Chapter 8

Potential Energy

Potential Energy

- Potential energy is the energy associated with the configuration of a system of objects that exert forces on each other
  - This can be used only with *conservative* forces
  - When conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy.
    - This is ***Conservation of Mechanical Energy***

Types of Potential Energy

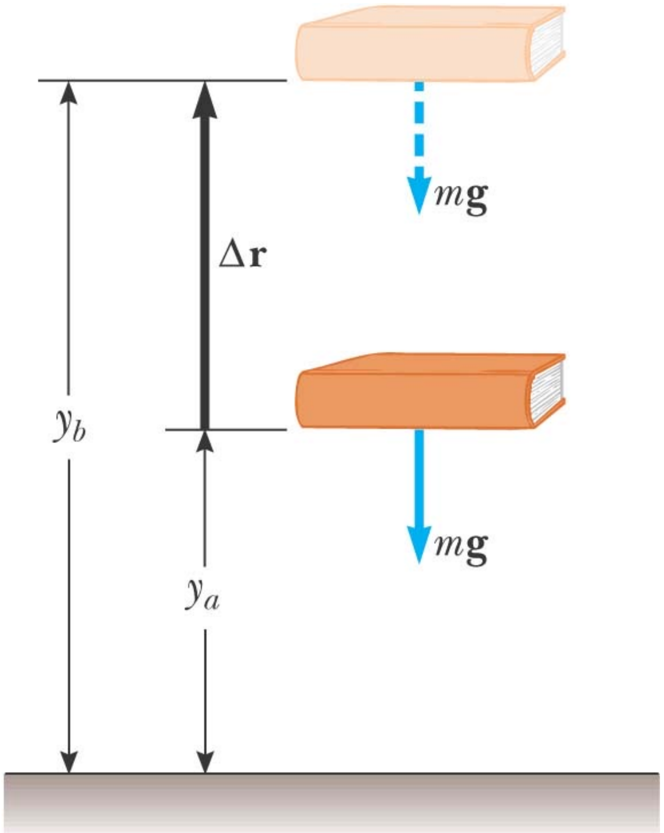
- There are many forms of potential energy, including:
  - Gravitational
  - Electromagnetic
  - Chemical
  - Nuclear
- One form of energy in a system can be converted into another

Systems with Multiple Particles

- We can extend our definition of a system to include multiple objects
- The force can be internal to the system
- The kinetic energy of the system is the algebraic sum of the kinetic energies of the individual objects
  - Sometimes, the kinetic energy of one of the objects may be negligible

System Example

- This system consists of Earth and a book
- Do work on the system by lifting the book through Δy
- The work done is $mgy_b - mgy_a$



Potential Energy

- The energy storage mechanism is called *potential energy*
- A potential energy can only be associated with specific types of forces
- Potential energy is always associated with a system of two or more interacting objects

Gravitational Potential Energy

- Gravitational Potential Energy is associated with an object at a given distance above Earth's surface
- Assume the object is in equilibrium and moving at constant velocity
- The work done on the object is done by $\mathbf{F}_{app}$ and the upward displacement is

$$\Delta \mathbf{r} = \Delta y \hat{\mathbf{j}}$$

Gravitational Potential Energy, cont

- $$W = \left(\mathbf{F}_{\text{app}}\right) \cdot \Delta\mathbf{r}$$
  $$W = (mg\hat{\mathbf{j}}) \cdot \left[\left(y_b - y_a\right)\hat{\mathbf{j}}\right]$$
  $$W = mgy_b - mgy_a$$
- The quantity *mgy* is identified as the gravitational potential energy, U_g
  - $U_g = mgy$
- Units are joules (J)

