

Energy and Energy Transfer

#### Introduction to Energy

- The concept of energy is one of the most important topics in science
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations
- Energy is not easily defined

#### **Energy Approach to Problems**

- The energy approach to describing motion is particularly useful when the force is not constant
- An approach will involve Conservation of Energy
  - This could be extended to biological organisms, technological systems and engineering situations

#### Systems

#### A system is a small portion of the Universe

- We will ignore the details of the rest of the Universe
- A critical skill is to identify the system

#### Valid System

- A valid system may
  - be a single object or particle
  - be a collection of objects or particles
  - be a region of space
  - vary in size and shape

#### **Problem Solving**

- Does the problem require the system approach?
  - What is the particular system and what is its nature?
- Can the problem be solved by the particle approach?
  - The particle approach is what we have been using to this time

#### Environment

- There is a system boundary around the system
  - The boundary is an imaginary surface
  - It does not necessarily correspond to a physical boundary
- The boundary divides the system from the *environment* 
  - The environment is the rest of the Universe

#### Work

The work, W, done on a system by an agent exerting a constant force on the system is the product of the magnitude, *F*, of the force, the magnitude  $\Delta r$  of the displacement of the point of application of the force, and  $\cos \theta$ , where  $\theta$  is the angle between the force and the displacement vectors



- $W = F \Delta r \cos \theta$ 
  - The displacement is that of the point of application of the force
  - A force does no work on the object if the force does not move through a displacement
  - The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application

#### Work Example

The normal force, n, and the gravitational force, mg, do no work on the object

•  $\cos \theta = \cos 90^\circ = 0$ 

The force F does do work on the object



#### More About Work

- The system and the environment must be determined when dealing with work
  - The environment does work on the system
    - Work by the environment on the system
- The sign of the work depends on the direction of F relative to ∆r
  - Work is positive when projection of F onto △r is in the same direction as the displacement
  - Work is negative when the projection is in the opposite direction

#### Units of Work

- Work is a scalar quantity
- The unit of work is a joule (J)
  - 1 joule = 1 newton · 1 meter

#### Work Is An Energy Transfer

- This is important for a system approach to solving a problem
- If the work is done on a system and it is positive, energy is transferred to the system
- If the work done on the system is negative, energy is transferred from the system

#### Work Is An Energy Transfer, cont

 If a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary

This will result in a change in the amount of energy stored in the system

#### Scalar Product of Two Vectors

- The scalar product of two vectors is written as A · B
  - It is also called the dot product
- $\mathbf{A} \cdot \mathbf{B} = A B \cos \theta$ 
  - $\theta$  is the angle between A and B



#### Scalar Product, cont

- The scalar product is commutative
  A · B = B · A
- The scalar product obeys the distributive law of multiplication
- $\bullet \mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$

# **Dot Products of Unit Vectors**

• 
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$
  
 $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$ 

Using component form with A and B:

$$A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$
  

$$B = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$
  

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

#### Work Done by a Varying Force

- Assume that during a very small displacement, ∆x, F is constant
- For that displacement,  $W \sim F \Delta x$
- For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



#### Work Done by a Varying Force, cont

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$
  
Therefore,  $W = \int_{x_i}^{x_f} F_x dx$ 

 The work done is equal to the area under the curve

Xc



#### Work Done By Multiple Forces

If more than one force acts on a system and the system can be modeled as a particle, the total work done on the system is the work done by the net force

$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

# Work Done by Multiple Forces, cont.

If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces

 $W_{\rm net} = \sum W_{\rm by individual forces}$ 



The force exerted by the spring is

$$F_s = -kx$$

- *x* is the position of the block with respect to the equilibrium position (x = 0)
- k is called the spring constant or force constant and measures the stiffness of the spring
- This is called Hooke's Law

#### Hooke's Law, cont.

- When x is positive (spring is stretched), F is negative
- When x is 0 (at the equilibrium position), F is 0
- When x is negative (spring is compressed), F is positive



#### Hooke's Law, final

- The force exerted by the spring is always directed opposite to the displacement from equilibrium
- F is called the *restoring force*
- If the block is released it will oscillate back and forth between -x and x

Work Done by a Spring

 $F_{\rm c}$ 

 $F_s = -kx$ 

Area =  $\frac{1}{2}kx_{\text{max}}^2$ 

 $x_{\rm max}$ 

(d)

 $kx_{\max}$ 

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- Identify the block as the system
- Calculate the work as the block moves from x<sub>i</sub>
  - $= x_{\max} \text{ to } x_f = 0$

$$W_{s} = \int_{x_{i}}^{x_{f}} F_{x} dx = \int_{-x_{\max}}^{0} \left(-kx\right) dx = \frac{1}{2} kx_{\max}^{2}$$

 The total work done as the block moves from

 $-x_{max}$  to  $x_{max}$  is zero

#### Spring with an Applied Force

- Suppose an external agent, *F*<sub>app</sub>, stretches the spring
- The applied force is equal and opposite to the spring force

$$F_{app} = -F_s = -(-kx) = kx$$

• Work done by  $F_{app}$  is equal to  $\frac{1}{2} kx^2_{max}$ 



#### Kinetic Energy

Kinetic Energy is the energy of a particle due to its motion

•  $K = \frac{1}{2} m v^2$ 

- *K* is the kinetic energy
- *m* is the mass of the particle
- *v* is the speed of the particle
- A change in kinetic energy is one possible result of doing work to transfer energy into a system

