1st Assignment (2020)

استعن بالله وكن على يقين بأن كل ما ورد في هذه الورقة تعرفه جيدا وقد تدربت عليه بما فيه الكفاية

Instruction and guides for the assignment:

1. This assignment is designed to guide you to understand fully the topics and practice covered in the 1st month of the course.
2. To give you plenty of time to review and apply the materials for the answer, the assignment duration is from **4:00pm** Thursday April 2,2020 until **12:00 midnight same day**
3. You can use the lecture notes, the text book, Excel for your answer.
4. Make sure to indicate the source of the information in you answer.
5. You are the guardian of your behavior in this assignment. This assignment is totally for your independent effort. **Do not attempt** to collaboration or communication with anyone about the questions of the assignment, it is totally not allowed by any means.
6. Write your answers on a word document and email the document on PDF and WORD format. Write the subject of the email as:

   OPER-441-Assignment#1 <<Section Number>> , << your name>> , <<your KSU ID >>
7. Put all you of answers in one document. If you have part of the answers on excel, capture the answer from the screen and insert it in the document.
8. Make your answers as **comprehensive** in information as much as you can, write all information related to the answers. There will be **Extra Marks** for that>
9. Make sure to make your document as **organized** as possible, there will be **Extra Marks** for the organization.

وفقكم الله ويسر لكم .. وحفظكم ورعاكم

Question #1:

I. Which of the following is an ***attribute*** and which one is ***a state variable***?

1. The age of the arrival to a supermarket	
2. Number of busy lines in a call center	
3. Number of passengers in an airport with First Class tickets	
4. The type of ticket (First, Business, Couch) for a passengers in airport	
5. The arrival time for a delivery	
6. The amount of money that a customer spend in a supermarket	

II. Which of the following is for ***mathematical*** model and which one is for ***Simulation*** model?

1. Always give the exact solution	
2. Give optimal solution	
3. Requires only parameters to get the final results	
4. Requires data analysis to get final results	
5. Every time you solve the model you get different value than last time	
6. Give approximate solution	
7. Require certain values to get solution	

III. Which of the following for Binomial, Exponential, Poisson or Normal distribution?

1. Has only one parameter	
2. The mean equals to the variance	
3. The mean equals to the standard dilation	
4. Is used for discrete random variables	
5. Is used for Continuous Random variables	
6. Always the mean equals to the median	
7. Used to model number of events during one unit of time	
8. Used to model number of successes	
9. Used to model time between events	
10. Used for random variable with all real values	
11. Used to build confidence intervals	
12. Has no mod for any value of the parameters	

Question #2:

Given the following set of numbers. Show that this sample of 100 numbers is taken from U[0,1] using Chi-Square Test and α = 95%.

	1	2	3	4	5
1	0.6588	0.2341	0.5034	0.8933	0.0734
2	0.9836	0.3975	0.7676	0.9718	0.7934
3	0.5979	0.7369	0.9671	0.6812	0.3737
4	0.8332	0.1914	0.6216	0.9358	0.5765
5	0.0510	0.1204	0.6725	0.7279	0.6876
6	0.3249	0.0338	0.7221	0.6088	0.9479
7	0.4156	0.8604	0.5586	0.9742	0.5802
8	0.4622	0.3520	0.6688	0.5766	0.9850
9	0.7781	0.6482	0.9550	0.8988	0.9829
10	0.8299	0.7629	0.5379	0.7976	0.9462
11	0.8877	0.6122	0.9555	0.0343	0.7382
12	0.9820	0.5124	0.6978	0.4838	0.9848
13	0.5480	0.6428	0.2258	0.7633	0.6936
14	0.6402	0.8372	0.9933	0.3995	0.8421
15	0.5499	0.8271	0.0881	0.4650	0.7375
16	0.9591	0.8020	0.2551	0.2723	0.5010
17	0.6473	0.6177	0.5775	0.4673	0.8282
18	0.9657	0.8981	0.3032	0.2239	0.6094
19	0.7749	0.6443	0.7578	0.8819	0.6007
20	0.7831	0.8771	0.8734	0.0778	0.7868

Question #3:

1. 1.Draw the imperical distribution for following sample.
2. Given the following set of numbers. Show that this sample of 100 numbers is taken from U[0,1] using Kolmogorov- Smirnov Test and α = 95%.