Gravitational Potential Energy, final

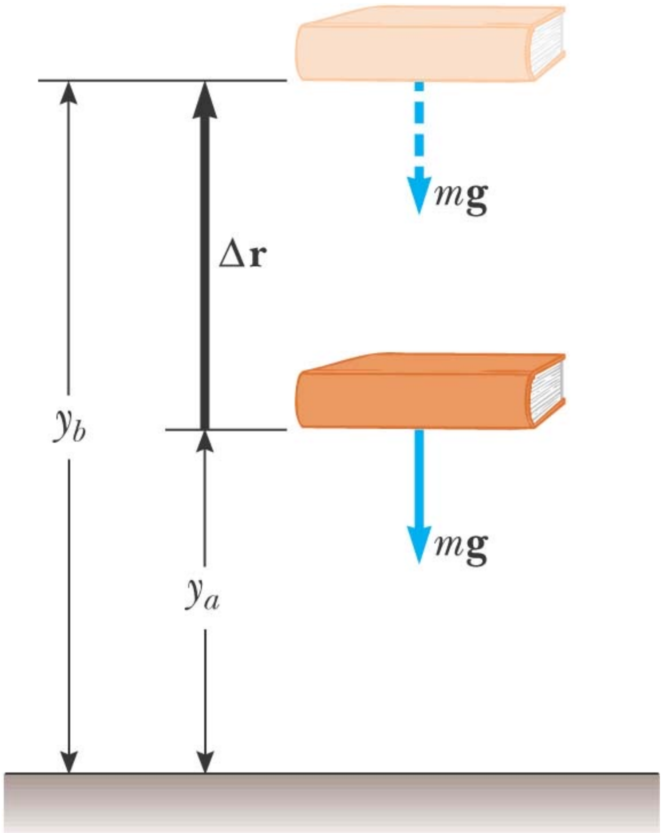
- The gravitational potential energy depends only on the vertical height of the object above Earth's surface
- In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero
  - The choice is arbitrary because you normally need the *difference* in potential energy, which is independent of the choice of reference configuration

Conservation of Mechanical Energy

- The mechanical energy of a system is the algebraic sum of the kinetic and potential energies in the system
  - $E_{\text{mech}} = K + U_g$
- The statement of Conservation of Mechanical Energy for an isolated system is $K_f + U_f = K_i + U_i$
  - An isolated system is one for which there are no energy transfers across the boundary

Conservation of Mechanical Energy, example

- Look at the work done by the book as it falls from some height to a lower height
- $W_{\text{on book}} = \Delta K_{\text{book}}$
- Also, $W = mgy_b - mgy_a$
- So, $\Delta K = -\Delta U_g$

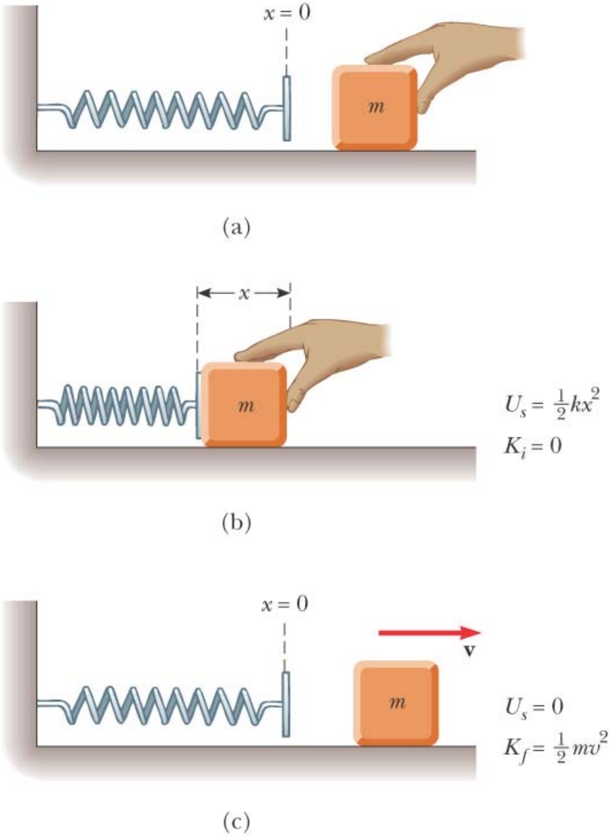


Elastic Potential Energy

- ***Elastic Potential Energy*** is associated with a spring
- The force the spring exerts (on a block, for example) is $F_s = -kx$
- The work done by an external applied force on a spring-block system is
  - $W = ½ kx_f^2 – ½ kx_i^2$
  - The work is equal to the difference between the initial and final values of an expression related to the configuration of the system

