Phys 103
Chapter 7
Energy and Energy Transfer

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LECTURE OUTLINE

7.2 Work Done by a Constant Force

7.3 The Scalar Product of Two Vectors

7.4 Work Done by a Varying Force

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

7.6 The Nonisolated System—Conservation of Energy

7.7 Situations Involving Kinetic Friction

7.8 Power

Introduction

The concept of energy is one of the most important topics in science and engineering.

In everyday life, we think of energy in terms of fuel for transportation and heating, electricity for lights and appliances, and foods for consumption. However, these ideas do not really define energy. They merely tell us that fuels are needed to do a job and that those fuels provide us with something we call energy.

The definitions of quantities such as position, velocity, acceleration, and force and associated principles such as Newton's second law have allowed us to solve a variety of problems. Some problems that could theoretically be solved with Newton's laws, however, are very difficult in practice. These problems can be made much simpler with a different approach. In this and the following chapters, we will investigate this new approach, which will include definitions of quantities that may not be familiar to you.

Introduction

Other quantities may sound familiar, but they may have more specific meanings in physics than in everyday life. We begin this discussion by exploring the notion of energy.

Energy is present in the Universe in various forms. *Every* physical process that occurs in the Universe involves energy and energy transfers or transformations. Unfortunately, despite its extreme importance, energy cannot be easily defined. The variables in previous chapters were relatively concrete; we have everyday experience with velocities and forces, for example. The notion of energy is more abstract, although we do have *experiences* with energy, such as running out of gasoline, or losing our electrical service if we forget to pay the utility bill.

Introduction

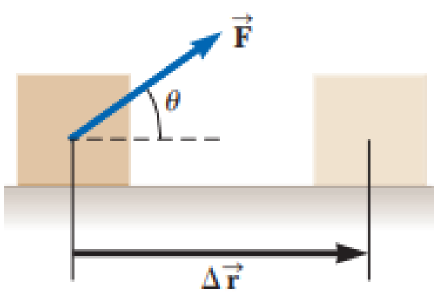
The concept of energy can be applied to the dynamics of a mechanical system without resorting to Newton's laws. This "energy approach" to describing motion is especially useful when the force acting on a particle is not constant; in such a case, the acceleration is not constant, and we cannot apply the constant acceleration equations that were developed in Chapter 2. Particles in nature are often subject to forces that vary with the particles' positions. These forces include gravitational forces and the force exerted on an object attached to a spring. We shall describe techniques for treating such situations with the help of an important concept called *conservation of energy*. This approach extends well beyond physics, and can be applied to biological organisms, technological systems, and engineering situations.

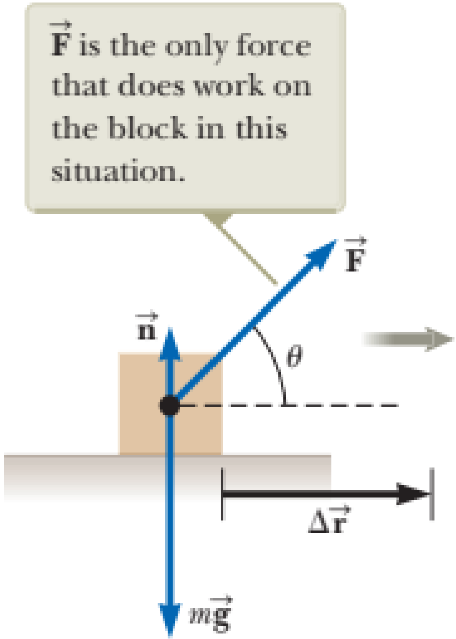
Our problem-solving techniques presented in earlier chapters were based on the motion of a particle or an object that could be modeled as a particle. This was called the *particle model*. We begin our new approach by focusing our attention on a *system* and developing techniques to be used in a *system model*.

7.2 Work Done by a Constant Force

The **work** W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and displacement vectors:

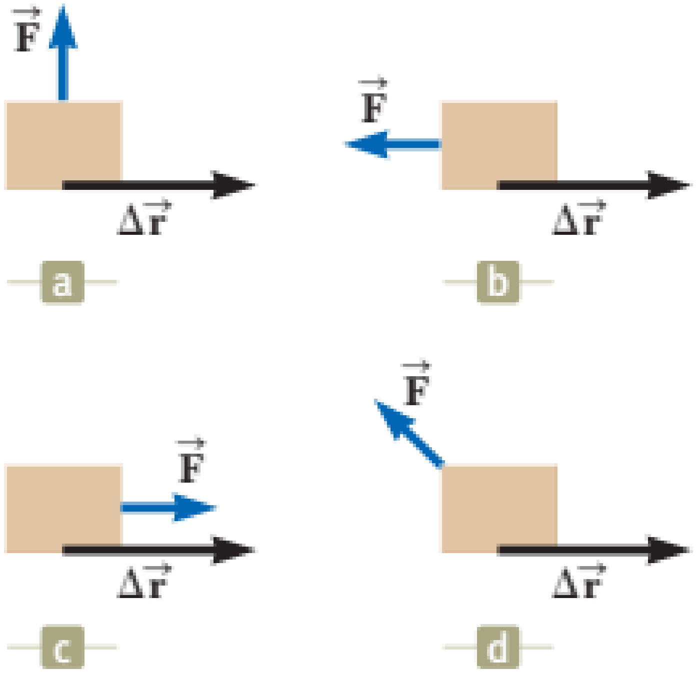
$$W \equiv F\,\Delta r \cos\theta \qquad (7.1)$$





$$W = F\Delta r$$

uick Quiz 7.1 The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Let us assume that the orbit is perfectly circular. The work done by this gravitational force during a short time interval in which the Earth moves through a displacement in its orbital path is **(a)** zero **(b)** positive **(c)** negative **(d)** impossible to determine uick Quiz 7.2 Figure 7.4 shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.