	1	2	3	4	5
1	0.6588	0.2341	0.5034	0.8933	0.0734
2	0.9836	0.3975	0.7676	0.9718	0.7934
3	0.5979	0.7369	0.9671	0.6812	0.3737
4	0.8332	0.1914	0.6216	0.9358	0.5765
5	0.0510	0.1204	0.6725	0.7279	0.6876
6	0.3249	0.0338	0.7221	0.6088	0.9479
7	0.4156	0.8604	0.5586	0.9742	0.5802
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20	0.7831	0.8771	0.8734	0.0778	0.7868

Question #4:

Conceder the following LCG generator: $R_n = (5\ R_{n-1} + 2)\ mod\ (21)$

Answer the following:

a) By looking to the LCG function, How many total U[0,1] random numbers this LCG can give you (Answer by giving the number ONLY, do not use the function)?
b) Without substitution in the LCG function above, What are all U[0,1] random numbers that you could get from this LCG by looking to the function.
c) Generate all possible random streams from the above LCG. Use any choice for starting values.

Question #5:

You are trying to build your own U(0,1) generator with 53 different numbers:

1. Give the parameters of an LCG function the provides 53 number with full cycle
2. Prove that your answer in (1) is true by theorem?
3. Give the random stream of your LCG starting from $X_0 = 6$?

Question #6:

Consider a car washing station. The station has **two** washing machines and one parking space for waiting. If any car enters the station and finds both machines busy servicing cars, then the new car waits in the parking space. If any car enters the station and finds another car waiting in parking space the new car is lost. The following data (in minutes) is given to you about 20 cars.

car #	Time betwn Arrival	Washing Time (min.)	Arrival Time (min.)	Car Enters?? (0,1)	Car Waits?? (0,1)	Service Starting Time	Service at Mach. #	Dep. Time	Idle Time Mach.1	Idle Time Mach.2
1	**4**	**7**								
2	**4**	**8**								
3	**2**	**12**								
4	**7**	**12**								
5	**8**	**10**								
6	**7**	**14**								
7	**7**	**12**								
8	**4**	**10**								
9	**4**	**13**								
10	**3**	**13**								
11	**5**	**7**								
12	**7**	**8**								
13	**2**	**10**								
14	**5**	**14**								
15	**4**	**8**								
16	**4**	**8**								
17	**6**	**9**								
18	**2**	**9**								
19	**8**	**14**								
20	**2**	**11**								

Do the discrete-event simulation and Complete the details of the car wash station with the following simulation output:

1. Arrival time (in minutes)
2. Explain your logic for computing the Arrival time.
3. Service time (in minutes)

4. The starting time.
5. Explain your logic for computing The starting time.
6. The car Enters? (0 = No, 1 = Yes)
7. Explain your logic for computing if the car Enters.
8. The car Waits? (0 = No, 1 = Yes)
9. Explain your logic for computing if the car Waits.
10. The Service Machine (**1 =** car is being served at Machine 1, **2 =** car is being served at Machine 2)
11. Departure Time
12. Explain your logic for computing The Departure Time
13. Waiting Time (in minutes)
14. Your output will be like the as shown in the table.

15. **After filling the hand simulation, compute the following from the simulation:**
    - The percentage of lost.
    - Average arrival rate per hour
    - Average time for service.
    - Average waiting time if car waits
    - Percentage of cars getting the service directly without waiting.
    - Percentage of time Machine 1 is in operation.
    - Probability empty parking (No one in the parking)

1st Assignment (2020) Solution

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Total Evaluation
180 Points

Question #1: (30 Points)

I. Which of the following is an ***attribute*** and which one is ***a state variable***?

(6 Points)

1. The age of the arrival to a supermarket	***attribute***
2. Number of busy lines in a call center	***state variable***
3. Number of passengers in an airport with First Class tickets	***state variable***
4. The type of ticket (First, Business, Couch) for a passengers in airport	***attribute***
5. The arrival time for a delivery	***attribute***
6. The amount of money that a customer spend in a supermarket	***attribute***

II. Which of the following is for ***mathematical*** model and which one is for ***Simulation*** model?

(7 Points)

