2nd Assignment (April 2020)

استعن بالله وكن على يقين بأن كل ما ورد في هذه الورقة تعرفه جيدا وقد تدربت عليه بما فيه الكفاية

Instruction and guides for the assignment:

1. This assignment is designed to guide you to understand fully the topics and practice covered in the 1st month of the course.
2. To give you plenty of time to review and apply the materials for the answer, the assignment duration is from **4:00pm** Thursday April 9,2020 until **12:00 noon Friday Apr. 10**
3. You can use the lecture notes, the text book, Excel for your answer.
4. Make sure to indicate the source of the information in you answer.
5. You are the guardian of your behavior in this assignment. This assignment is totally for your independent effort. **Do not attempt** to collaboration or communication with anyone about the questions of the assignment, it is totally not allowed by any means.
6. Write your answers on a word document and email the document on PDF and WORD format. Write the subject of the email as:

   OPER-441-Assignment#1 <<Section Number>> , << your name>> , <<your KSU ID >>
7. Put all you of answers in one document. If you have part of the answers on excel, capture the answer from the screen and insert it in the document.
8. Make your answers as **comprehensive** in information as much as you can, write all information related to the answers. There will be **Extra Marks** for that>
9. Make sure to make your document as **organized** as possible, there will be **Extra Marks** for the organization.

وفقكم الله ويسر لكم .. وحفظكم ورعاكم

Question #1:

A traffic department of a small city wants to simulate the occurrence time for the accident in an intersection using some different choices of distributions to estimate the number of accidents per month (30 days). Use the following U(0,1) numbers to compute the simulation under the following assumptions.

1. Estimate the number of accidents in the 1st month assuming the time between accident is an exponential distribution with mean 5 days.
2. Estimate the number of accidents in the 1st month assuming The time between accident is an Erlang distribution with parameter r = 3 and λ = 0.2 accident/day
3. Estimate the number of accidents in the 1st month assuming The time between accident is shifted Binomial distribution with minimum value 3 and parameters n = 5 and p = 0.6.
4. Estimate the number of accidents in the 1st month assuming that the time between accident is taken as in part (**1**) with chance 40% or as in part (**2**) with chance 60%.

Use uniform numbers (as needed) by columns until you finish all numbers in the column then move to the next.

↓ 0.420	↓ 0.245	↓ 0.299	↓ 0.464	↓ 0.534	↓ 0.141	↓ 0.293	↓ 0.689	↓ 0.349
0.737	0.454	0.516	0.922	0.405	0.965	0.686	0.623	0.327
0.293	0.046	0.239	0.356	0.686	0.577	0.234	0.439	0.588
0.136	0.024	0.034	0.134	0.534	0.648	0.244	0.525	0.340
0.848	0.162	0.032	0.224	0.209	0.441	0.493	0.850	0.607
0.692	0.359	0.946	0.607	0.420	0.058	0.197	0.336	0.353
0.727	0.908	0.385	0.181	0.683	0.067	0.856	0.736	0.328
0.116	0.287	0.537	0.196	0.087	0.297	0.772	0.564	0.633
0.074	0.980	0.383	0.485	0.909	0.061	0.201	0.356	0.361
0.262	0.253	0.671	0.545	0.765	0.651	0.030	0.839	0.546
0.385	0.160	0.498	0.090	0.432	0.187	0.588	0.248	0.954
0.317	0.815	0.328	0.604	0.038	0.668	0.818	0.296	0.995
0.923	0.500	0.336	0.742	0.122	0.464	0.612	0.502	0.063
0.057	0.872	0.600	0.181	0.654	0.211	0.889	0.320	0.166
0.441	0.993	0.965	0.536	0.816	0.794	0.193	0.876	0.924
0.528	0.393	0.173	0.405	0.013	0.666	0.297	0.056	0.809
0.478	0.457	0.183	0.660	0.106	0.977	0.718	0.547	0.045
0.203	0.602	0.125	0.486	0.879	0.871	0.891	0.167	0.066
0.553	0.430	0.227	0.172	0.507	0.103	0.362	0.029	0.584
0.409	0.010	0.694	0.017	0.068	0.420	0.742	0.653	0.021
0.454	0.550	0.640	0.390	0.432	0.714	0.259	0.799	0.207
0.995	0.685	0.975	0.992	0.896	0.237	0.929	0.270	0.524
0.400	0.419	0.955	0.286	0.108	0.183	0.466	0.721	0.793
0.922	0.909	0.257	0.684	0.131	0.321	0.868	0.428	0.160

Question #2:

An insurance company has a customer care calling center to process their claims. Assume that claims arrive to the calling center according to a random process. The time between calls (in minutes) is assumed to follow the Geometric distribution with mean 25 minutes.

$$f(x) = (1-p)p^x \quad ; \qquad x = 0, 1, 2, \ldots$$

From past data, it is found that the claim value distributed as follows:

Claim Amount	$5000 \le X < 10000$	$10000 \le X < 15000$	$15000 \le X < 20000$	$20000 \le X$
Percentage	30%	40%	20%	10%

The exact amount within any category is assumed to be discrete uniform. The work hours for the call center is between 6 am to 6 pm every day.