Elastic Potential Energy, cont

- This expression is the elastic potential energy: $U_s = \frac{1}{2} kx^2$
- The elastic potential energy can be thought of as the energy stored in the deformed spring
- The stored potential energy can be converted into kinetic energy



Elastic Potential Energy, final

- The elastic potential energy stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$)
  - The energy is stored in the spring only when the spring is stretched or compressed
- The elastic potential energy is a maximum when the spring has reached its maximum extension or compression
- The elastic potential energy is always positive
  - x^2 will always be positive

Problem Solving Strategy – Conservation of Mechanical Energy

- Define the isolated system and the initial and final configuration of the system
  - The system may include two or more interacting particles
  - The system may also include springs or other structures in which elastic potential energy can be stored
  - Also include all components of the system that exert forces on each other

Problem-Solving Strategy, 2

- Identify the configuration for zero potential energy
  - Include both gravitational and elastic potential energies
  - If more than one force is acting within the system, write an expression for the potential energy associated with each force

Problem-Solving Strategy, 3

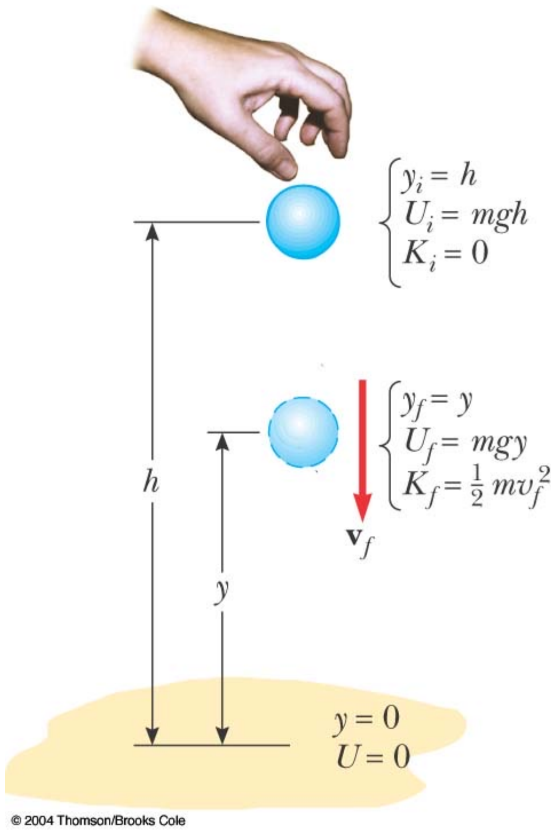
- If friction or air resistance is present, mechanical energy of the system is not conserved
- Use energy with non-conservative forces instead

Problem-Solving Strategy, 4

- If the mechanical energy of the system is conserved, write the total energy as
  - $E_i = K_i + U_i$ for the initial configuration
  - $E_f = K_f + U_f$ for the final configuration
- Since mechanical energy is conserved, $E_i = E_f$ and you can solve for the unknown quantity

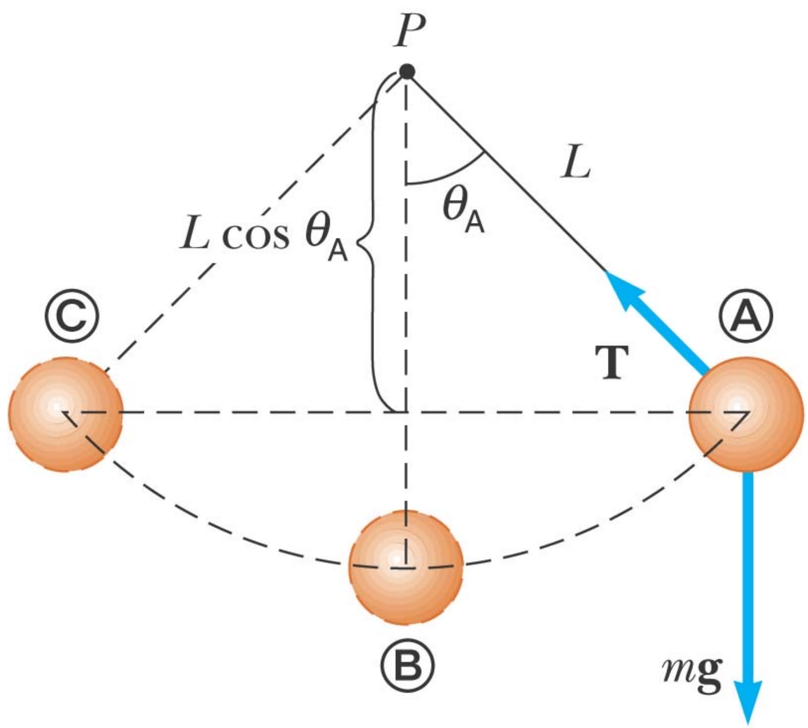
Conservation of Energy, Example 1 (Drop a Ball)