7.2 Work Done by a Constant Force

The work W done on a system by an agent exerting a constant force on the system is the product of the magnitude **F**of the force, the magnitude Δ**r** of the displacement of the point of application of the force, and cosθ, where θ is the angle between the force and displacement vectors:

$$W = F\Delta r \ \cos\theta$$

if θ= 90°, then W = 0 because cos 90o= 0

If an applied force **F** is in the same direction as the displacement Δ**r**, then θ= 0 and cos 0 = 1. In this case, Equation 7.1 gives:

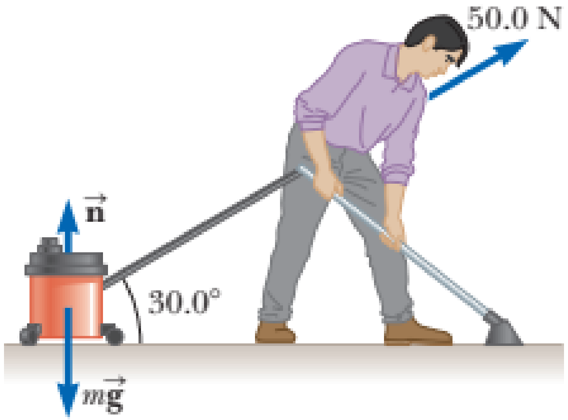
$$W = F\Delta r$$

Work is a *scalar quantity*, and its units are force multiplied by length. Therefore, the SI unit of work is the newton.meter (N. m). This combination of units is used so frequently that it has been given a name of its own: the *joule*(J).

Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.5). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

$$W = F\,\Delta r\cos\theta = (50.0\text{ N})(3.00\text{ m})(\cos 30.0°)$$
$$= 130\text{ J}$$



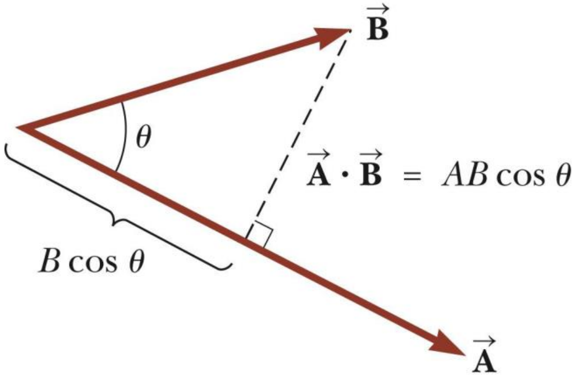
7.3 The Scalar Product of Two Vectors

Because of the way the force and displacement vectors are combined in Equation 7.1, it is helpful to use a convenient mathematical tool called the scalar product of two vectors.

In general; for any two vectors **A**and **B**; Scalar product is defined as:

$$A.B = AB\cos\theta$$

$$W = F\Delta r\ \cos\theta = F.\Delta r$$



In other words, **F.Δr**("F dot Δr") is a shorthand notation for *F*Δr*cos* θ*.*

7.3 The Scalar Product of Two Vectors

Dot Products

Note that the scalar product is commutative.

That is:

$$A.B = B.A$$

Although (7.3) defines the work in terms of two vectors, *work is a scalar*. *All types of* energy and energy transfer are scalars. This is a major advantage of the energy approach. We don't need vector calculations!

Dot Products of Unit Vectors

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$$

Using component form with vectors:

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$$

In the special case where

$$\vec{\mathbf{A}} = \vec{\mathbf{B}};$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Example 7.2 The Scalar Product

The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given by $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.

(A) Determine the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$.

$$\begin{aligned}\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \\ &= -2\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} + 2\hat{\mathbf{i}} \cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} + 3\hat{\mathbf{j}} \cdot 2\hat{\mathbf{j}} \\ &= -2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = \boxed{4}\end{aligned}$$

(B) Find the angle θ between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = \boxed{60.3^\circ}$$

Example 7.3 Work Done by a Constant Force

A particle moving in the *xy* plane undergoes a displacement given by $\Delta\vec{\mathbf{r}} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$ m as a constant force $\vec{\mathbf{F}} = (5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$ N acts on the particle. Calculate the work done by $\vec{\mathbf{F}}$ on the particle.

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\ \text{N}] \cdot [(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\ \text{m}]$$

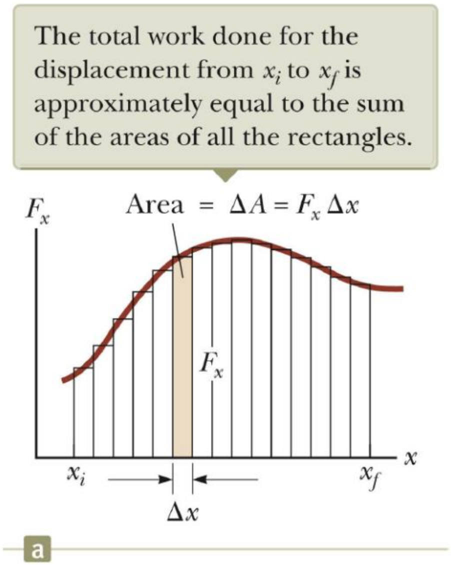
$$= (5.0\hat{\mathbf{i}} \cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}} \cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}} \cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} \cdot 3.0\hat{\mathbf{j}})\ \text{N} \cdot \text{m}$$

