# Chapter 9

Linear Momentum and Collisions

# Linear Momentum

- The **linear momentum** of a particle or an object that can be modeled as a particle of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:
  - $\mathbf{p} = m\mathbf{v}$
    - The terms momentum and linear momentum will be used interchangeably in the text

# Linear Momentum, cont

- Linear momentum is a vector quantity
  - Its direction is the same as the direction of **v**
- The dimensions of momentum are ML/T
- The SI units of momentum are kg · m / s
- Momentum can be expressed in component form:
  - $p_x = m\ v_x \qquad p_y = m\ v_y \qquad p_z = m\ v_z$

# Newton and Momentum

- Newton called the product $m\mathbf{v}$ the *quantity of motion* of the particle
- Newton's Second Law can be used to relate the momentum of a particle to the resultant force acting on it

$$\Sigma\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d\left(m\mathbf{v}\right)}{dt} = \frac{d\mathbf{p}}{dt}$$

with constant mass

# Conservation of Linear Momentum

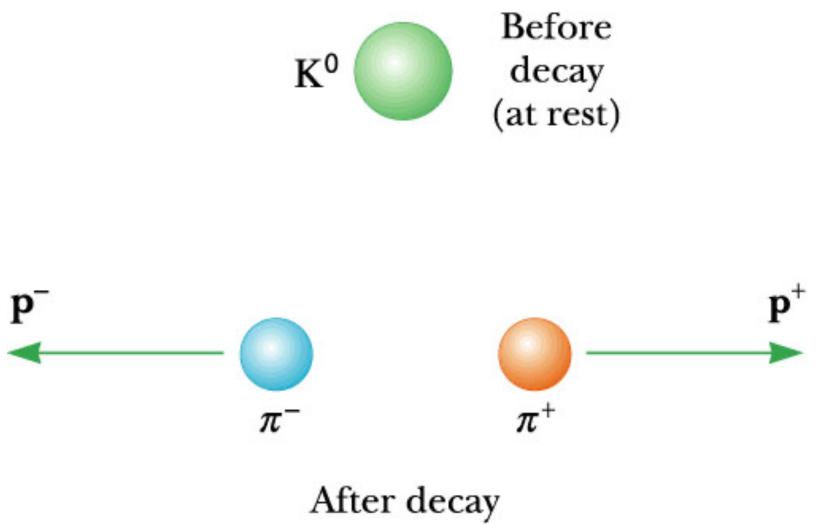
- Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant
  - The momentum of the *system* is conserved, not necessarily the momentum of an individual particle
  - This also tells us that the total momentum of an isolated system equals its initial momentum

# Conservation of Momentum, 2

- Conservation of momentum can be expressed mathematically in various ways
  - $\mathbf{p}_{total} = \mathbf{p}_1 + \mathbf{p}_2 =$ constant
  - $\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$
- In component form, the total momenta in each direction are independently conserved
  - $p_{ix} = p_{fx}$ $\quad p_{iy} = p_{fy}$ $\quad p_{iz} = p_{fz}$
- Conservation of momentum can be applied to systems with any number of particles

# Conservation of Momentum, Kaon Example

- The kaon decays into a positive $\pi$ and a negative $\pi$ particle
- Total momentum before decay is zero
- Therefore, the total momentum after the decay must equal zero
  - $p^+ + p^- = 0$ or $p^+ = -p^-$