- Initial conditions:
  - $E_i = K_i + U_i = mgh$
  - The ball is dropped, so $K_i = 0$
- The configuration for zero potential energy is the ground
- Conservation rules applied at some point y above the ground gives
  - $\frac{1}{2} mv_f^2 + mgy = mgh$



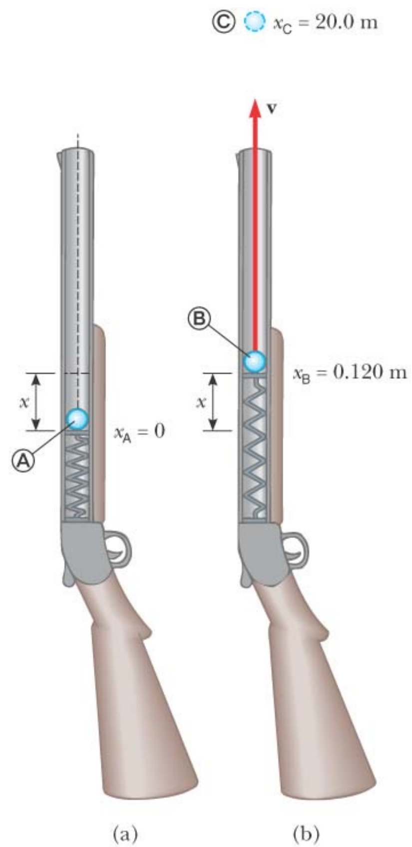
Conservation of Energy, Example 2 (Pendulum)

- As the pendulum swings, there is a continuous change between potential and kinetic energies
- At A, the energy is potential
- At B, all of the potential energy at A is transformed into kinetic energy
  - Let zero potential energy be at B
- At C, the kinetic energy has been transformed back into potential energy



Conservation of Energy, Example 3 (Spring Gun)

- Choose point A as the initial point and C as the final point
- $E_A = E_C$
  - $K_A + U_{gA} + U_{sA} = K_A + U_{gA} + U_{sA}$
  - $\frac{1}{2} kx^2 = mgh$



Conservative Forces

- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle
- The work done by a conservative force on a particle moving through any closed path is zero
  - A closed path is one in which the beginning and ending points are the same

Conservative Forces, cont

- Examples of conservative forces:
  - Gravity
  - Spring force
- We can associate a potential energy for a system with any conservative force acting between members of the system
  - This can be done only for conservative forces
  - In general: $W_C = -\Delta U$