1. Write the algorithm for simulating the arrival of a call for a claim.
2. Write the algorithm for simulating the amount of the claim
3. Simulate the call arrival time and the claim amount for 5 days using the following uniform (0,1) numbers (*use as needed*) and your answers in (**1,2**).

Day-1			Day-2			Day-3			Day-4			Day-5		
U1	U2	U3	U1	U2	U3	U1	U2	U3	U1	U2	U3	U1	U2	U3
0.590	0.190	0.766	0.994	0.162	0.957	0.817	0.830	0.937	0.473	0.749	0.937	0.355	0.315	0.728
0.019	0.421	0.845	0.603	0.123	0.245	0.993	0.281	0.821	0.325	0.233	0.821	0.792	0.230	0.938
0.023	0.932	0.380	0.362	0.022	0.998	0.630	0.979	0.193	0.818	0.291	0.193	0.382	0.904	0.760
0.058	0.541	0.630	0.800	0.373	0.795	0.686	0.946	0.651	0.064	0.263	0.651	0.638	0.266	0.679
0.164	0.063	0.383	0.891	0.252	0.375	0.925	0.982	0.262	0.370	0.444	0.262	0.556	0.702	0.513
0.839	0.485	0.668	0.633	0.748	0.700	0.893	0.723	0.105	0.814	0.800	0.105	0.102	0.237	0.352
0.571	0.614	0.319	0.548	0.154	0.799	0.001	0.124	0.704	0.618	0.076	0.704	0.513	0.740	0.332
0.502	0.939	0.510	0.323	0.684	0.385	0.278	0.608	0.588	0.264	0.313	0.588	0.230	0.278	0.141
0.644	0.032	0.085	0.221	0.500	0.701	0.024	0.609	0.414	0.306	0.233	0.414	0.202	0.124	0.846
0.091	0.024	0.215	0.803	0.939	0.554	0.693	0.617	0.776	0.211	0.296	0.776	0.218	0.307	0.525
0.675	0.458	0.089	0.895	0.584	0.447	0.871	0.760	0.642	0.712	0.962	0.642	0.361	0.690	0.468
0.991	0.728	0.009	0.733	0.382	0.985	0.661	0.508	0.682	0.267	0.961	0.682	0.308	0.043	0.152
0.419	0.687	0.485	0.502	0.136	0.834	0.478	0.163	0.692	0.096	0.736	0.692	0.454	0.399	0.104
0.278	0.666	0.547	0.707	0.198	0.340	0.517	0.362	0.221	0.287	0.159	0.221	0.374	0.051	0.841
0.737	0.726	0.772	0.301	0.498	0.755	0.919	0.592	0.695	0.229	0.380	0.695	0.420	0.179	0.264

Question #3:

Consider a continuous variable X in days that represents the length of stay (LOS) for patients in a hospital. It is assumed that LOS (in days) is a continuous random variable following the Weibull distribution with parameters α=2 and β=4 having the following *pdf*:

$$f(x) = \frac{2}{16}\, x\, e^{-\frac{x^2}{16}} \quad ; x > 0$$

1. Find the CDF function of $f(x)$ and then find the inverse transform for generating random numbers from $f(x)$.
2. Compute the average and standard deviation of LOS, by simulating the length of stay for 50 patients using the following uniform (0,1) numbers (*use as needed*) and your answer in (**1**) to compute.

1	2	3	4	5	6	7	8	9	10
0.590	0.190	0.994	0.162	0.817	0.839	0.893	0.748	0.633	0.485
0.019	0.421	0.603	0.123	0.993	0.571	0.001	0.154	0.548	0.614
0.023	0.932	0.362	0.022	0.630	0.502	0.278	0.684	0.323	0.939
0.058	0.541	0.800	0.373	0.686	0.644	0.024	0.500	0.221	0.032
0.164	0.063	0.891	0.252	0.925	0.091	0.693	0.939	0.803	0.024

3. Now, assume that LOS for any patient is at least half a day. Find the new inverse transform for generating random numbers from $f(x)$. Then, us it to compute the length of stay for 50 patients using the following uniform (0,1) numbers (*use as needed*).

