} = -\frac{(6.00\ \text{m/s})^2}{500\ \text{m}} = -0.072\,0\ \text{m/s}^2$$

$$\sqrt{a_r^2 + a_t^2} = \sqrt{(-0.072\,0\ \text{m/s}^2)^2 + (0.300\ \text{m/s}^2)^2}$$

$$= \boxed{0.309\ \text{m/s}^2}$$

$$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.072\,0\ \text{m/s}^2}{0.300\ \text{m/s}^2}\right) = \boxed{-13.5^\circ}$$