1. Always give the exact solution	***mathematical***
2. Give optimal solution	***mathematical***
3. Requires only parameters to get the final results	***mathematical***
4. Requires data analysis to get final results	***Simulation***
5. Every time you solve the model you get different value than last time	***Simulation***
6. Give approximate solution	***Simulation***
7. Require certain values to get solution	***mathematical***

III. Which of the following for Binomial, Exponential, Poisson or Normal distribution?

(15 Points) ***each distribution is a point***

1. Has only one parameter	***Poisson, Exponential***
2. The mean equals to the variance	***Poisson***
3. The mean equals to the standard dilation	***Exponential***
4. Is used for discrete random variables	***Poisson, Binomial***
5. Is used for Continuous Random variables	***Normal, Exponential***
6. Always the mean equals to the median	***Normal***
7. Used to model number of events during one unit of time	***Poisson***
8. Used to model number of successes	***Binomial***
9. Used to model time between events	***Exponential***
10. Used for random variable with all real values	***Normal***
11. Used to build confidence intervals	***Normal***
12. Has no mod for any value of the parameters	***Exponential***

Question #2: (30 Points)

Given the following set of numbers. Show that this sample of 100 numbers is taken from U[0,1] using Chi-Square Test and α = 95%.

	1	2	3	4	5
1	0.6588	0.2341	0.5034	0.8933	0.0734
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18	0.9657	0.8981	0.3032	0.2239	0.6094
19	0.7749	0.6443	0.7578	0.8819	0.6007
20	0.7831	0.8771	0.8734	0.0778	0.7868

(20 Points)

i	b(i-1)	bi	ci	pi	Ei = n*pi	$x_0^2 = \frac{(ci-Ei)^2}{Ei}$
1	0.05	0.1	6	0.1	10	**1.6**
2	0.15	0.2	2	0.1	10	**6.4**
3	0.25	0.3	5	0.1	10	**2.5**
4	0.35	0.4	6	0.1	10	**1.6**
5	0.45	0.5	5	0.1	10	**2.5**
6	0.55	0.6	12	0.1	10	**0.4**
7	0.65	0.7	18	0.1	10	**6.4**
8	0.75	0.8	15	0.1	10	**2.5**
9	0.85	0.9	15	0.1	10	**2.5**
10	0.95	1	16	0.1	10	**3.6**
			100	1	100	30

Now we test : $H_{0=U_i}\ follow\ \ U(0,1)\ \ Vs\ \ \ \ H_{1=U_i}\ doesn't\ follow\ U(0.1)$

From χ^2 table, we have $X^2_{9,0.05} = 16.92$

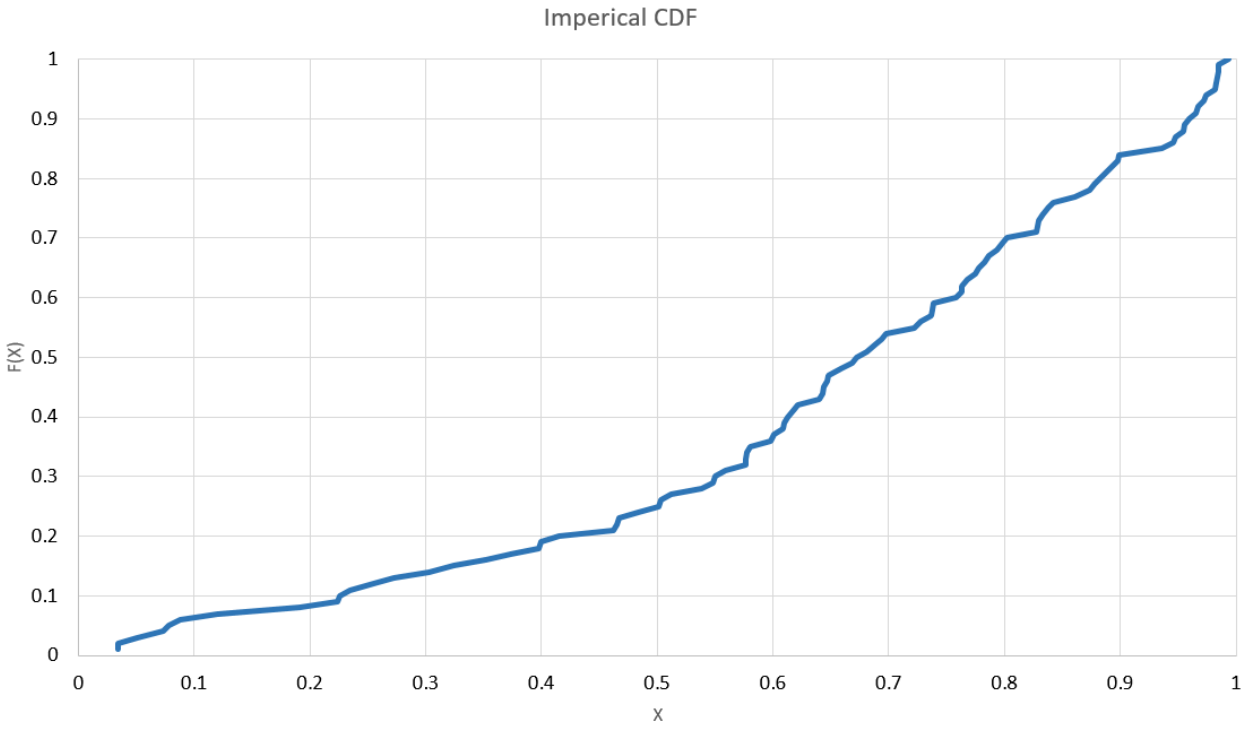
(10 Points)