1	2	3	4	5	6	7	8	9	10
0.675	0.865	0.584	0.774	0.639	0.871	0.759	0.458	0.895	0.881
0.991	0.107	0.382	0.751	0.122	0.661	0.908	0.728	0.733	0.296
0.419	0.043	0.136	0.495	0.976	0.478	0.550	0.687	0.502	0.658
0.278	0.101	0.198	0.635	0.248	0.517	0.783	0.666	0.707	0.959
0.737	0.158	0.498	0.252	0.310	0.919	0.044	0.726	0.301	0.172

4. Assume that LOS for any patient is at least Y days. Where Y is a random variable following the Exponential distribution with mean half a day. Find the new inverse transform for generating random numbers from $f(x)$. Then, compute the average and standard deviation of LOS, by simulating the length of stay for 50 patients using the uniform (0,1) numbers in part (**2**) for X and the uniform random numbers in part (**3**) for Y.

5. Some patients are known to have a medical condition that requires them to stay a random length of stay between a minimum of 0.5 day up to 5 days maximum. Write the inverse transform for generating random numbers from $f(x)$ under this condition.
6. Compute the average LOS of patients in part (**5**), by simulating the length of stay for 50 patients using the following uniform (0,1) numbers (*use as needed*) and your answer in (**5**) to compute.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0.796	0.957	0.622	0.074	0.539	0.012	0.377	0.972	0.256	0.697
0.216	0.937	0.891	0.456	0.623	0.192	0.278	0.304	0.654	0.043
0.374	0.003	0.759	0.389	0.638	0.313	0.019	0.952	0.643	0.854
0.058	0.541	0.800	0.373	0.686	0.644	0.024	0.500	0.221	0.032
0.164	0.063	0.891	0.252	0.925	0.091	0.693	0.939	0.803	0.024

Question #4:

Consider a continuous random variable with the following pdf:

$$f(x) = \begin{cases} \frac{1}{2} & 0 \le x \le 1 \\ \frac{3}{4} - \frac{x}{4} & 1 \le x \le 3 \end{cases}$$

(a) Construct the algorithm for obtaining random numbers from $f(x)$ using the inverse transform method.

(b) Using your answer in (a), give 10 random numbers from $f(x)$ using the following uniform numbers.

1	2	3	4	5	6	7	8	9	10
0.387	0.336	0.466	0.074	0.184	0.336	0.900	0.875	0.475	0.636

(c) Assume that the variable X represents the time between arrival to a service station (in hours). The station work hours are from 8 am to 3 pm. The owner wants to determine the expected number arrivals per day using simulation output for 5 days. Using the following uniform numbers for each day, estimate the average number of arrivals per day.

	Day-1	Day-2	Day-3	Day-4	Day-5
1	0.194	0.489	0.029	0.381	0.068
2	0.790	0.300	0.800	0.353	0.256
3	0.084	0.791	0.764	0.906	0.737
4	0.111	0.918	0.553	0.579	0.327
5	0.954	0.638	0.452	0.365	0.856
6	0.723	0.890	0.744	0.760	0.434
7	0.551	0.079	0.139	0.488	0.778
8	0.919	0.926	0.281	0.799	0.930
9	0.337	0.310	0.384	0.245	0.755
10	0.831	0.086	0.100	0.088	0.589

Question #5:

Consider a continuous random variable with the following pdf:

$$f(x) = \begin{cases} \frac{x}{8} - \frac{1}{4} & 2 \le x \le 4 \\ \frac{10}{24} - \frac{x}{24} & 4 \le x \le 10 \end{cases}$$

(a) Construct the algorithm for obtaining random numbers from $f(x)$ using the acceptance–rejection method using majorizing function g(x) = max f(x).

(b) Using your answer in (a), give 10 random numbers from $f(x)$ using the following uniform numbers (as needed).

	u	v
1	0.194	0.381
2	0.790	0.353
3	0.084	0.906
4	0.111	0.579
5	0.954	0.365
6	0.723	0.760
7	0.551	0.488
8	0.919	0.799
9	0.337	0.245
10	0.831	0.088
11	0.489	0.029
12	0.300	0.800
13	0.791	0.764
14	0.918	0.553
15	0.638	0.452
16	0.890	0.744
17	0.079	0.139
18	0.926	0.281
19	0.310	0.384
20	0.086	0.100

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Question #1:

A traffic department of a small city wants to simulate the occurrence time for the accident in an intersection using some different choices of distributions to estimate the number of accidents per month (30 days). Use the following U (0,1) numbers to compute the simulation under the following assumptions.