Nonconservative Forces

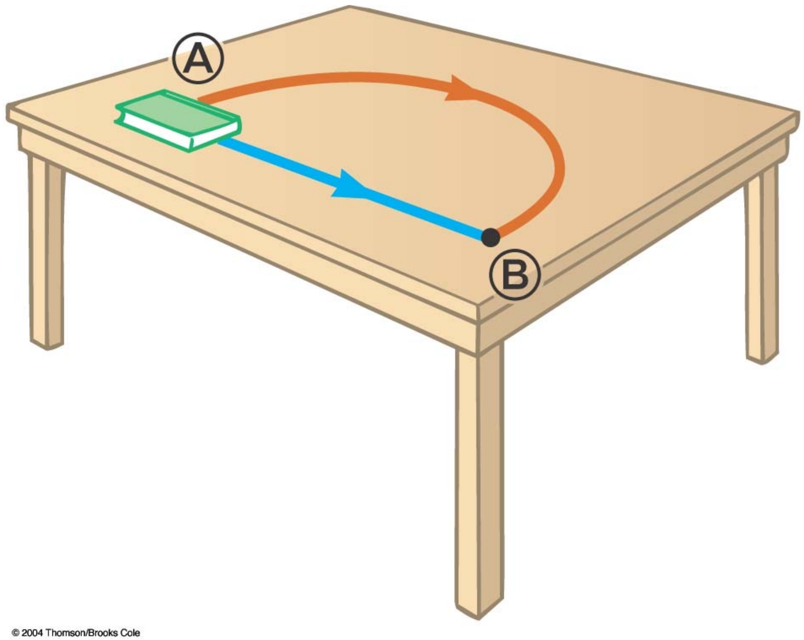
- A nonconservative force does not satisfy the conditions of conservative forces
- Nonconservative forces acting in a system cause a *change* in the mechanical energy of the system

Mechanical Energy and Nonconservative Forces

- In general, if friction is acting in a system:
  - $\Delta E_{mech} = \Delta K + \Delta U = -f_k d$
  - ΔU is the change in all forms of potential energy
  - If friction is zero, this equation becomes the same as Conservation of Mechanical Energy

Nonconservative Forces, cont

- The work done against friction is greater along the red path than along the blue path
- Because the work done depends on the path, friction is a nonconservative force



Problem Solving Strategies – Nonconservative Forces

- Define the isolated system and the initial and final configuration of the system
- Identify the configuration for zero potential energy
  - These are the same as for Conservation of Energy
- The difference between the final and initial energies is the change in mechanical energy due to friction

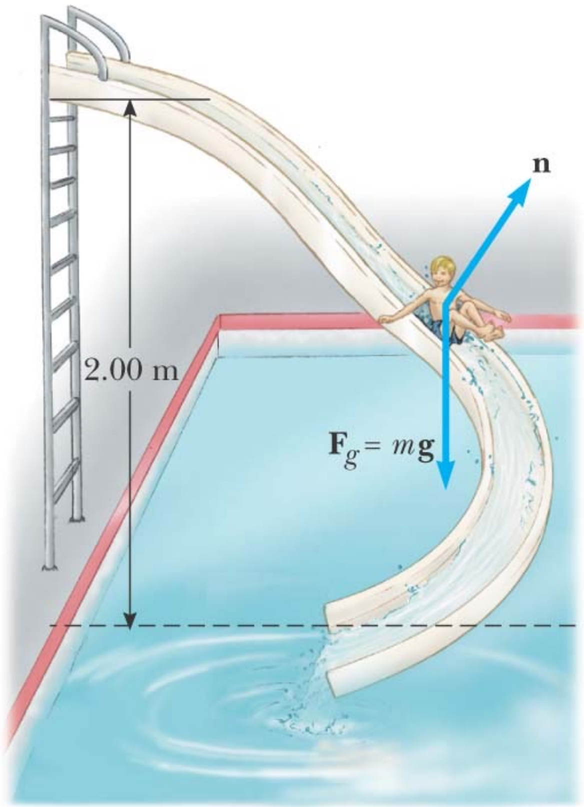
Nonconservative Forces, Example 1 (Slide)