Sinve χ^2 calculation = 30 $> \chi^2$ table $= 16.92$ then we REJECT H_o
Then the data does not follow U(0,1)

Question #3: (30 Points)

1. Draw the empirical distribution for following sample.
2. Given the following set of numbers. Show that this sample of 100 numbers is taken from U[0,1] using Kolmogorov-Smirnov Test and α = 95%.

(10 Points)



(10 Points)

i	xi	i/n	(i-1)/n	F(xi)	(i/n)-F(xi)	F(Xi)-((1-i)/n))
1	0.0338	0.01	0	0.0338	-0.0238	0.0338
2	0.0343	0.02	0.01	0.0343	-0.0143	0.0243
3	0.051	0.03	0.02	0.051	-0.021	0.031
4	0.0734	0.04	0.03	0.0734	-0.0334	0.0434
5	0.0778	0.05	0.04	0.0778	-0.0278	0.0378
6	0.0881	0.06	0.05	0.0881	-0.0281	0.0381
7	0.1204	0.07	0.06	0.1204	-0.0504	0.0604

8	0.1914	0.08	0.07	0.1914	-0.1114	0.1214
9	0.2239	0.09	0.08	0.2239	-0.1339	0.1439
10	0.2258	0.1	0.09	0.2258	-0.1258	0.1358
11	0.2341	0.11	0.1	0.2341	-0.1241	0.1341
12	0.2551	0.12	0.11	0.2551	-0.1351	0.1451
13	0.2723	0.13	0.12	0.2723	-0.1423	0.1523
14	0.3032	0.14	0.13	0.3032	-0.1632	0.1732
15	0.3249	0.15	0.14	0.3249	-0.1749	0.1849
16	0.352	0.16	0.15	0.352	-0.192	0.202
17	0.3737	0.17	0.16	0.3737	-0.2037	0.2137
18	0.3975	0.18	0.17	0.3975	-0.2175	0.2275
19	0.3995	0.19	0.18	0.3995	-0.2095	0.2195
20	0.4156	0.2	0.19	0.4156	-0.2156	0.2256
21	0.4622	0.21	0.2	0.4622	-0.2522	0.2622
22	0.465	0.22	0.21	0.465	-0.245	0.255
23	0.4673	0.23	0.22	0.4673	-0.2373	0.2473
24	0.4838	0.24	0.23	0.4838	-0.2438	0.2538
25	0.501	0.25	0.24	0.501	-0.251	0.261
26	0.5034	0.26	0.25	0.5034	-0.2434	0.2534
27	0.5124	0.27	0.26	0.5124	-0.2424	0.2524
28	0.5379	0.28	0.27	0.5379	-0.2579	0.2679
29	0.548	0.29	0.28	0.548	-0.258	0.268
30	0.5499	0.3	0.29	0.5499	-0.2499	0.2599
31	0.5586	0.31	0.3	0.5586	-0.2486	0.2586
32	0.5765	0.32	0.31	0.5765	-0.2565	0.2665
33	0.5766	0.33	0.32	0.5766	-0.2466	0.2566
34	0.5775	0.34	0.33	0.5775	-0.2375	0.2475
35	0.5802	0.35	0.34	0.5802	-0.2302	0.2402
36	0.5979	0.36	0.35	0.5979	-0.2379	0.2479
37	0.6007	0.37	0.36	0.6007	-0.2307	0.2407
38	0.6088	0.38	0.37	0.6088	-0.2288	0.2388
39	0.6094	0.39	0.38	0.6094	-0.2194	0.2294
40	0.6122	0.4	0.39	0.6122	-0.2122	0.2222
41	0.6177	0.41	0.4	0.6177	-0.2077	0.2177
42	0.6216	0.42	0.41	0.6216	-0.2016	0.2116
43	0.6402	0.43	0.42	0.6402	-0.2102	0.2202
44	0.6428	0.44	0.43	0.6428	-0.2028	0.2128
45	0.6443	0.45	0.44	0.6443	-0.1943	0.2043
46	0.6473	0.46	0.45	0.6473	-0.1873	0.1973
47	0.6482	0.47	0.46	0.6482	-0.1782	0.1882