1. Estimate the number of accidents in the $1^{st\ month}$ assuming the time between accident is an exponential distribution with mean 5 days.

   $T_n \sim \text{Exp}(1/5)$

   $F^{-1}(u)=-5\ln(1-u)$

   $AT_n=T_n+AT_{n-1}$

U	Tn	ATn
0.42	2.7236	2.723636
0.737	6.678	9.401642
0.293	1.7336	11.13527
0.136	0.7309	11.86618
0.848	9.4194	21.28555
0.692	5.8883	27.17383
0.727	6.4914	33.66525

2. Estimate the number of accidents in the $1^{st\ month}$ assuming The time between accident is an Erlang distribution with parameter r = 3 and □= 0.2 accident/day.

   $T_n \sim \text{Erlang}(3,0.2)$

   $F^{-1}(u)= -\frac{1}{2}\ln(1-u_1) + -\frac{1}{2}\ln(1-u_2) + -\frac{1}{2}\ln(1-u_3)$

   $AT_n=T_n+AT_{n-1}$

U1	U2	U3	Tn	ATn
0.116	0.074	0.262	2.519954	2.519954
0.385	0.317	0.923	17.15672	19.67667
0.057	0.441	0.528	6.955355	26.63203
0.478	0.203	0.553	8.410925	35.04295

2. Estimate the number of accidents in the $1^{st\ month}$ assuming The time between accident is shifted Binomial distribution with minimum value 3 and parameters n = 5 and p = 0.6.

Let x ~ Binomial (n=5, p=0.6)

$$F^{-1}(u)=\begin{cases}0 & 0 \le u \le 0.01024\\ 1 & 0.01024 < u \le 0.08704\\ 2 & 0.08704 < u \le 0.31744\\ 3 & 0.31744 < u \le 0.66304\\ 4 & 0.66304 < u \le 0.92224\\ 5 & 0.92224 < u \le 1\end{cases}$$

$T_n = 3+x$

$AT_n=T_n+AT_{n-1}$

U	Tn	ATn
0.409	6	6
0.454	6	12
0.995	8	20
0.4	6	26
0.922	7	33

4. Estimate the number of accidents in the 1st month assuming that the time between accident is taken as in part (**1**) with chance 40% or as in part (**2**) with chance 60%.

If $0 \le u_1 \le 0.4$ $\quad T_n \sim \text{Exp}(1/5)$, $F^{-1}(u)=-5\ln(1-u_2)$

If $0.4 \le u_1 \le 1$ $\quad T_n \sim \text{Erlang}(3,0.2)$, $F^{-1}(u)= -\frac{1}{2}\ln(1-u_2) + -\frac{1}{2}\ln(1-u_3) + -\frac{1}{2}\ln(1-u_4)$

U1	Dist	U2	U3	U4	Tn	ATn
0.245	Exp	0.454			3.025682	3.025682
0.046	Exp	0.024			0.121463	3.147145
0.162	Exp	0.359			2.223629	5.370774
0.908	Erlang	0.287	0.98	0.253	22.70993	28.08071
0.16	Exp	0.815			8.436997	36.51771
0.5	Erlang					

Question #2:

An insurance company has a customer care calling center to process their claims. Assume that claims arrive to the calling center according to a random process. The time between calls (in minutes) is assumed to follow the Geometric distribution with mean 25 minutes.

$$(x) = (1-p)^x p; \; x = 0, 1, 2, \ldots$$

From past data, it is found that the claim value distributed as follows:

Claim Amount	$5000 \le X < 10000$	$10000 \le X < 15000$	$15000 \le X < 20000$	$20000 \le X$
Percentage	30%	40%	20%	10%

The exact amount within any category is assumed to be discrete uniform. The work hours for the call center is between 6 am to 6 pm every day.

1. Write the algorithm for simulating the arrival of a call for a claim.

$$\frac{1-P}{P} = 25 \quad \rightarrow P = 0.038$$

TBC~ Geometric (0.038)

$F^{-1}(u)=\left\lfloor\frac{LN(1-U)}{LN(1-0.038)}\right\rfloor$

$AT_n = TBC + AT_{n-1}$

2. Write the algorithm for simulating the amount of the claim
Amount of claim ~ Discrete uniform
$F^{-1}(u) = a + (b-a) *u$

$$\text{Interval}=\begin{cases}1 & 0 \le u \le 0.3\\ 2 & 0.3 < u \le 0.7\\ 3 & 0.7 < u \le 0.9\\ 4 & 0.9 < u \le 1\end{cases}$$

$$\text{Amount of claim} = \begin{cases}5000+5000u & Interval = 1\\ 10000+5000u & Interval = 2\\ 15000+5000u & Interval = 3\\ 20000+5000u & Interval = 4\end{cases}$$

3. Simulate the call arrival time and the claim amount for 5 days using the following uniform (0,1) numbers (*use as needed*) and your answers in (**1,2**).