$$= [10 + 0 + 0 + 6]\ \text{N} \cdot \text{m} = 16\ \text{J}$$

7.4 Work Done by a Varying Force

If a force $\mathbf{F}_x$ is varying with position, *x*, we can express the work done by $\mathbf{F}_x$ as the particle moves from x_i to x_f as:

$$W \approx \sum_{x_i}^{x_f} F_x \, \Delta x$$

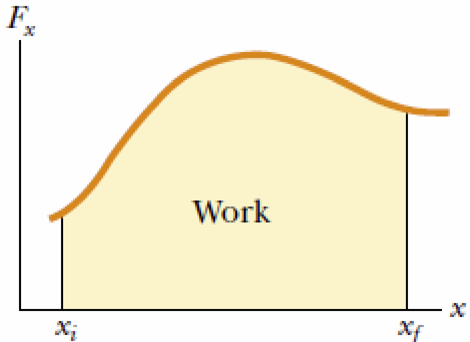


or

$$W = \int_{x_i}^{x_f} F_x \, dx$$

The work done by the component $\mathbf{F}_x$ *of* the varying force as the particle moves from x_i to x_f exactly equal to the area under this curve.

$$W = \int_{x_i}^{x_f} F_x \, dx$$



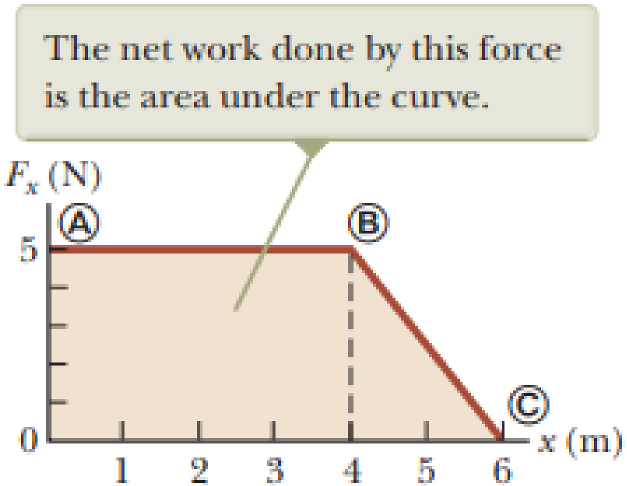
Example 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with x as shown in Figure 7.8. Calculate the work done by the force on the particle as it moves from $x = 0$ to $x = 6.0$ m.

$$W_{Ⓐ \text{ to } Ⓑ} = (5.0 \text{ N})(4.0 \text{ m}) = 20 \text{ J}$$

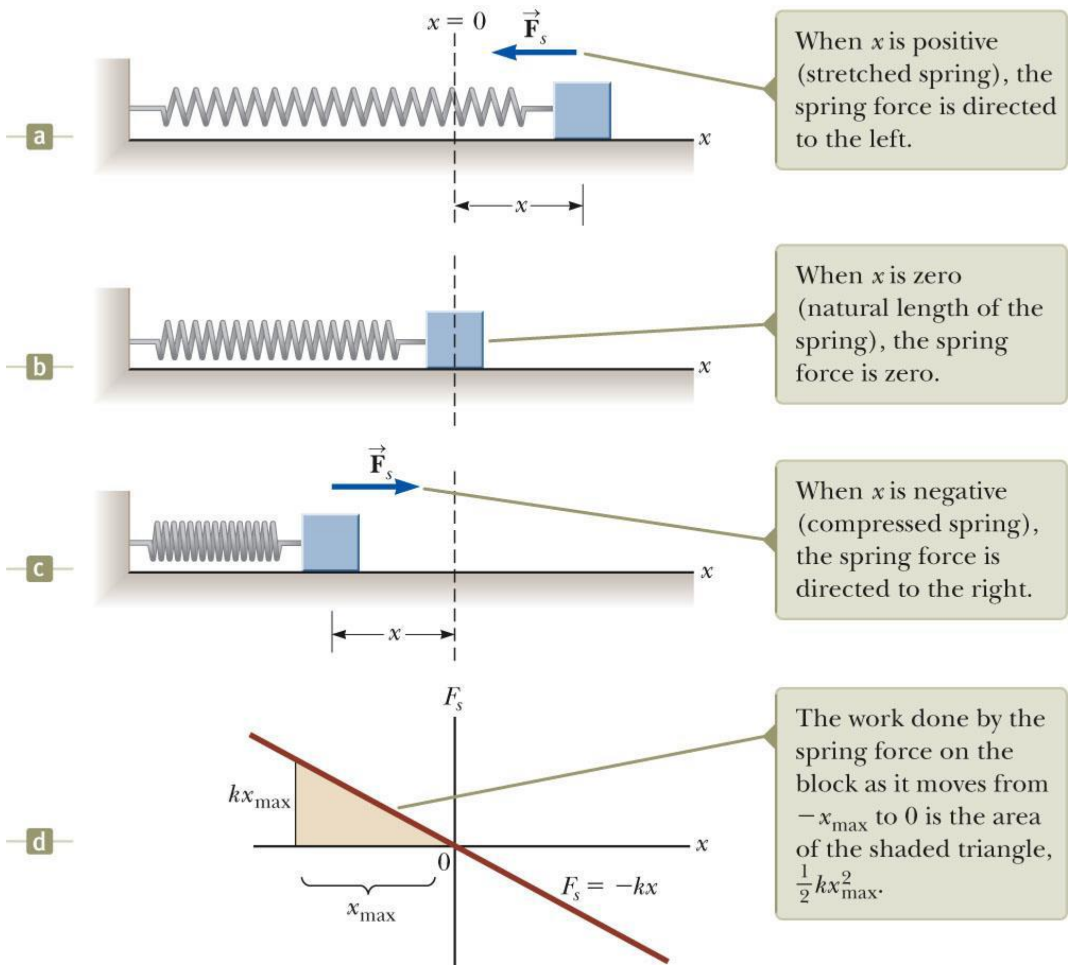
$$W_{Ⓑ \text{ to } Ⓒ} = \tfrac{1}{2}(5.0 \text{ N})(2.0 \text{ m}) = 5.0 \text{ J}$$