# Impulse and Momentum

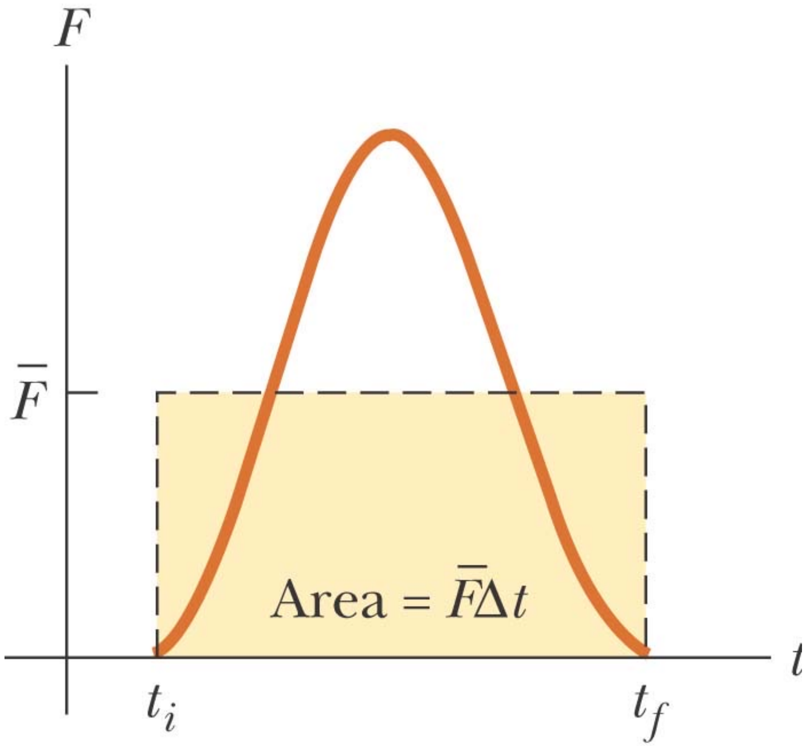
- From Newton's Second Law, $\mathbf{F} = d\mathbf{p}/dt$
- Solving for $d\mathbf{p}$ gives $d\mathbf{p} = \mathbf{F}dt$
- Integrating to find the change in momentum over some time interval

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \int_{t_i}^{t_f} \mathbf{F}dt = \mathbf{I}$$

- The integral is called the *impulse*, $\boldsymbol{I}$, of the force **F** acting on an object over $\Delta t$

# Impulse, Final

- The impulse can also be found by using the time averaged force
- $\mathbf{I} = \overline{F}\Delta t$
- This would give the same impulse as the time-varying force does



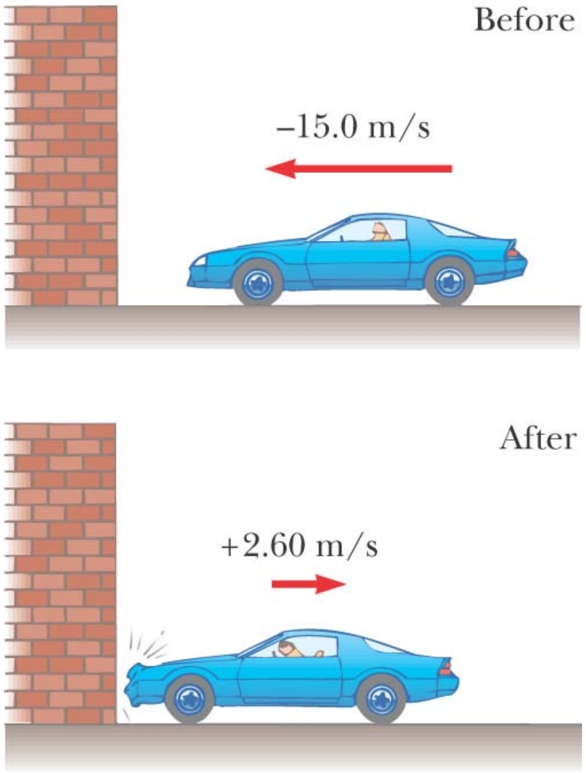
(b)

# Impulse Approximation

- In many cases, one force acting on a particle will be much greater than any other force acting on the particle
- When using the Impulse Approximation, we will assume this is true
- The force will be called the impulse force
- $\mathbf{p}_f$ and $\mathbf{p}_i$ represent the momenta immediately before and after the collision
- The particle is assumed to move very little during the collision

# Impulse-Momentum: Crash Test Example

- The momenta before and after the collision between the car and the wall can be determined ($\mathbf{p} = m\,\mathbf{v}$)
- Find the impulse:
  - $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$
  - $\mathbf{F} = \Delta\mathbf{p} / \Delta t$