48	0.6588	0.48	0.47	0.6588	-0.1788	0.1888
49	0.6688	0.49	0.48	0.6688	-0.1788	0.1888
50	0.6725	0.5	0.49	0.6725	-0.1725	0.1825
51	0.6812	0.51	0.5	0.6812	-0.1712	0.1812
52	0.6876	0.52	0.51	0.6876	-0.1676	0.1776
53	0.6936	0.53	0.52	0.6936	-0.1636	0.1736
54	0.6978	0.54	0.53	0.6978	-0.1578	0.1678
55	0.7221	0.55	0.54	0.7221	-0.1721	0.1821
56	0.7279	0.56	0.55	0.7279	-0.1679	0.1779
57	0.7369	0.57	0.56	0.7369	-0.1669	0.1769
58	0.7375	0.58	0.57	0.7375	-0.1575	0.1675
59	0.7382	0.59	0.58	0.7382	-0.1482	0.1582
60	0.7578	0.6	0.59	0.7578	-0.1578	0.1678
61	0.7629	0.61	0.6	0.7629	-0.1529	0.1629
62	0.7633	0.62	0.61	0.7633	-0.1433	0.1533
63	0.7676	0.63	0.62	0.7676	-0.1376	0.1476
64	0.7749	0.64	0.63	0.7749	-0.1349	0.1449
65	0.7781	0.65	0.64	0.7781	-0.1281	0.1381
66	0.7831	0.66	0.65	0.7831	-0.1231	0.1331
67	0.7868	0.67	0.66	0.7868	-0.1168	0.1268
68	0.7934	0.68	0.67	0.7934	-0.1134	0.1234
69	0.7976	0.69	0.68	0.7976	-0.1076	0.1176
70	0.802	0.7	0.69	0.802	-0.102	0.112
71	0.8271	0.71	0.7	0.8271	-0.1171	0.1271
72	0.8282	0.72	0.71	0.8282	-0.1082	0.1182
73	0.8299	0.73	0.72	0.8299	-0.0999	0.1099
74	0.8332	0.74	0.73	0.8332	-0.0932	0.1032
75	0.8372	0.75	0.74	0.8372	-0.0872	0.0972
76	0.8421	0.76	0.75	0.8421	-0.0821	0.0921
77	0.8604	0.77	0.76	0.8604	-0.0904	0.1004
78	0.8734	0.78	0.77	0.8734	-0.0934	0.1034
79	0.8771	0.79	0.78	0.8771	-0.0871	0.0971
80	0.8819	0.8	0.79	0.8819	-0.0819	0.0919
81	0.8877	0.81	0.8	0.8877	-0.0777	0.0877
82	0.8933	0.82	0.81	0.8933	-0.0733	0.0833
83	0.8981	0.83	0.82	0.8981	-0.0681	0.0781
84	0.8988	0.84	0.83	0.8988	-0.0588	0.0688
85	0.9358	0.85	0.84	0.9358	-0.0858	0.0958
86	0.9462	0.86	0.85	0.9462	-0.0862	0.0962
87	0.9479	0.87	0.86	0.9479	-0.0779	0.0879