Day-1				Day-2				Day-3				Day-4				Day-5			
U1	U2	U3		U1	U2	U3		U1	U2	U3		U1	U2	U3		U1	U2	U3	
0.59	0.19	0.766		0.994	0.162	0.957		0.817	0.83	0.937		0.473	0.749	0.937		0.355	0.315	0.728	
0.019	0.421	0.845		0.603	0.123	0.245		0.993	0.281	0.821		0.325	0.233	0.821		0.792	0.23	0.938	
0.023	0.932	0.38		0.362	0.022	0.998		0.63	0.979	0.193		0.818	0.291	0.193		0.382	0.904	0.76	
0.058	0.541	0.63		0.8	0.373	0.795		0.686	0.946	0.651		0.064	0.263	0.651		0.638	0.266	0.679	
0.164	0.063	0.383		0.891	0.252	0.375		0.925	0.982	0.262		0.37	0.444	0.262		0.556	0.702	0.513	
0.839	0.485	0.668		0.633	0.748	0.7		0.893	0.723	0.105		0.814	0.8	0.105		0.102	0.237	0.352	
0.571	0.614	0.319		0.548	0.154	0.799		0.001	0.124	0.704		0.618	0.076	0.704		0.513	0.74	0.332	
0.502	0.939	0.51		0.323	0.684	0.385		0.278	0.608	0.588		0.264	0.313	0.588		0.23	0.278	0.141	
0.644	0.032	0.085		0.221	0.5	0.701		0.024	0.609	0.414		0.306	0.233	0.414		0.202	0.124	0.846	
0.091	0.024	0.215		0.803	0.939	0.554		0.693	0.617	0.776		0.211	0.296	0.776		0.218	0.307	0.525	
0.675	0.458	0.089		0.895	0.584	0.447		0.871	0.76	0.642		0.712	0.962	0.642		0.361	0.69	0.468	
0.991	0.728	0.009		0.733	0.382	0.985		0.661	0.508	0.682		0.267	0.961	0.682		0.308	0.043	0.152	
0.419	0.687	0.485		0.502	0.136	0.834		0.478	0.163	0.692		0.096	0.736	0.692		0.454	0.399	0.104	
0.278	0.666	0.547		0.707	0.198	0.34		0.517	0.362	0.221		0.287	0.159	0.221		0.374	0.051	0.841	
0.737	0.726	0.772		0.301	0.498	0.755		0.919	0.592	0.695		0.229	0.38	0.695		0.42	0.179	0.264	
TBC	AT	wich category?	claim Amount	TBC	AT	wich category?	claim Amount	TBC	AT	wich category?	claim Amount	TBC	AT	wich category?	claim Amount	TBC	AT	wich category?	claim Amount
22	22	1	8830	130	130	1	9785	43	43	3	19685	16	16	3	19685	11	11	2	13640
0	22	2	14225	23	153	1	6225	126	169	1	9105	10	26	1	9105	40	51	1	9690
0	22	4	21900	11	164	1	9990	25	194	4	20965	43	69	1	5965	12	63	4	23800
1	23	2	13150	41	205	2	13975	29	223	4	23255	1	70	1	8255	25	88	1	8395
4	27	1	6915	56	261	1	6875	66	289	4	21310	11	81	2	11310	20	108	3	17565
46	73	2	13340	25	286	3	18500	56	345	3	15525	42	123	3	15525	2	110	1	6760
21	94	2	11595	20	306	1	8995	0	345	1	8520	24	147	1	8520	18	128	3	16660
17	111	4	22550	9	315	2	11925	8	353	2	12940	7	154	2	12940	6	134	1	5705
26	137	1	5425	6	321	2	13505	0	353	2	12070	9	163	1	7070	5	139	1	9230
2	139	1	6075	41	362	4	22770	30	383	2	13880	6	169	1	8880	6	145	2	12625
28	167	2	10445	57	419	2	12235	52	435	3	18210	31	200	4	23210	11	156	2	12340
120	287	3	15045	33	452	2	14925	27	462	2	13410	7	207	4	23410	9	165	1	5760
13	300	2	12425	17	469	1	9170	16	478	1	8460	2	209	3	18460	15	180	2	10520
8	308	2	12735	31	500	1	6700	18	496	2	11105	8	217	1	6105	11	191	1	9205
34	342	3	18860	9	509	2	13775	64	560	2	13475	6	223	2	13475	13	204	1	6320