$$W_{Ⓐ \text{ to } Ⓒ} = W_{Ⓐ \text{ to } Ⓑ} + W_{Ⓑ \text{ to } Ⓒ} = 20 \text{ J} + 5.0 \text{ J} = \boxed{25 \text{ J}}$$



Work Done By A Spring

A model of a common physical system for which the force varies with position. The block is on a horizontal, frictionless surface. Observe the motion of the block with various values of the spring constant.



Spring Force (Hooke's Law)

The force exerted by the spring is $F_s = -kx$

x is the position of the block with respect to the equilibrium position (x = 0).

k is called the spring constant or force constant and measures the stiffness of the spring.

k measures the stiffness of the spring. This is called Hooke's Law.

When x is positive (spring is stretched), F is negative

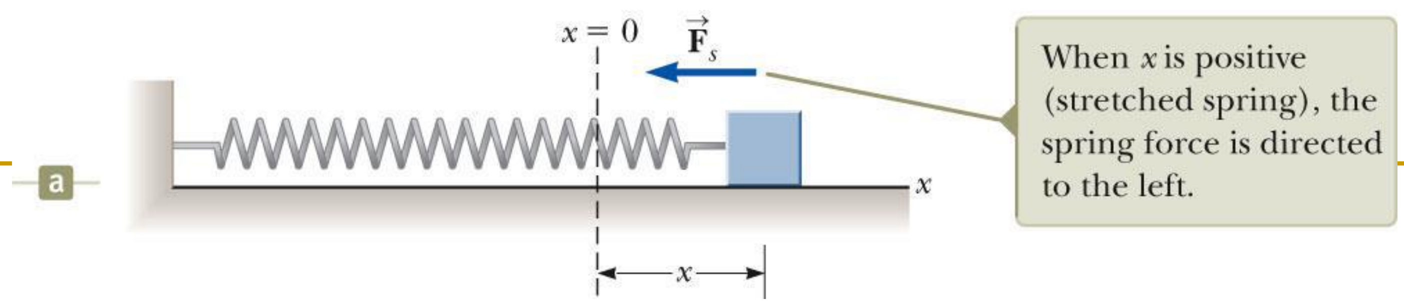
When x is 0 (at the equilibrium position), F is 0

When x is negative (spring is compressed), F is positive

The force exerted by the spring is always directed opposite to the displacement from equilibrium.

The spring force is sometimes called the restoring force.

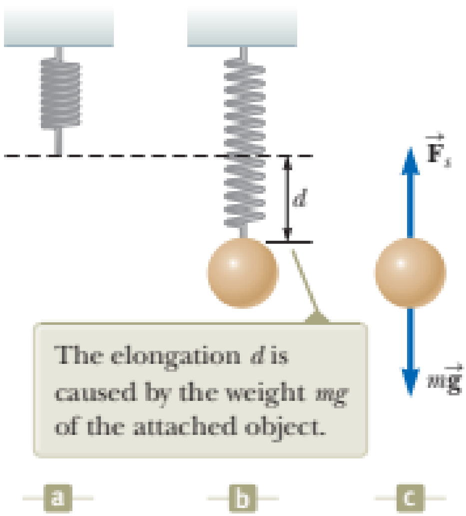
If the block is released it will oscillate back and forth between –x and x.



Example 7.5 Measuring k for a Spring AM

A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.11. The spring is hung vertically (Fig. 7.11a), and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position (Fig. 7.11b).

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?



$$\vec{\mathbf{F}}_s + m\vec{\mathbf{g}} = 0 \quad \rightarrow \quad F_s - mg = 0 \quad \rightarrow \quad F_s = mg$$

$$k = \frac{mg}{d} = \frac{(0.55\text{ kg})(9.80\text{ m/s}^2)}{2.0 \times 10^{-2}\text{ m}} = 2.7 \times 10^2\text{ N/m}$$

(B) How much work is done by the spring on the object as it stretches through this distance?

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\text{ N/m})(2.0 \times 10^{-2}\text{ m})^2$$

$$= -5.4 \times 10^{-2}\text{ J}$$

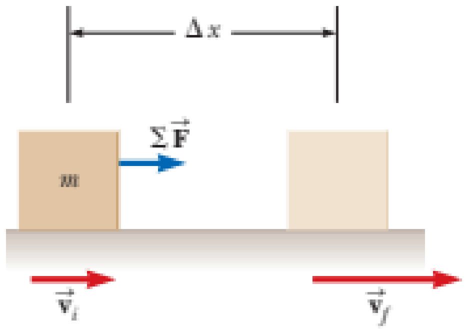
7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

