Question 1 : [7pts]

1. Let A, B, C and D be matrices of order 3 such that $AB + AC - D = 0$, $|D| = 6$, $B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ -1 & 1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & -1 & 3 \end{pmatrix}$.
   Find $|A|$.
2. Let R and S be matrices of order 3 such that $RS + R - 2I = 0$.
   Find R^{-1} if $S = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 2 & 5 \end{pmatrix}$.

Question 1 : [6 pts]

a) Let A be a matrix of order 3 such that $|A| = 3$ and $|A^2 + I| = 2$.
Find $|A + A^{-1}|$.

a) If $\boldsymbol{A^{-1}} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$, then find $adj(adj(A))$.

b) Find the values of k that makes the matrix $\begin{bmatrix} \mathbf{2} & \mathbf{3k-2} \\ \mathbf{k^2} & \mathbf{-1} \end{bmatrix}$ symmetric.

c) Let $\boldsymbol{B} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -5 & 4 \\ 0 & 0 & 6 \end{bmatrix}$. Explain! Why the matrix $\boldsymbol{B}$ can be expressed as a product of elementary matrices?

(c) Find $\left|3(adjA)^{-1}+A\right|$ where A is a matrix of size 4×4 such that $|A|=3$.

1. Let A, B be matrices of size $(3,3)$ such that A is not invertible and $|B| = 2$. Find $\left|A\text{adj}(A) + 2B^{-1}\right|$.

2. Compute the following determinant $\begin{vmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix}$.

3. Compute the inverse matrix of the matrix A, where $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$.

1. Consider the matrices A and B such that $A=\begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ and $AB = A + 2I_3$.

   Find the matrices A^{-1} and B.

2. Consider the matrices $C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ and $D = \begin{pmatrix} -2 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$.

   If E is a 4×4 matrix such that $EC^2 + ED = 2I_4$, then find E^{-1}.

1. a) Find the matrix $\mathrm{adj}(A)$ if $A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ -2 & 4 & -2 \end{pmatrix}$.

   b) Find $\mathrm{adj}(A).A$.

a) **Find** the *values* of λ for which the *matrix* $\begin{bmatrix} 1 & 0 & \lambda \\ 2 & 1 & 2+\lambda \\ 2 & 3 & \lambda^2 \end{bmatrix}$ is *invertible*.

b) **By using properties of the determinants**, **show** that:

$$\begin{vmatrix} a+b+c & b & a \\ d+e+f & e & d \\ g+h+i & h & g \end{vmatrix} = \begin{vmatrix} c & b & a \\ f & e & d \\ i & h & g \end{vmatrix}.$$

c) **Let** $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$. **Find** *adj*$(A)$ and A^{-1}.