Question #3:

Consider a continuous variable X in days that represents the length of stay (LOS) for patients in a hospital. It is assumed that LOS (in days) is a continuous random variable following the Weibull distribution with parameters □=2 and □=4 having the following *pdf*:

$$f(x) = \frac{2}{16}\, e^{-\frac{x}{16}^{2}} \text{ ; x>0}$$

1. Find the CDF function of $f(x)$ and then find the inverse transform for generating random numbers from $f(x)$.

$$F(x) \int_0^x \frac{2}{16}\, e^{-\frac{x}{16}^{2}} = 1 - e^{-\frac{x}{16}^{2}}$$

$F^{-1}(u) = 4\,(-\ln(1-u))^{0.5}$

2. Compute the average and standard deviation of LOS, by simulating the length of stay for 50 patients using the following uniform (0,1) numbers (*use as needed*) and your answer in (**1**) to compute.

U(0,1)										
1	2	3	4	5	6	7	8	9	10	
0.59	0.19	0.994	0.162	0.817	0.839	0.893	0.748	0.633	0.485	**Average**
0.019	0.421	0.603	0.123	0.993	0.571	0.001	0.154	0.548	0.614	3.434
0.023	0.932	0.362	0.022	0.63	0.502	0.278	0.684	0.323	0.939	**Std.dev**
0.058	0.541	0.8	0.373	0.686	0.644	0.024	0.5	0.221	0.032	2.169
0.164	0.063	0.891	0.252	0.925	0.091	0.693	0.939	0.803	0.024	
LOS										
3.777	1.836	9.047	1.682	5.213	5.406	5.980	4.696	4.005	3.258	
0.554	2.957	3.845	1.449	8.910	3.680	0.127	1.636	3.564	3.903	
0.610	6.558	2.682	0.597	3.988	3.340	2.283	4.293	2.498	6.690	
0.978	3.530	5.075	2.733	4.305	4.065	0.623	3.330	1.999	0.721	
1.693	1.020	5.955	2.155	6.438	1.236	4.347	6.690	5.098	0.623	

3. Now, assume that LOS for any patient is at least half a day. Find the new inverse transform for generating random numbers from *f*(*x*). Then, us it to compute the length of stay for 50 patients using the following uniform (0,1) numbers (*use as needed*).

$F^{-1}(u) = 0.5 + 4\,(-\ln(1-u))^{0.5}$

U(0,1)									
1	2	3	4	5	6	7	8	9	10
0.675	0.865	0.584	0.774	0.639	0.871	0.759	0.458	0.895	0.881
0.991	0.107	0.382	0.751	0.122	0.661	0.908	0.728	0.733	0.296
0.419	0.043	0.136	0.495	0.976	0.478	0.55	0.687	0.502	0.658
0.278	0.101	0.198	0.635	0.248	0.517	0.783	0.666	0.707	0.959
0.737	0.158	0.498	0.252	0.31	0.919	0.044	0.726	0.301	0.172
LOS (minimum 0.5 day)									
4.741	6.160	4.246	5.378	4.538	6.224	5.272	3.630	6.505	6.336
9.182	1.846	3.275	5.216	1.943	4.660	6.679	5.064	5.097	2.870
3.448	1.339	2.029	3.806	8.225	3.725	4.074	4.811	3.840	4.643
2.783	1.805	2.379	4.516	2.635	3.912	5.444	4.689	4.932	7.649
5.123	2.159	3.821	2.655	2.937	6.841	1.349	5.051	2.894	2.238

4. Assume that LOS for any patient is at least *Y* days. Where *Y* is a random variable following the Exponential distribution with mean half a day. Find the new inverse transform for generating random numbers from *f*(*x*). Then, compute the average and standard deviation of LOS, by simulating the length of stay for 50 patients using the uniform (0,1) numbers in part (**2**) for *X* and the uniform random numbers in part (**3**) for *Y*.