When work (W) is applied on a system; its kinetic energy (K) changes from initial value (K_i) to final value (K_f) so that:

$$W = K_f - K_i$$

We define k as: $K = \frac{1}{2}mv^2$

$$W = \frac{1}{2}m({v^2}_f - {v^2}_i)$$



The **work–kinetic energy theorem** is defined as:

$$W = K_f - K_i = \Delta K$$

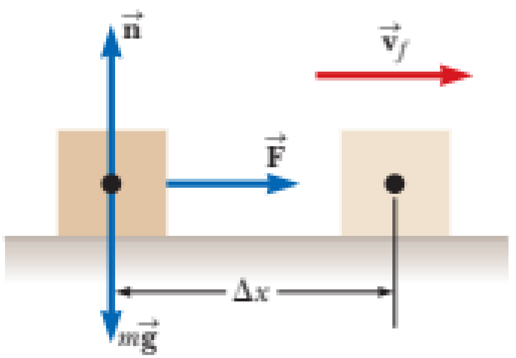
This theorem indicates that the speed of a particle will **increase** if the net work done on it is **positive**, because the final kinetic energy will be greater than the initial kinetic energy. The speed will **decrease** if the net work is **negative.**

Remember work is a scalar, so this is the algebraic sum.

Example 7.6 A Block Pulled on a Frictionless Surface AM

A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of magnitude 12 N. Find the block's speed after it has moved through a horizontal distance of 3.0 m.

$$W = F\Delta x = (12\,\text{N})(3.0\,\text{m}) = 36\,\text{J}$$



$$W_{\text{ext}} = \Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0 = \tfrac{1}{2}mv_f^2$$

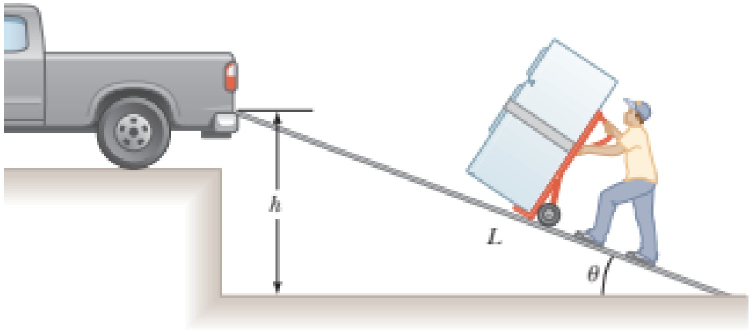
$$v_f = \sqrt{\frac{2W_{\text{ext}}}{m}} = \sqrt{\frac{2F\,\Delta x}{m}}$$

$$v_f = \sqrt{\frac{2(12\text{ N})(3.0\text{ m})}{6.0\text{ kg}}} = 3.5\text{ m/s}$$

Conceptual Example 7.7

A man wishes to load a refrigerator onto a truck using a ramp at angle θ as shown in Figure 7.14. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his claim valid?

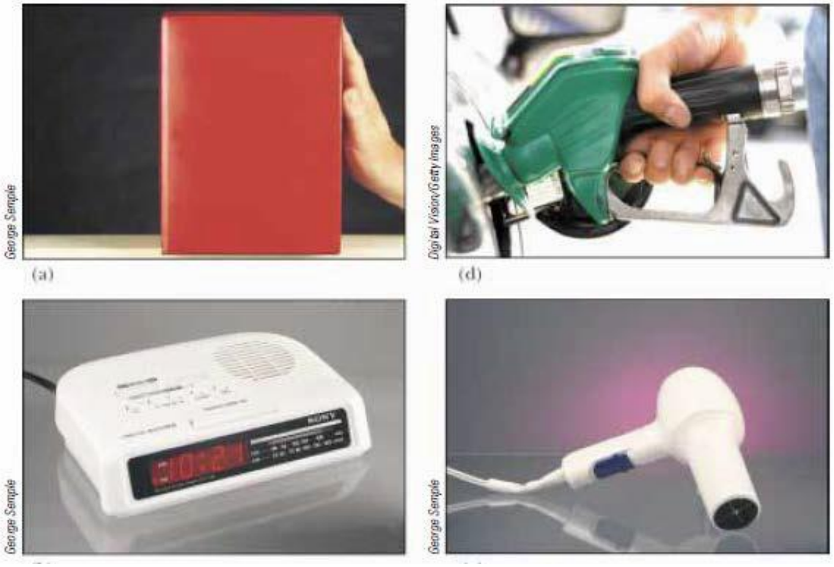
$$W_{ext} = W_{by\ man} + W_{by\ gravity} = 0$$



$$W_{by\ man} = -W_{by\ gravity} = -(mg)(L)[\cos(\theta + 90°)]$$
$$= mgL \sin\theta = mgh$$

7.6 The Nonisolated System-Conservation of Energy

- A particle, that is acted on by various forces, resulting in a change in its kinetic energy is an example of nonisolated system.
- Another example: when a body slides on a surface, heat will be generated although kinetic energy of the surface has not changed.
- Methods of Energy Transfer:Work
- Mechanical Waves
- Heat
- Matter transfer
- Electrical Transmission
- Electromagnetic radiation



7.6 The Nonisolated System-Conservation of Energy

We can neither create nor destroy energy—energy is always conserved. Thus, if the total amount of energy in a system changes, it can only be due to the fact that energy has crossedtheboundaryofthesystembyatransfermechanismsuchasoneofthe methodslistedabove. This is a general statement of the principle of conservation of energy.

$$\Delta E_{system} = \sum T$$

Change in the total energy of the system = the amount of energy transferred across the system boundary by some mechanism

7.7 Situations Involving Kinetic Friction

- Change in Kinetic energy is linked to the work done by a frictional force as:

$$-f_k d = \Delta K \qquad (7.20)$$

or :

$$\Delta E_{\text{int}} = f_k d \qquad (7.22)$$

- *the result of a friction force is to transform kinetic energy into internal energy, and the increase in internal energy is equal to the decrease in kinetic energy.*