88	0.955	0.88	0.87	0.955	-0.075	0.085
89	0.9555	0.89	0.88	0.9555	-0.0655	0.0755
90	0.9591	0.9	0.89	0.9591	-0.0591	0.0691
91	0.9657	0.91	0.9	0.9657	-0.0557	0.0657
92	0.9671	0.92	0.91	0.9671	-0.0471	0.0571
93	0.9718	0.93	0.92	0.9718	-0.0418	0.0518
94	0.9742	0.94	0.93	0.9742	-0.0342	0.0442
95	0.982	0.95	0.94	0.982	-0.032	0.042
96	0.9829	0.96	0.95	0.9829	-0.0229	0.0329
97	0.9836	0.97	0.96	0.9836	-0.0136	0.0236
98	0.9848	0.98	0.97	0.9848	-0.0048	0.0148
99	0.985	0.99	0.98	0.985	0.005	0.005
100	0.9933	1	0.99	0.9933	0.0067	0.0033

(10 Points)

n	100
a	0.05
D+	0.0067
D-	0.268
D_{100} = max{ D+, D- }	0.268
$D_{0.05}$	0.136

Now we test : $H_{0=U_i}$ $follow$ $U(0,1)$ Vs $H_{1=U_i}$ $doesn't\ follow$ $U(0.1)$

From K-S table, we have $D_{0.05} = 0.136$

Since D_{100} calculation = 0.268 > D_0 table $= 0.136$ then we EJECT H_o

Then the data does not follow U(0,1)

Question #4: (20 Points)

Conceder the following LCG generator: $R_n = (5\ R_{n-1} + 2)\ mod\ (21)$

Answer the following:

a) By looking to the LCG function, How many total U[0,1] random numbers this LCG can give you (Answer by giving the number ONLY, do not use the function)?

b) Without substitution in the LCG function above, What are all U[0,1] random numbers that you could get from this LCG by looking to the function.

c) Generate all possible random streams from the above LCG. Use any choice for starting values.

Solution

LCG generator: $R_n = (5\ R_{n-1} + 2)\ mod\ (21)$ Answer the following:

a) By looking to the LCG function, the U[0,1] random numbers this LCG can give you

(2 Points)

M= 21

b) Without substitution in the LCG function above, all U[0,1] random numbers that this LCG gives are:

(5 Points)

$$U_i=\frac{R_i}{M}\ ,\ M-1\geq R_i\geq 0$$

$$U=0,\frac{1}{21},\frac{2}{21},\frac{3}{21},\frac{4}{21},\frac{5}{21},\frac{6}{21},\frac{7}{21},\frac{8}{21},\frac{9}{21},\frac{10}{21},\frac{11}{21},\frac{12}{21},\frac{13}{21},\frac{14}{21},\frac{15}{21},\frac{16}{21},\frac{17}{21},\frac{18}{21},\frac{19}{21},\frac{20}{21}$$

c) All possible random streams from the above LCG

(13 Points)

	Xn	X(n+1)	U(n+1)
0	0	2	0.095238
1	2	12	0.571429
2	12	20	0.952381
3	20	18	0.857143
4	18	8	0.380952
5	8	0	0
6	0	2	0.095238

X_0=0 ,K=6<21=m

N	Xn	X(n+1)	U(n+1)
0	1	7	0.333333
1	7	16	0.761905
2	16	19	0.904762
3	19	13	0.619048
4	13	4	0.190476
5	4	1	0.047619
6	1	7	0.333333

X_0=1 , K=6+6=12<21=m

N	Xn	X(n+1)	U(n+1)
0	3	17	0.809524
1	17	3	0.142857
2	3	17	0.809524

X_0=3 ,K=12+2=14<21=m

N	Xn	X(n+1)	U(n+1)
0	5	6	0.285714
1	6	11	0.52381
2	11	15	0.714286
3	15	14	0.666667
4	14	9	0.428571
5	9	5	0.238095
6	5	6	0.285714

X_0=5 , K=14+6=20<21

n	Xn	X(n+1)	U(n+1)
0	10	10	0.47619
1	10	10	0.47619

X_0=10 , K=20+1=21= m

Question #5: (20 Points)

You are trying to build your own U(0,1) generator with 53 different numbers:

1. Give the parameters of an LCG function that provides a stream with period of least 52.
2. Prove that your answer in (1) is true by theorem?
3. Give the random stream of your LCG starting from X_0 =6?