Y ~ Exp (2)

Y = -0.5 ln (1-u2)

X ~ Weibull (2,4)

$X = Y + 4\,(-\ln(1-u))^{0.5}$

U(0,1)										
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
0.675	0.865	0.584	0.774	0.639	0.871	0.759	0.458	0.895	0.881	
0.991	0.107	0.382	0.751	0.122	0.661	0.908	0.728	0.733	0.296	
0.419	0.043	0.136	0.495	0.976	0.478	0.55	0.687	0.502	0.658	
0.278	0.101	0.198	0.635	0.248	0.517	0.783	0.666	0.707	0.959	
0.737	0.158	0.498	0.252	0.31	0.919	0.044	0.726	0.301	0.172	
y										
0.562	1.001	0.439	0.744	0.509	1.024	0.711	0.306	1.127	1.064	
2.355	0.057	0.241	0.695	0.065	0.541	1.193	0.651	0.660	0.175	
0.272	0.022	0.073	0.342	1.865	0.325	0.399	0.581	0.349	0.536	
0.163	0.053	0.110	0.504	0.143	0.364	0.764	0.548	0.614	1.597	
0.668	0.086	0.345	0.145	0.186	1.257	0.022	0.647	0.179	0.094	
U(0,1)										
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
0.59	0.19	0.994	0.162	0.817	0.839	0.893	0.748	0.633	0.485	
0.019	0.421	0.603	0.123	0.993	0.571	0.001	0.154	0.548	0.614	
0.023	0.932	0.362	0.022	0.63	0.502	0.278	0.684	0.323	0.939	
0.058	0.541	0.8	0.373	0.686	0.644	0.024	0.5	0.221	0.032	
0.164	0.063	0.891	0.252	0.925	0.091	0.693	0.939	0.803	0.024	
X : LOS (minimum Y day)										
4.339	2.837	9.486	2.425	5.722	6.430	6.691	5.002	5.132	4.323	**Average**
2.909	3.013	4.085	2.144	8.975	4.221	1.320	2.287	4.225	4.078	3.981
0.882	6.580	2.755	0.938	5.853	3.665	2.682	4.874	2.847	7.226	**Std.dev**
1.141	3.583	5.185	3.237	4.448	4.429	1.387	3.879	2.613	2.318	2.083
2.361	1.106	6.300	2.301	6.623	2.492	4.369	7.337	5.277	0.718	

5. Some patients are known to have a medical condition that requires them to stay a random length of stay between a minimum of 0.5 day up to 5 days' maximum. Write the inverse transform for generating random numbers from $f(x)$ under this condition.

W= F(a) + (F(b)-F(a)) *u = F (0.5) + (F (5)-F (0.5)) *u = 0.0155 + 0.77488 u

$X = 4\ (-\ln(1-w))^{0.5}$

6. Compute the average LOS of patients in part (**5**), by simulating the length of stay for 50 patients using the following uniform (0,1) numbers (*use as needed*) and your answer in (**5**) to compute.

U(0,1)										
1	2	3	4	5	6	7	8	9	10	
0.796	0.957	0.622	0.074	0.539	0.012	0.377	0.972	0.256	0.697	
0.216	0.937	0.891	0.456	0.623	0.192	0.278	0.304	0.654	0.043	
0.374	0.003	0.759	0.389	0.638	0.313	0.019	0.952	0.643	0.854	
0.058	0.541	0.8	0.373	0.686	0.644	0.024	0.5	0.221	0.032	
0.164	0.063	0.891	0.252	0.925	0.091	0.693	0.939	0.803	0.024	
W										
0.632	0.757	0.497	0.073	0.433	0.025	0.308	0.769	0.214	0.556	
0.183	0.742	0.706	0.369	0.498	0.164	0.231	0.251	0.522	0.049	
0.305	0.018	0.604	0.317	0.510	0.258	0.030	0.753	0.514	0.677	
0.060	0.435	0.635	0.305	0.547	0.515	0.034	0.403	0.187	0.040	
0.143	0.064	0.706	0.211	0.732	0.086	0.552	0.743	0.638	0.034	
LOS (minimum 0.5 day and maximum 5 days)										
4.001	4.758	3.318	1.100	3.014	0.634	2.425	4.840	1.962	3.602	
1.798	4.653	4.425	2.714	3.322	1.695	2.050	2.151	3.438	0.895	
2.414	0.536	3.848	2.470	3.378	2.185	0.701	4.731	3.397	4.254	**Average**
0.999	3.021	4.018	2.411	3.560	3.400	0.745	2.873	1.819	0.811	2.723
1.569	1.031	4.425	1.946	4.592	1.200	3.587	4.663	4.031	0.745	

Question #4:

Consider a continuous random variable with the following pdf:

$$f(x) = \begin{cases} \frac{1}{2} & 0 \le x \le 1 \\ \frac{3}{4} - \frac{x}{4} & 1 \le x \le 3 \end{cases}$$

(a) Construct the algorithm for obtaining random numbers from $f(x)$ using the inverse transform method.

$$F(x) = \begin{cases} 0 & x < 0 \\ \int_0^x \frac{1}{2}\, dx = \frac{1}{2}x & 0 \le x \le 1 \\ \int_0^1 x + \int_1^x \frac{3}{4} - \frac{x}{4}\, dx = 1 - \frac{(3-x)^2}{8} & 1 \le x \le 3 \\ 1 & x \ge 3 \end{cases}$$

$$F^{-1}(u) = \begin{cases} 2u & 0 \le u \le 0.5 \\ 3 - \sqrt{8(1-u)} & 0 < u \le 1 \end{cases}$$

(b) Using your answer in (a), give 10 random numbers from f(x) using the following uniform numbers.