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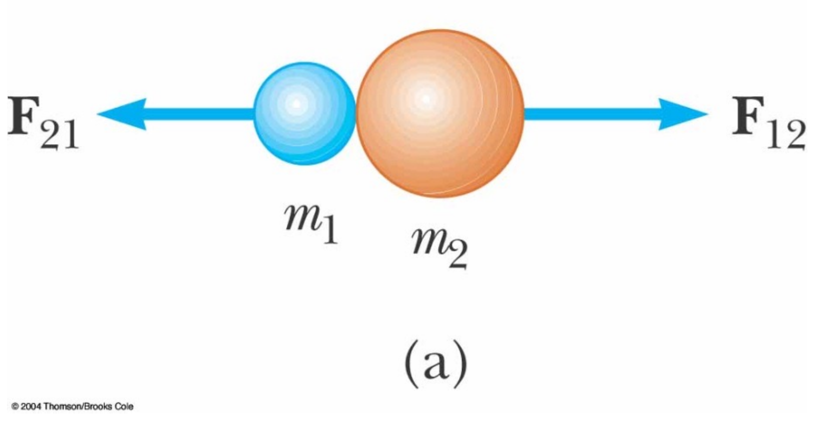
# Collisions – Characteristics

- We use the term **collision** to represent an event during which two particles come close to each other and interact by means of forces
- The time interval during which the velocity changes from its initial to final values is assumed to be short
- The interaction force is assumed to be much greater than any external forces present
  - This means the impulse approximation can be used

# Collisions – Example 1

- Collisions may be the result of direct contact
- The impulsive forces may vary in time in complicated ways
  - This force is internal to the system
- Momentum is conserved



# Types of Collisions

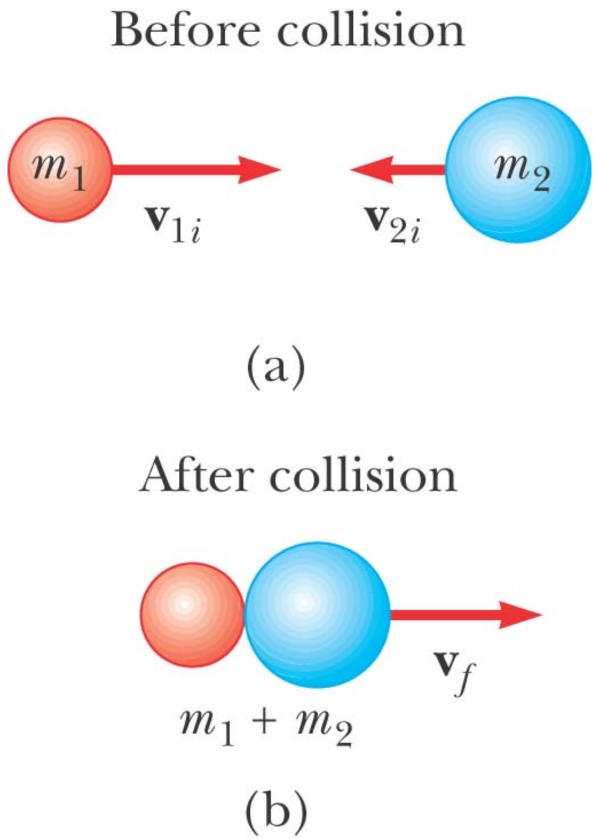
- In an ***elastic*** collision, momentum and kinetic energy are conserved
  - Perfectly elastic collisions occur on a microscopic level
  - In macroscopic collisions, only approximately elastic collisions actually occur
- In an ***inelastic*** collision, kinetic energy is not conserved although momentum is still conserved
  - If the objects stick together after the collision, it is a ***perfectly inelastic*** collision

# Collisions, cont

- In an inelastic collision, some kinetic energy is lost, but the objects do not stick together
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types
- Momentum is conserved in all collisions

# Perfectly Inelastic Collisions

- Since the objects stick together, they share the same velocity after the collision
- $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\,\mathbf{v}_f$