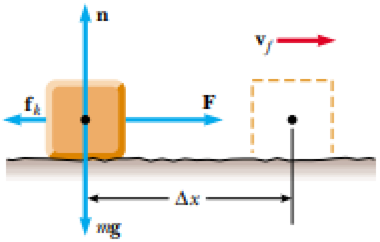
$$\Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$K_f = K_i - f_k d + \sum W_{\text{other forces}}$$

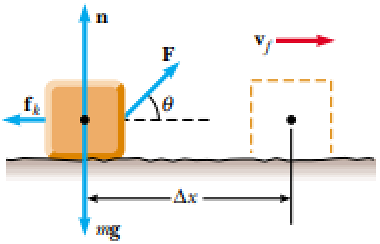
Example 7.9 A Block Pulled on a Rough Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15. (This is Example 7.7, modified so that the surface is no longer frictionless.)



(a)



(b)

$$W = F\Delta x = (12\text{ N})(3.0\text{ m}) = 36\text{ J}$$

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

$$\Delta K_{\text{friction}} = -f_k d = -(8.82\text{ N})(3.0\text{ m}) = -26.5\text{ J}$$

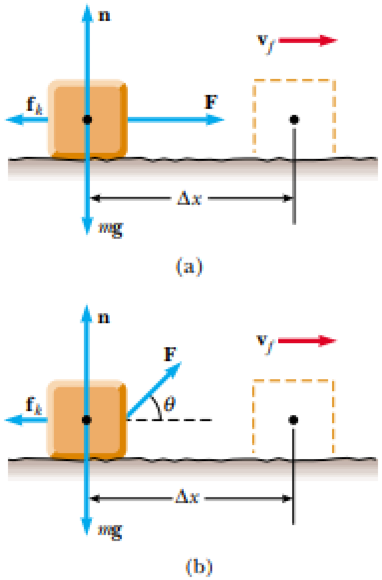
$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 - f_k d + \sum W_{\text{other forces}}$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}\left(-f_k d + \sum W_{\text{other forces}}\right)}$$

$$= \sqrt{0 + \frac{2}{6.0\text{ kg}}(-26.5\text{ J} + 36\text{ J})}$$

$$= 1.8\text{ m/s}$$

(B) Suppose the force **F** is applied at an angle θ as shown in Figure 7.18b. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?



$$W = F\Delta x \cos\theta = Fd\cos\theta$$

$$\sum F_y = n + F\sin\theta - mg = 0$$

$$n = mg - F\sin\theta$$

Because $K_i = 0$, Equation 7.21b can be written,

$$\begin{aligned} K_f &= -f_k d + \sum W_{\text{other forces}} \\ &= -\mu_k n d + Fd\cos\theta \\ &= -\mu_k (mg - F\sin\theta)\, d + Fd\cos\theta \end{aligned}$$

$$\frac{d(K_f)}{d\theta} = -\mu_k(0 - F\cos\theta)\, d - Fd\sin\theta = 0$$

$$\mu_k\cos\theta - \sin\theta = 0$$

$$\tan\theta = \mu_k$$

For $\mu_k = 0.15$, we have,

$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = 8.5^\circ$$

Example 7.11 A Block–Spring System

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0×10^3 N/m, as shown in Figure 7.10. The spring is compressed 2.0 cm and is then released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

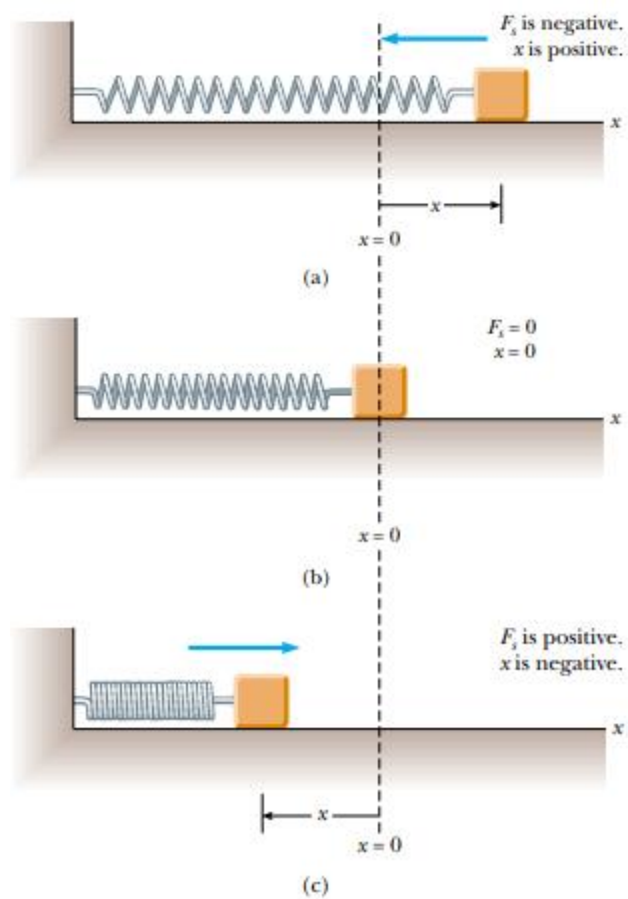
$$W_s = \tfrac{1}{2}kx_{max}^2 = \tfrac{1}{2}(1.0 \times 10^3\ \text{N/m})(-2.0 \times 10^{-2}\ \text{m})^2 = 0.20\ \text{J}$$

