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| --- | --- |
| Student Name:  **Old Mid Term 02** | Mid Term 02 |
| Student Roll Number:  Max. Time: 60min | **Date:**  **Max. Marks: 20** |

**Instructions:**

* Attempt all questions.
* Write in the spaces provided (you can use the back side of the page as well).
* No extra time will be given.

**Question No. 1 (Fast Fourier Transform)**

**[4+1 marks]**

Given a sequence for, where , and,

1. Determine its DFT using decimation-in-frequency FFT method ?
2. Determine the number of complex multiplication in doing part (a).

**Solution:**

1. According to the FFT (decimation-in-frequency method) for calculating DFT of a give sequence

where, and

|  |
| --- |
|  |
|  |

The DFT of the given sequence, using FFT (decimation-in-frequency method) is given in the following diagram:

Therefore, the DFT coefficients are , and.

1. Number of complex multiplications in .

**Question No. 2 (z-Transform using Table of z-Transform Pairs) [5 marks]**

Using Table 1 (given at the end of the paper) find out the z-transform for the following sequences:

**Solution:**

1. From Line 3 in Table 1, we get
2. From Line 9 in Table 1, we obtain
3. From Line 6 in Table 1, we get
4. From Line 11 in Table 1, we get
5. From Line 14 in Table 1, we get

**Question No. 3 (inverse z-Transform using partial fractions) [5 marks]**

Find the inverse of the following z-transform

**Solution:**

Multiplying the numerator and the denominator by we get

Dividing both sides by , we have

We notice that the right hand side of the above equation is a proper rational polynomial of . Also we notice that the denominator of the right hand side has distinct poles, therefore, we right into partial fractions as,

To find out the unknown constants and, we use:

Substituting the values, we have,

Or it can be written as (by multiplying both sides by )

Taking the inverse z-transform of both sides and using Table 1, we have

**Question No. 4 (Difference Equations Solution using z-Transform) [5 marks]**

A relaxed (zero initial conditions) DSP system is described by the difference equation

Determine the impulse response due to the impulse sequence.

**Solution:**

Taking z-transform of both sides of the given equation, we get

|  |  |
| --- | --- |
|  | (1) |

We have

Using shift theorem, we have

Also we can apply sift theorem for in case of zero initial conditions, i.e.,

Putting these values in Equation (1), we have

|  |  |
| --- | --- |
|  |  |
|  |  |

As therefore, (from Table 1), The above equation can now be written as

|  |  |
| --- | --- |
|  |  |

Multiplying both the numerator and the denominator with we get

|  |  |
| --- | --- |
|  |  |

The denominator can be factorized as

|  |  |
| --- | --- |
|  |  |
|  | (2) |

The right hand side of the above equation is a proper rational polynomial, with the denominator polynomial having distinct poles, therefore, it can be written into partial fractions as

|  |  |
| --- | --- |
|  | (3) |

To find out the unknown constants and, we use:

Equation (3) becomes:

|  |  |
| --- | --- |
|  |  |
|  |  |

Taking inverse z-transform of both sides

|  |  |
| --- | --- |
|  |  |

Thus the output signal is

**Table 5.1 Table of z-transform pairs (for causal sequences)**

|  |  |  |  |
| --- | --- | --- | --- |
| Line No. | Signal | z-Transform | Region of Convergence |
| 1 |  |  |  |
| 2 |  | **1** | **Entire z-plane** |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| 13 |  |  |  |
| 14 |  |  |  |
| 15 | **where and are complex constants defined by**  **,** |  |  |

**Shift Theorem:**