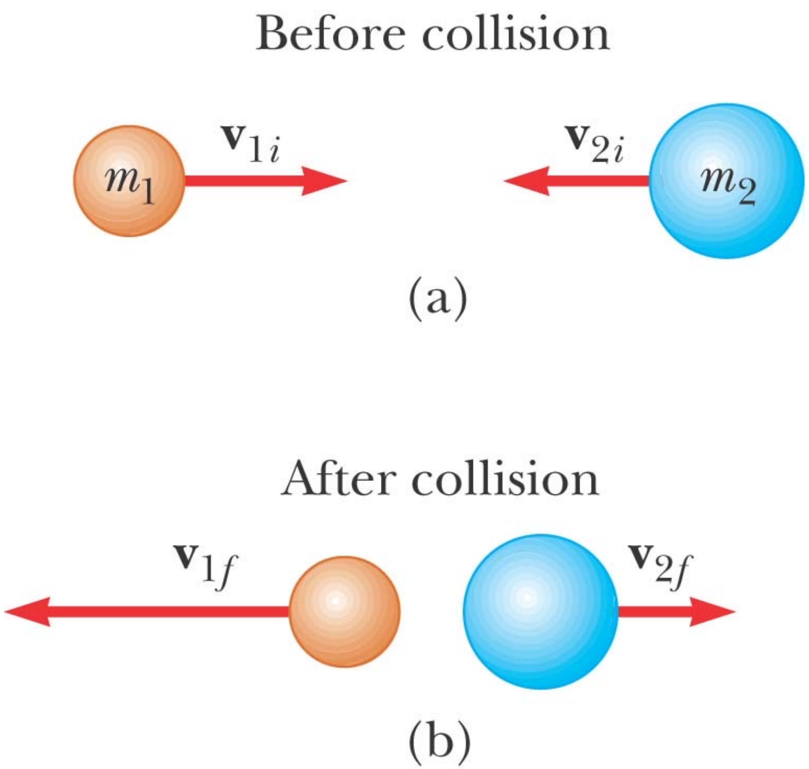


# Elastic Collisions

- Both momentum and kinetic energy are conserved

$$m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} =$$
$$m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f}$$

$$\frac{1}{2} m_1 \mathbf{v}_{1i}^2 + \frac{1}{2} m_2 \mathbf{v}_{2i}^2 =$$
$$\frac{1}{2} m_1 \mathbf{v}_{1f}^2 + \frac{1}{2} m_2 \mathbf{v}_{2f}^2$$



# Elastic Collisions, cont

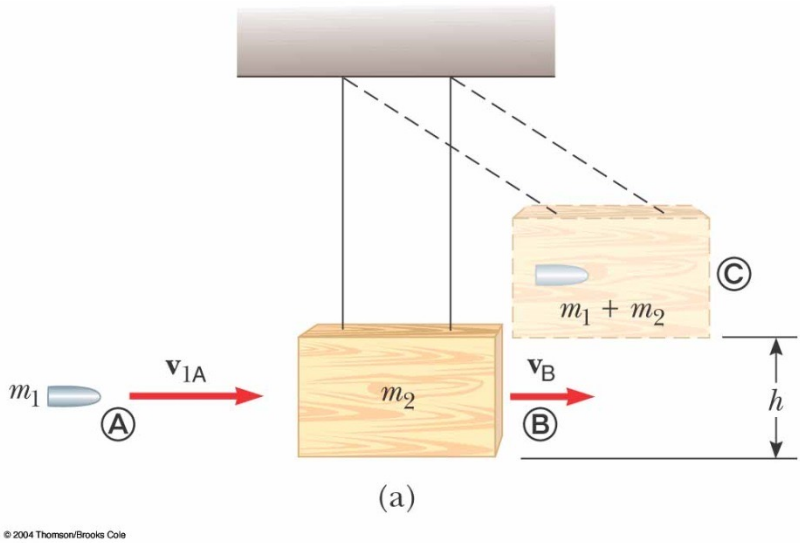
- Typically, there are two unknowns to solve for and so you need two equations
- The kinetic energy equation can be difficult to use
- With some algebraic manipulation, a different equation can be used

$$v_{1i} - v_{2i} = v_{1f} + v_{2f}$$

- This equation, along with conservation of momentum, can be used to solve for the two unknowns
    - It can only be used with a one-dimensional, elastic collision between two objects

# Collision Example – Ballistic Pendulum

- Perfectly inelastic collision – the bullet is embedded in the block of wood
- Momentum equation will have two unknowns
- Use conservation of energy from the pendulum to find the velocity just after the collision
- Then you can find the speed of the bullet