Solution

(5 Points) 1. Let m = 53, and a = 5 and c = 1 ➔ this give a random stream with period 52

2. Theorem:

(5 Points)

- The only positive integer that (exactly) divides both m and c is 1 ➔ TRUE
- If *q* prime number of m then it should divide (*a*-1) ➔ FALSE: 53 is prime
- 4 is not a divisor for *m*➔ no need

3. Give the random stream of your LCG starting from X_0 =6?

(10 Points)

n	R(n)	R(n+1)	U(n+1)	n	R(n)	R(n+1)	U(n+1)	n	R(n)	R(n+1)	U(n+1)
0	6	31	**0.5849**	**19**	33	7	**0.1321**	**38**	24	15	**0.2830**
1	31	50	**0.9434**	**20**	7	36	**0.6792**	**39**	15	23	**0.4340**
2	50	39	**0.7358**	**21**	36	22	**0.4151**	**40**	23	10	**0.1887**
3	39	37	**0.6981**	**22**	22	5	**0.0943**	**41**	10	51	**0.9623**
4	37	27	**0.5094**	**23**	5	26	**0.4906**	**42**	51	44	**0.8302**
5	27	30	**0.5660**	**24**	26	25	**0.4717**	**43**	44	9	**0.1698**
6	30	45	**0.8491**	**25**	25	20	**0.3774**	**44**	9	46	**0.8679**
7	45	14	**0.2642**	**26**	20	48	**0.9057**	**45**	46	19	**0.3585**
8	14	18	**0.3396**	**27**	48	29	**0.5472**	**46**	19	43	**0.8113**
9	18	38	**0.7170**	**28**	29	40	**0.7547**	**47**	43	4	**0.0755**
10	38	32	**0.6038**	**29**	40	42	**0.7925**	**48**	4	21	**0.3962**
11	32	2	**0.0377**	**30**	42	52	**0.9811**	**49**	21	0	**0.0000**
12	2	11	**0.2075**	**31**	52	49	**0.9245**	**50**	0	1	**0.0189**
13	11	3	**0.0566**	**32**	49	34	**0.6415**	**51**	1	6	**0.1132**
14	3	16	**0.3019**	**33**	34	12	**0.2264**				
15	16	28	**0.5283**	**34**	12	8	**0.1509**				
16	28	35	**0.6604**	**35**	8	41	**0.7736**				
17	35	17	**0.3208**	**36**	41	47	**0.8868**				
18	17	33	**0.6226**	**37**	47	24	**0.4528**				

Question #6: (50 Points)

Consider a car washing station. The station has **two** washing machines and one parking space for waiting. If any car enters the station and finds both machines busy servicing cars, then the new car waits in the parking space. If any car enters the station and finds another car waiting in parking space the new car is lost. The following data (in minutes) is given to you about 20 cars.

(12 Points)

car #	Time btn Arriv.	Wash. Time (min.)	Arriv. Time (min.)	Car In?? (0,1)	Car Waits? (0,1)	Wait Time	Srvc. Start Time	Srvc. Mach. #	Dep. Time	Idle Time Mach.1	Idle Time Mach.2
1	4	7	4	1	0	0	4	1	11	4	4
2	4	8	8	1	0	0	8	2	16	0	4
3	2	12	10	1	1	1	11	1	23	0	0
4	7	12	17	1	0	1	16	2	29	0	0
5	8	10	25	1	0	0	25	1	35	2	0
6	7	14	32	1	0	0	32	2	46	0	3
7	7	12	39	1	0	0	39	1	51	4	4
8	4	10	43	1	1	3	46	2	56	0	0
9	4	13	47	1	1	4	51	1	64	0	0
10	3	13	50	0	lost	0	lost	lost	lost	lost	lost
11	5	7	55	1	1	1	56	2	63	0	0
12	7	8	62	1	1	1	64	2	71	0	1
13	2	10	64	1	0	0	64	1	74	0	0
14	5	14	69	1	1	2	72	2	85	0	1
15	4	8	73	1	1	1	74	1	82	0	0
16	4	8	77	1	1	5	82	1	90	0	0
17	6	9	83	1	1	2	85	2	94	0	0
18	2	9	85	1	1	5	90	1	99	0	0
19	8	14	93	1	0	1	94	2	108	0	0
20	2	11	95	1	0	1	99	1	110	0	0