#### Kinetic Energy, cont

Calculating the work:

$$W = \int_{x_i}^{x_f} \sum F \, dx = \int_{x_i}^{x_f} ma \, dx$$
$$W = \int_{v_i}^{v_f} mv \, dv$$
$$\sum W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$



#### Work-Kinetic Energy Theorem

- The Work-Kinetic Energy Principle states  $\Sigma W$ =  $K_f - K_i = \Delta K$
- In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.
- We can also define the kinetic energy

•  $K = \frac{1}{2} m v^2$ 

#### Work-Kinetic Energy Theorem – Example

 The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement

•  $W = F \Delta x$ 

•  $W = \Delta K = \frac{1}{2} m v_f^2 - 0$ 



#### Nonisolated System

- A nonisolated system is one that interacts with or is influenced by its environment
  - An *isolated system* would not interact with its environment
- The Work-Kinetic Energy Theorem can be applied to nonisolated systems

#### **Internal Energy**

- The energy associated with an object's temperature is called its *internal energy*, E<sub>int</sub>
- In this example, the surface is the system
- The friction does work and increases the internal energy of the surface



### **Potential Energy**

- Potential energy is energy related to the configuration of a system in which the components of the system interact by forces
- Examples include:
  - elastic potential energy stored in a spring
  - gravitational potential energy
  - electrical potential energy

Ways to Transfer Energy Into or Out of A System

- Work transfers by applying a force and causing a displacement of the point of application of the force
- Mechanical Waves allow a disturbance to propagate through a medium
- Heat is driven by a temperature difference between two regions in space

More Ways to Transfer Energy Into or Out of A System

- Matter Transfer matter physically crosses the boundary of the system, carrying energy with it
- *Electrical Transmission* transfer is by electric current
- Electromagnetic Radiation energy is transferred by electromagnetic waves

## Examples of Ways to Transfer Energy

a) Work

- b) Mechanical Waves
- c) Heat











(c)

#### Examples of Ways to Transfer Energy, cont.

- d) Matter transfer
- e) Electrical
   Transmission
- f) Electromagnetic radiation







#### **Conservation of Energy**

#### Energy is conserved

- This means that energy cannot be created or destroyed
- If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer

#### Conservation of Energy, cont.

- Mathematically,  $\Sigma E_{system} = \Sigma T$ 
  - *E*<sub>system</sub> is the total energy of the system
  - T is the energy transferred across the system boundary
    - Established symbols:  $T_{work} = W$  and  $T_{heat} = Q$
- The Work-Kinetic Energy theorem is a special case of Conservation of Energy

#### Power

- The time rate of energy transfer is called *power*
- The average power is given by

$$\overline{P} = \frac{W}{\Delta t}$$

when the method of energy transfer is work

#### **Instantaneous** Power

# The *instantaneous power* is the limiting value of the average power as ∆t approaches zero

$$P =_{\Delta t \to 0}^{\lim} \frac{W}{\Delta t} = \frac{dW}{dt}$$

This can also be written as

$$P = \frac{dW}{dt} = F \cdot \frac{dr}{dt} = F \cdot v$$

#### **Power Generalized**

- Power can be related to any type of energy transfer
- In general, power can be expressed as

$$P = \frac{dE}{dt}$$

*dE*/ *dt* is the rate rate at which energy is crossing the boundary of the system for a given transfer mechanism

#### **Units of Power**

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg  $\cdot$  m<sup>2</sup> / s<sup>2</sup>
- A unit of power in the US Customary system is horsepower

■ 1 hp = 746 W

- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 x10<sup>6</sup> J

#### Energy and the Automobile

- The concepts of energy, power, and friction help to analyze automobile fuel consumption
- About 67% of the energy available from the fuel is lost in the engine
- About 10% is lost due to friction in the transmission, drive shaft, bearings, etc.
  - About 6% goes to internal energy and 4% to operate the fuel and oil pumps and accessories
- This leaves about 13% to actually propel the car

#### Friction in a Car

The magnitude of the total friction force is the sum of the rolling friction force and air resistance

•  $f_t = f_r + f_a$ 

- At low speeds, rolling friction predominates
- At high speeds, air drag predominates

#### Friction in a Car, cont

#### Table 7.2

Friction Forces and Power Requirements for a Typical Car<sup>a</sup>

v(mi/h)	v(m/s)	$n(\mathbf{N})$	$f_r(\mathbf{N})$	$f_a(\mathbf{N})$	$f_t(\mathbf{N})$	$\mathcal{P} = f_t v(\mathbf{kW})$
0	0	14 200	227	0	227	0
20	8.9	$14\ 100$	226	48	274	2.4
40	17.9	$13\ 900$	222	192	414	7.4
60	26.8	13 600	218	431	649	17.4
80	35.8	13 200	211	767	978	35.0
100	44.7	12 600	202	1 199	1 400	62.6

<sup>a</sup> In this table, *n* is the normal force,  $f_r$  is rolling friction,  $f_a$  is air friction,  $f_t$  is total friction, and  $\mathcal{P}$  is the power delivered to the wheels.