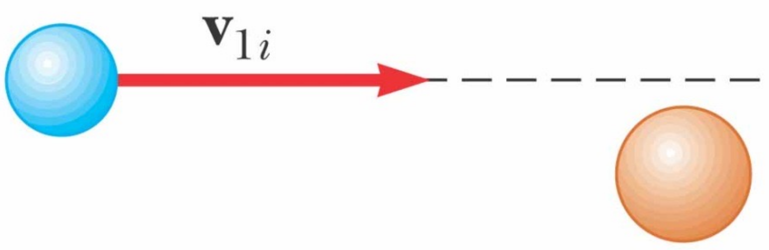


# Two-Dimensional Collisions

- The momentum is conserved in all directions
- Use subscripts for
  - identifying the object
  - indicating initial or final values
  - the velocity components
- If the collision is elastic, use conservation of kinetic energy as a second equation
  - Remember, the simpler equation can only be used for one-dimensional situations

# Two-Dimensional Collision, example

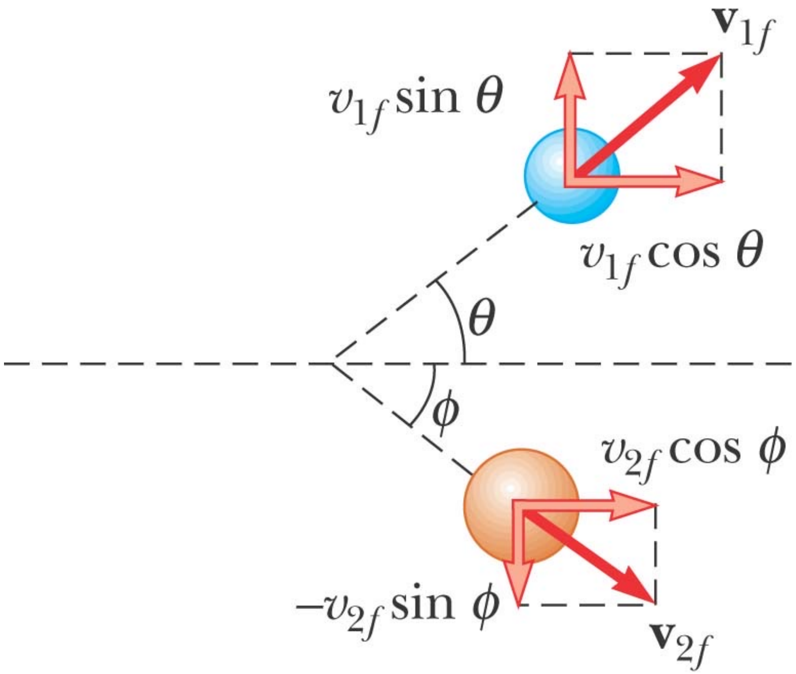
- Particle 1 is moving at velocity $\mathbf{v}_{1i}$ and particle 2 is at rest
- In the *x*-direction, the initial momentum is $m_1 v_{1i}$
- In the *y*-direction, the initial momentum is 0



(a) Before the collision

# Two-Dimensional Collision, example cont

- After the collision, the momentum in the *x*-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
- After the collision, the momentum in the *y*-direction is $m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$



(b) After the collision

# Problem-Solving Strategies – Two-Dimensional Collisions

- Set up a coordinate system and define your velocities with respect to that system
  - It is usually convenient to have the *x*-axis coincide with one of the initial velocities
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information

# Problem-Solving Strategies – Two-Dimensional Collisions, 2

- Write expressions for the *x*- and *y*-components of the momentum of each object before and after the collision
  - Remember to include the appropriate signs for the components of the velocity vectors
- Write expressions for the total momentum of the system in the *x*-direction before and after the collision and equate the two. Repeat for the total momentum in the *y*-direction.

# Problem-Solving Strategies – Two-Dimensional Collisions, 3

- If the collision is inelastic, kinetic energy of the system is not conserved, and additional information is probably needed
- If the collision is perfectly inelastic, the final velocities of the two objects are equal.  Solve the momentum equations for the unknowns.

# Two-Dimensional Collision Example

- Before the collision, the car has the total momentum in the *x*-direction and the van has the total momentum in the *y*-direction
- After the collision, both have *x*- and *y*-components