Do the discrete-event simulation and Complete the details of the car wash station with the following simulation output:

1. Arrival time (in minutes)
2. Explain your logic for computing the Arrival time.

3. Service time (in minutes)
4. The starting time.
5. Explain your logic for computing The starting time.
6. The car Enters? (0 = No, 1 = Yes)
7. Explain your logic for computing if the car Enters.
8. The car Waits? (0 = No, 1 = Yes)
9. Explain your logic for computing if the car Waits.
10. The Service Machine (**1 =** car is being served at Machine 1, **2 =** car is being served at Machine 2)
11. Departure Time
12. Explain your logic for computing The Departure Time
13. Waiting Time (in minutes)
14. Your output will be like the as shown in the table.

15. After filling the hand simulation, compute the following from the simulation:

- The percentage of lost.
- Average arrival rate per hour
- Average time for service.
- Average waiting time if car waits
- Percentage of cars getting the service directly without waiting.
- Percentage of time Machine 1 is in operation.
- Probability empty parking (No one in the parking)

==========

1. Arrival time (in minutes) ➔ in table

2. Explain your logic for computing the Arrival time.

(3 Points)

- Arrival time = Arrival time of the previous + time between arrivals

3. Service time (in minutes) ➔ in table

4. The starting time. ➔ in table

5. Explain your logic for computing The starting time.

(4 Points)

- If arrival finds no one in the system, then his service start immediately (the same as the arrival time) at 1st Machine
- If arrival finds one in service, then his service starts immediately (the same as the arrival time) at the empty Machine

- If arrival finds two in service, then his service starts after the minimum of departure time from any machine.

6. The car Enters? (0 = No, 1 = Yes) ➔ in table

7. Explain your logic for computing if the car Enters.

(4 Points)

- If arrival finds empty system, the car enters, value = 1
- arrival finds one in system, the car enters, value = 1
- If arrival finds two in system, the car enters , value = 1
- If arrival finds three in system, the car blocked, value = 0

8. The car Waits? (0 = No, 1 = Yes) ➔ in table
9. Explain your logic for computing if the car Waits.

(4 Points)

- If arrival finds empty system, the car doesn't wait, value = 0
- If arrival finds one in system the car doesn't wait, value = 0
- If arrival finds two in system the car doesn't wait, value = 1

10. The Service Machine (**1 =** car is being served at Machine 1, **2 =** car is being served at Machine 2) ➔ in table

11. Departure Time ➔ in table

12. Explain your logic for computing The Departure Time

(2 points)

- If car enters then the departure time = Arrival time + Service time + waiting time

13. Waiting Time (in minutes) ➔ in table
14. Your output will be like the as shown in the table.

15. **After filling the hand simulation, compute the following from the simulation:**

- **The percentage of lost.**

(3 Points)

$$\frac{number\ of\ lost\ cars}{total\ cars} = \frac{1}{20}$$

- **Average arrival rate per hour**

(3 Points)

$$\frac{number\ of\ cars}{samulation\ time} = \frac{20 * 60}{110} = 10.91\ car/hr$$

- **Average time for service.**

(3 Points)

$$\frac{sum\ of service\ times\ for\ cars\ entered}{number\ of\ cars\ entered} = \frac{209 - 13}{19} = 10.3$$

- **Average waiting time if car waits**

(3 Points)

$$\frac{sum\ of waiting\ time}{number\ of\ cars\ waited} = \frac{30}{12} = 2.5\ \ min$$

- **Percentage of cars getting the service directly without waiting.**

(3 Points)

$$\frac{number\ of\ cars\ that\ did\ not\ wait}{total\ number\ of\ cars} = \frac{7}{20} = 0.35$$

- **Percentage of time Machine 1 is in operation.**

(3 Points)

$$\frac{time\ of\ cars\ in\ maching 1}{samlution\ time} = \frac{100}{110} = 0.909$$

- **Probability empty parking (No one in the parking)**

(3 Points)

$$\frac{number\ of\ time\ empty\ parking}{sumlation\ time} = \frac{69}{110} = 0.67$$