$$\Delta E_{mech} = \Delta K + \Delta U$$

$$\Delta E_{mech} = (K_f - K_i) + (U_f - U_i)$$

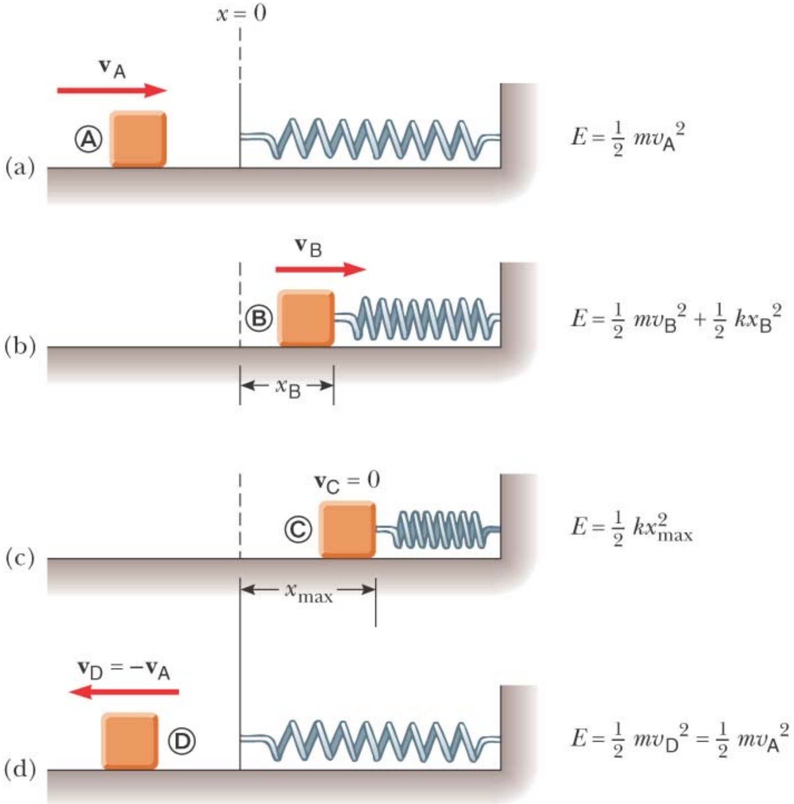
$$\Delta E_{mech} = (K_f + U_f) - (K_i + U_i)$$

$$\Delta E_{mech} = \frac{1}{2} m v_f^2 - mgh = -f_k d$$



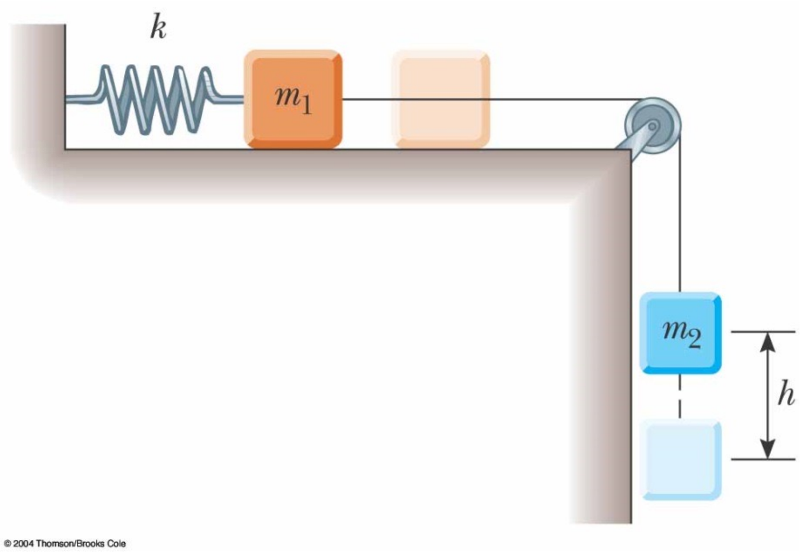
Nonconservative Forces, Example 2 (Spring-Mass)

- Without friction, the energy continues to be transformed between kinetic and elastic potential energies and the total energy remains the same
- If friction is present, the energy decreases
  - $\Delta E_{mech} = -f_k d$



Nonconservative Forces, Example 3 (Connected Blocks)

- The system consists of the two blocks, the spring, and Earth
- Gravitational and potential energies are involved
- The kinetic energy is zero if our initial and final configurations are at rest



Connected Blocks, cont

- Block 2 undergoes a change in gravitational potential energy
- The spring undergoes a change in elastic potential energy
- The coefficient of kinetic energy can be measured

Conservative Forces and Potential Energy

- Define a potential energy function, *U*, such that the work done by a conservative force equals the decrease in the potential energy of the system
- The work done by such a force, *F*, is

$$W_C = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

  - ΔU is negative when *F* and *x* are in the same direction

Conservative Forces and Potential Energy

- The conservative force is related to the potential energy function through

$$F_x = -\frac{dU}{dx}$$

- The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x*

Conservative Forces and Potential Energy – Check

- Look at the case of a deformed spring

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

  - This is Hooke's Law