**Question 2 :** [5pts]
Find the values of $m$ for which the following linear system

$$\begin{cases} x + my + 2z & = & 3 \\ 4x + (6+m)y - mz & = & 13 - m \\ x + 2(m-1)y + (m+4)z & = & m+2 \end{cases}$$

a) has a unique solution.
b) has infinite solutions.
c) has no solution.

**Question 2** [4+3 marks]**:**

a) Solve of the linear system of equations with augmented matrix:

$$[A:B]=\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 1 \\ -1 & 2 & 0 & -2 & 2 \\ 3 & 1 & 0 & 3 & -1 \end{array}\right].$$

b) Solve the following linear system of equations by Cramer's Rule:

$$\begin{aligned} x - y \quad\quad &= 1 \\ -2x + 3y - 4z &= 0 \\ -2x + 3y - 3z &= 1 \end{aligned}$$

Question 1: [3+2+2 Marks]

$(a)$ Let $\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 + \alpha x_3 &= 3 \\ x_1 + \alpha x_2 + 3x_3 &= 2 \end{aligned}$ be a given system of linear equations.

For what values of $\alpha$ does the system have

$(i)$ a unique solution $(ii)$ infinitely many solutions $(iii)$ no solution?

**Question 2 :** [8pts]

1. Given the linear system $\begin{cases} x - y + 3z + 2t = a \\ -x + 8z + 3t = a \\ -2x + y + 5z + t = b \\ 3x - 2y - 2z + t = c \end{cases}$

   Find the conditions on $a, b, c$ such that the system is consistent.

2. Given the linear system: $\begin{cases} ax + by - 3z = -3 \\ -2x - by + cz = -1 \\ ax + 3y - cz = -3 \end{cases}$

   (i) Find the values of $a, b$, and $c$ so that the system has the solution $x = 1$, $y = -1$, and $z = 2$.

   (ii) Solve the system for the values of $a, b, c$ found in (i).

2. Find the conditions on $a, b$ such that the following linear system is consistent

$$\begin{cases} x + 2y + z + 3t = a \\ 2x + y + 3z + 2t = b \\ -x + 7y - 4z + 9t = 1 \end{cases}$$

b) **Solve** the following *system of linear equations* **by using the Cramer's rule**:

$$\begin{aligned} x - y + z &= 0 \\ x + y + z &= 2 \\ x + 2y + 4z &= 3. \end{aligned}$$

**Solution:** $|A| = 6$, $|A_x| = 6$, $|A_y| = 6$ and $|A_z| = 0$. Hence, $x = \frac{|A_x|}{|A|} = \frac{6}{6} = 1$, $y = \frac{|A_y|}{|A|} = \frac{6}{6} = 1$ and $z = \frac{|A_z|}{|A|} = \frac{0}{6} = 0$.

c) **Use any of the elimination methods to show that the following** *system of linear equations* is *inconsistent*:

$$-x + 2y - 5z = 3$$

$$x - 3y + z = 4$$

$$5x - 13y + 13z = 8.$$

**Solution**: Since $\left[\begin{array}{ccc|c} -1 & 2 & -5 & 3 \\ 1 & -3 & 1 & 4 \\ 5 & -13 & 13 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 0 & 2 \end{array}\right]$, the given linear system is inconsistent.

**Question 5** [Marks: 2]**:**

By using the Cramer's rule, find values of $x$ and $y$ in solution of the following linear system:

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 5y + 3z &= 2 \\ x \qquad + 8z &= 0. \end{aligned}$$

**Answer:**

**Question 6** [Marks: 2]:

Find condition on $a$, $b$ and $c$ for which the following linear system has infinitely many solutions:

$$\begin{aligned} x - 2y + 5z &= a \\ 4x - 5y + 8z &= b \\ -3x + 3y - 3z &= c \end{aligned}$$

**Answer:**

Augmented matrix $\left[\begin{array}{ccc|c} 1 & -2 & 5 & a \\ 4 & -5 & 8 & b \\ -3 & 3 & -3 & c \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 5 & a \\ 0 & 3 & -12 & b-4a \\ 0 & -3 & 12 & c+3a \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 5 & a \\ 0 & 3 & -12 & b-4a \\ 0 & 0 & 0 & b+c-a \end{array}\right]$

The System has infinitely many Solutions if and only if

$$\boxed{b+c-a=0}$$

**Question 2:** [Marks: 3+4+3]:

a) **Explain! Why** the following system of linear equations has no non-trivial solution?

$$\begin{aligned} x - \ y + z &= 0 \\ x + \ y + z &= 0 \\ 4x + 2y + z &= 0. \end{aligned}$$

**Solution:** Since $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{vmatrix} = -6 \neq 0$, the matrix of coefficients is invertible.

Hence, the given system has the unique solution $(0, 0, 0)$; no non-trivial solution.

b) **Find** the values of $\lambda$ for which the following system of equations has a unique solution:

$$\begin{aligned} 2x_1 + 3x_2 + \quad\quad\quad x_3 &= -1 \\ x_1 + 2x_2 + \quad\quad\quad x_3 &= 0 \\ 3x_1 + \quad x_2 + (\lambda^2 - 6)\, x_3 &= \lambda - 3. \end{aligned}$$

**Solution:** Augmented matrix $\left[\begin{array}{ccc|c} 2 & 3 & 1 & -1 \\ 1 & 2 & 1 & 0 \\ 3 & 1 & \lambda^2-6 & \lambda-3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \lambda^2-4 & \lambda-2 \end{array}\right].$

Hence, the given system has a unique solution for all $\lambda \in \mathbb{R} - \{\mathbf{-2}, \mathbf{2}\}$.