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}W_s}$$

$$= \sqrt{0 + \frac{2}{1.6\ \text{kg}}(0.20\ \text{J})}$$

$$= 0.50\ \text{m/s}$$



(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

$$\Delta K = -f_k d = -(4.0\text{ N})(2.0 \times 10^{-2}\text{ m}) = -0.080\text{ J}$$

$$K_f = 0.20\text{ J} - 0.080\text{ J} = 0.12\text{ J} = \tfrac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(0.12\text{ J})}{1.6\text{ kg}}} = 0.39\text{ m/s}$$

What If? **What if the friction force were increased to 10.0 N? What is the block's speed at $x = 0$?**

Answer In this case, the loss of kinetic energy as the block moves to $x = 0$ is

$$\Delta K = -f_k d = -(10.0\text{ N})(2.0 \times 10^{-2}\text{ m}) = -0.20\text{ J}$$

7.8 Power

- Average power is defined as:

$$\overline{p} = \frac{W}{\Delta t} \qquad (7.23)$$

- instantaneous power is:

$$p = \frac{dW}{dt}$$

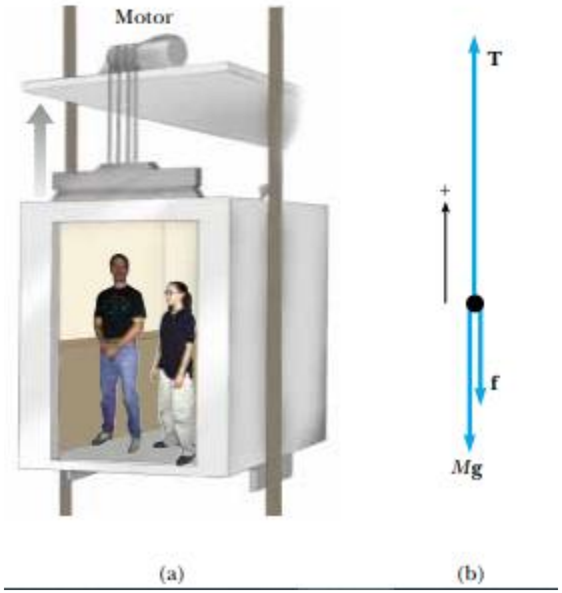
$$\because dW = \boldsymbol{F}.d\boldsymbol{r}$$

$$\rightarrow p = \frac{\boldsymbol{F}.d\boldsymbol{r}}{dt} = \boldsymbol{F}.\frac{d\boldsymbol{r}}{dt} = \boldsymbol{F}.\boldsymbol{v} \qquad (7.23)$$

- instantaneous power is: *Applied force × velocity*
- The SI unit of power is joules per second (J/s), also called the watt (W)
- *Or horsepower: 1hp = 746 W*

Example 7.12 Power Delivered by an Elevator Motor

An elevator car has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion upward, as shown in Figure 7.19a.



(A) What power delivered by the motor is required to lift the elevator car at a constant speed of 3.00 m/s?

$$\sum F_y = T - f - Mg = 0$$

where M is the *total* mass of the system (car plus passengers), equal to 1 800 kg. Therefore,

$$\begin{aligned} T &= f + Mg \\ &= 4.00 \times 10^3\ \text{N} + (1.80 \times 10^3\ \text{kg})(9.80\ \text{m/s}^2) \\ &= 2.16 \times 10^4\ \text{N} \end{aligned}$$

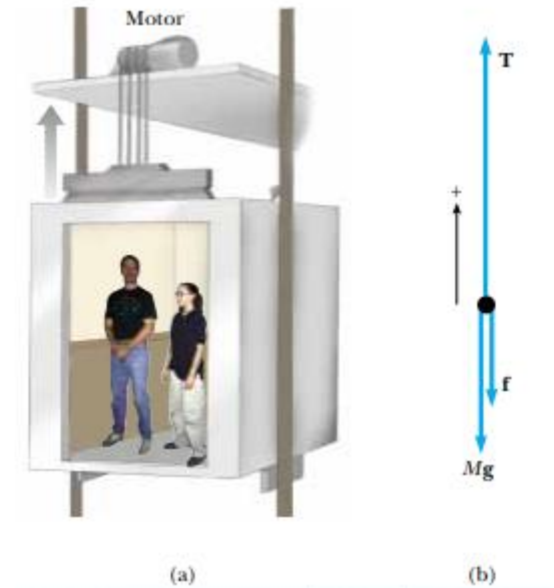
$$\mathcal{P} = \mathbf{T} \cdot \mathbf{v} = Tv$$

$$= (2.16 \times 10^4\ \text{N})(3.00\ \text{m/s}) = 6.48 \times 10^4\ \text{W}$$

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s²?

$$\sum F_y = T - f - Mg = Ma$$

$$\begin{aligned} T &= M(a + g) + f \\ &= (1.80 \times 10^3 \text{ kg})(1.00 \text{ m/s}^2 + 9.80 \text{ m/s}^2) \\ &\quad + 4.00 \times 10^3 \text{ N} \\ &= 2.34 \times 10^4 \text{ N} \end{aligned}$$



Lecture Summary

The work W done on a system by anagentexertinga constant force on the system is the product of the magnitude **F** of the force, the magnitude Δr of the displacement of the point of application of the force, and cosθ, where θ is the angle between the force and displacement vectors:

$$W = F\Delta r\ \cos\theta = F.\Delta r$$

The scalar product (dot product) of two vectors A and B is defined by the relationship:

$$A.B = AB\cos\theta$$

If aforce $\mathbf{F_x}$ is varying with position, x, we can express the work done by $\mathbf{F_x}$ as the particle moves from x_i to x_f as:

$$W = \int_{x_i}^{x_f} F_x\, dx$$

PROBLEMS

Section 7.2 Work Done by a Constant Force

1. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, and (c) the gravitational force. (d) Determine the total work done on the block.