	1	2	3	4	5	6	7	8	9	10
u	0.387	0.336	0.466	0.074	0.184	0.34	0.9	0.875	0.475	0.64
x	0.774	0.672	0.932	0.148	0.368	0.672	2.106	2.000	0.950	1.294

(c) Assume that the variable X represents the time between arrival to a service station (in hours). The station work hours are from 8 am to 3 pm. The owner wants to determine the expected number arrivals per day using simulation output for 5 days. Using the following uniform numbers for each day, estimate the average number of arrivals per day.

	Day-1		Day-2		Day-3		Day-4		Day-5	
X	**TBA**	**AT**	**TBA**	**AT**	**TBA**	**AT**	**TBA**	**AT**	**TBA**	**AT**
1	0.388	0.388	0.978	0.978	0.058	0.058	0.762	0.762	0.136	0.136
2	1.704	2.092	0.600	1.578	1.735	1.793	0.706	1.468	0.512	0.648
3	0.168	2.260	1.707	3.285	1.626	3.419	2.133	3.601	1.549	2.197
4	0.222	2.482	2.190	5.475	1.109	4.528	1.165	4.766	0.654	2.851
5	2.393	4.875	1.298	6.773	0.904	5.432	0.730	5.496	1.927	4.778
6	1.511	6.387	2.062		1.569		1.614		0.868	5.646
7	1.105		0.158		0.278		0.976		1.667	
8	2.195		2.231		0.562		1.732		2.252	
9	0.674		0.620		0.768		0.490		1.600	
10	1.837		0.172		0.200		0.176		1.187	
No. of arrivals		6		5		5		5		6
Average	**5.4**									

Question #5:

Consider a continuous random variable with the following pdf:

$$f(x) = \begin{cases} \frac{x}{8} - \frac{1}{4} & 2 \le x \le 4 \\ \frac{10}{24} - \frac{x}{24} & 4 \le x \le 10 \end{cases}$$

(a) Construct the algorithm for obtaining random numbers from f(x) using the acceptance–rejection method using majorizing function g(x) = max f(x).

g(x) = max f(x) = 0.25

c = $\int_2^{10} 0.25 = 2$

w(x) = $\frac{0.25}{2} = \frac{1}{8}$

W(x) = $\int_2^{x} \frac{1}{8} = \frac{x-2}{8}$

W^{-1}(x) = 8u+ 2

(b) Using your answer in (a), give 10 random numbers from $f(x)$ using the following uniform numbers (as needed).

	U	W	f(w)	g(w)	f(w)/g(w)	v	Acc/rej
1	0.194	3.552	0.194	0.25	0.776	0.381	Acc
2	0.79	8.32	0.07	0.25	0.28	0.353	Rej
3	0.084	2.672	0.084	0.25	0.336	0.906	Rej
4	0.111	2.888	0.111	0.25	0.444	0.579	Rej
5	0.954	9.632	0.01533	0.25	0.06133	0.365	Rej
6	0.723	7.784	0.09233	0.25	0.36933	0.76	Rej
7	0.551	6.408	0.14967	0.25	0.59867	0.488	Acc
8	0.919	9.352	0.027	0.25	0.108	0.799	Rej
9	0.337	4.696	0.221	0.25	0.884	0.245	Acc
10	0.831	8.648	0.05633	0.25	0.22533	0.088	Acc
11	0.489	5.912	0.17033	0.25	0.68133	0.029	Acc
12	0.3	4.4	0.23333	0.25	0.93333	0.8	Acc
13	0.791	8.328	0.06967	0.25	0.27867	0.764	Rej
14	0.918	9.344	0.02733	0.25	0.10933	0.553	Rej
15	0.638	7.104	0.12067	0.25	0.48267	0.452	Acc
16	0.89	9.12	0.03667	0.25	0.14667	0.744	Rej
17	0.079	2.632	0.079	0.25	0.316	0.139	Acc
18	0.926	9.408	0.02467	0.25	0.09867	0.281	Rej
19	0.31	4.48	0.23	0.25	0.92	0.384	Acc
20	0.086	2.688	0.086	0.25	0.344	0.1	Acc