SOLUTIONS TO PROBLEM:

$W = F\Delta r\cos\theta$ = a16.0 Nfa2.20 mfcos 25.0°=31.9 J

The normal force and the weight are both at 90° to the displacement in any time interval. Both do 0 work.

PROBLEMS

Section 7.2 Work Done by a Constant Force

4. A raindrop of mass 3.35 * 10&5 kg falls vertically at constant speed under the influence of gravity and air resistance.

Model the drop as a particle. As it falls 100 m, what is the work done on the raindrop (a) by the gravitational force and (b) by air resistance?

SOLUTIONS TO PROBLEM:

$W = mgh$

Since $R = mg$

$W_{\text{air resistance}} = -W$

PROBLEMS

Section 7.3 The Scalar Product of Two Vectors

7. A force **F** = (6ˆ**i** & 2ˆ**j**) N acts on a particle that undergoes a displacement Δ **r =** (3ˆ**i**)ˆ**j**) m. Find (a) the work done by the force on the particle and (b) the angle between **F** and Δ **r**.

SOLUTIONS TO PROBLEM:

$W = \mathbf{F}\cdot\Delta\mathbf{r} = F_x x + F_y y = 6.00(3.00)\ \text{N}\cdot\text{m} + (-2.00)(1.00)\ \text{N}\cdot\text{m} = 16.0\ \text{J}$

$$\boldsymbol{\theta} = \cos^{-1}\left(\frac{\boldsymbol{F}.\Delta\boldsymbol{r}}{F\Delta r}\right) = \cos^{-1}\left(\frac{\mathbf{16}}{\sqrt{\boldsymbol{F_x}^2 + \boldsymbol{F_y}^2}\sqrt{\boldsymbol{x}^2 + \boldsymbol{y}^2}}\right)$$

PROBLEMS

Section 7.4 Work Done by a Varying Force

13. A particle is subject to a force F_x that varies with position as in Figure P7.13. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 5.00$ m, (b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

SOLUTIONS TO PROBLEM:

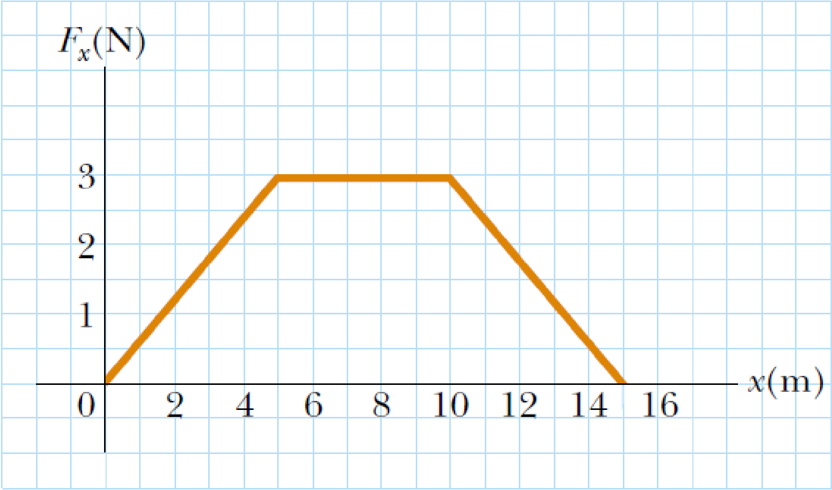


Figure P7.13 Problems 13 and 28.

PROBLEMS

Section 7.4 Work Done by a Varying Force

14. A force **F** = (4*x*ˆ**i**) 3*y*ˆ**j**) N acts on an object as the object moves in the *x* direction from the origin to $x = 5.00$ m. Find the work

$$W = \int F\, dr$$

done on the object by the force.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

15. When a 4.00-kg object is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg object is removed, (a) how far will the spring stretch if a 1.50-kg block is hung on it, and (b) how much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

16. An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work does the archer do in pulling the bow?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

19. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

21. A light spring with spring constant 1 200 N/m is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant 1 800 N/m.

An object of mass 1.50 kg is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system. We describe these springs as *in series*.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

24. A 0.600-kg particle has a speed of 2.00 m/s at point A And kinetic energy of 7.50 J at point B*.* What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

25. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) **What If?** If its speed were doubled, what would be its kinetic energy?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

26. A 3.00-kg object has a velocity (6.00ˆi - 2.00ˆj) m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to (8.00ˆi + 4.00ˆj) m/s. (*Note:* From the definition of the dot product, *v*2 = **v** . **v**.)

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

28. A 4.00-kg particle is subject to a total force that varies with position as shown in Figure P7.13. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m, (b) $x = 10.0$ m, (c) $x = 15.0$ m?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

31. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.300, find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor system due to friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

32. A 2.00-kg block is attached to a spring of force constant 500 N/m as in Figure 7.10. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

33. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate–incline system due to friction. (c) How much work is done by the 100-N force on the crate? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

35. A sled of mass *m* is given a kick on a frozen pond. The kick imparts to it an initial speed of 2.00 m/s. The coefficient of kinetic friction between sled and ice is 0.100. Use energy considerations to find the distance the sled moves before it stops.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.8 Power

37. A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.8 Power

40. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

SOLUTIONS TO PROBLEM: