## Second Midterm Exam Academic Year 1440-1441 Hijri- First Semester



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## Question \# 1:

Consider a car insurance company collected the following report about their clients:

|  | Male |  |  |  | Female |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age | Total \# | \#accidents | Min <br> Claim | Max <br> Claim | Total \# | \#accidents | Min <br> Claim | Max <br> Claim |
| $18-28$ | 113 | 284 | 350 | 9000 | 312 | 435 | 550 | 7500 |
| $28-38$ | 258 | 314 | 950 | 13000 | 271 | 361 | 1050 | 11300 |
| $38-48$ | 546 | 395 | 1300 | 6500 | 687 | 302 | 1450 | 9300 |
| $48-58$ | 420 | 150 | 2000 | 11000 | 348 | 129 | 2800 | 11800 |
| $58-68$ | 302 | 145 | 4000 | 20000 | 354 | 159 | 3700 | 32000 |
| More 68 | 93 | 23 | 15000 | 80000 | 12 | 43 | 21000 | 65000 |
| Total | $\mathbf{1 7 3 2}$ | $\mathbf{1 3 1 1}$ |  |  | $\mathbf{1 9 8 4}$ | $\mathbf{1 4 2 9}$ |  |  |

You are asked to simulation the information of clients based on the above table:

1. List the variables that you will simulate from the table for each client.
2. Write the algorithm of each variable you will simulate.

## Question \# 2:

Consider an investment with a monthly return on investment. The monthly percentage of return on investment is a random variable $X \%$ given by the following probability function:

$$
f(x)= \begin{cases}\frac{|x|}{10}, & -2 \leq x \leq 4 \\ 0, & \text { otherwise }\end{cases}
$$

Answer the following:

1) Write the inverse algorithm for generating the monthly percentage of return on investment.
2) Using simulation and the following $U[0,1]$ numbers, evaluate the results of the investment for one year with initial budget of 100,000 SR.

| 0.032 | 0.823 | 0.865 | 0.732 | 0.940 | 0.618 | 0.574 | 0.570 | 0.910 | 0.833 |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.138 | 0.776 | 0.911 | 0.259 | 0.458 | 0.343 | 0.105 | 0.940 | 0.188 | 0.343 |
| 0.623 | 0.306 | 0.797 | 0.238 | 0.897 | 0.020 | 0.434 | 0.135 | 0.219 | 0.328 |
| 0.776 | 0.613 | 0.623 | 0.652 | 0.110 | 0.813 | 0.629 | 0.269 | 0.077 | 0.376 |
| 0.301 | 0.120 | 0.491 | 0.145 | 0.448 | 0.048 | 0.049 | 0.846 | 0.590 | 0.509 |
| 0.691 | 0.684 | 0.880 | 0.963 | 0.526 | 0.716 | 0.495 | 0.981 | 0.840 | 0.467 |

3) From simulation, compute the average and standard deviation of the monthly percentage of return on investment.
4) From simulation compute the probability that the company will pay more than 3000 SR.
5) From simulation output, what is the probability of losing.

## Question \# 3:

An insurance policy pays for the insured 1000 SR per day spent admitted in a hospital for up to three days. If the insured spend more than three days, the insurance company pays 500 SR per day for each extra day of hospitalization thereafter. The number of days that an insured customer spends in the hospital, X , is a discrete random variable with probability function:

$$
P[X=k]= \begin{cases}\frac{6-k}{15}, & k=1,2,3,4,5 \\ 0, & \text { otherwise }\end{cases}
$$

Answer the following:

1) Write the inverse algorithm for generating number of days for the insured clients
2) Assume that the company wants to analyze the insurance policy using simulation for 15 hospitalized clients. Draw the flow chart for the simulation.
3) Using simulation algorithm and the following $U[0,1]$ numbers, give the results for the 15 insured clients.

| 0.032 | 0.823 | 0.865 | 0.732 | 0.940 | 0.618 | 0.574 | 0.570 | 0.910 | 0.833 |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.138 | 0.776 | 0.911 | 0.259 | 0.458 | 0.343 | 0.105 | 0.940 | 0.188 | 0.343 |
| 0.623 | 0.306 | 0.797 | 0.238 | 0.897 | 0.020 | 0.434 | 0.135 | 0.219 | 0.328 |
| 0.776 | 0.613 | 0.623 | 0.652 | 0.110 | 0.813 | 0.629 | 0.269 | 0.077 | 0.376 |
| 0.301 | 0.120 | 0.491 | 0.145 | 0.448 | 0.048 | 0.049 | 0.846 | 0.590 | 0.509 |
| 0.691 | 0.684 | 0.880 | 0.963 | 0.526 | 0.716 | 0.495 | 0.981 | 0.840 | 0.467 |
| 0.006 | 0.730 | 0.534 | 0.494 | 0.984 | 0.974 | 0.149 | 0.024 | 0.484 | 0.469 |
| 0.413 | 0.904 | 0.120 | 0.849 | 0.329 | 0.575 | 0.417 | 0.354 | 0.484 | 0.334 |
| 0.936 | 0.191 | 0.072 | 0.079 | 0.575 | 0.264 | 0.591 | 0.620 | 0.405 | 0.788 |
| 0.423 | 0.092 | 0.898 | 0.611 | 0.772 | 0.879 | 0.774 | 0.626 | 0.039 | 0.471 |

4) Compute the average and standard deviation of amount that the insurance company will pay per claim.
5) Compute the average and standard deviation of number of days that a client spends in the hospital.
6) From simulation compute the probability that the company will pay more than 3000 SR.

## Question \#4:

Patients arrive to a hospital's emergency room according to a Poisson process with rate 8 patients per hour. Patients come in three different health conditions. The patients are categorized according to their condition as critical, serious, or stable. In the past year, statistics show that:
i. $10 \%$ of the emergency room patients were critical; and take random amount of treatment of Erlang with parameters $\alpha=2$ and $\lambda=0.5$ per hour
ii. $30 \%$ of the emergency room patients were serious; and take random amount of treatment of Exponential with average time of 2 hours.
iii. the rest of the emergency room patients were stable; and take random amount of treatment of integer uniform between 15 min and 30 min .
After treatment at the ER, statistics show that:

- $40 \%$ of the critical patients died;
- $10 \%$ of the serious patients died; and
- $1 \%$ of the stable patients died.


## Answer the following:

1. Write the Step for simulation of this process
2. Starting from 6:00 am and using the $U[0,1]$ number below, do the simulation for the ER for 15 patients and show the details of each arrival: arrival time, patient's category, treatment time, and patient's exit condition.
3. What is the probability that any patient enter the Emergency room will live?
4. Given that a patient survived, calculate from simulation the probability that the patient was categorized as serious upon arrival.

| 0.032 | 0.823 | 0.865 | 0.732 | 0.940 | 0.618 | 0.574 | 0.570 | 0.910 | 0.833 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.138 | 0.776 | 0.911 | 0.259 | 0.458 | 0.343 | 0.105 | 0.940 | 0.188 | 0.343 |
| 0.623 | 0.306 | 0.797 | 0.238 | 0.897 | 0.020 | 0.434 | 0.135 | 0.219 | 0.328 |
| 0.776 | 0.613 | 0.623 | 0.652 | 0.110 | 0.813 | 0.629 | 0.269 | 0.077 | 0.376 |
| 0.301 | 0.120 | 0.491 | 0.145 | 0.448 | 0.048 | 0.049 | 0.846 | 0.590 | 0.509 |
| 0.691 | 0.684 | 0.880 | 0.963 | 0.526 | 0.716 | 0.495 | 0.981 | 0.840 | 0.467 |
| 0.006 | 0.730 | 0.534 | 0.494 | 0.984 | 0.974 | 0.149 | 0.024 | 0.484 | 0.469 |
| 0.413 | 0.904 | 0.120 | 0.849 | 0.329 | 0.575 | 0.417 | 0.354 | 0.484 | 0.334 |
| 0.936 | 0.191 | 0.072 | 0.079 | 0.575 | 0.264 | 0.591 | 0.620 | 0.405 | 0.788 |
| 0.423 | 0.092 | 0.898 | 0.611 | 0.772 | 0.879 | 0.774 | 0.626 | 0.039 | 0.471 |

## Question \# 5:

Consider the following probability density function:

$$
f(x)= \begin{cases}0.5003 e^{-x / 2}, & 0<x<15 \\ 0, & \text { otherwise }\end{cases}
$$

Write the algorithm to generate random numbers from $f(x)$ using acceptance/rejection method with majorizing function $g(x)$ fixed function.

| $\mathbf{n}$ | $\mathbf{U 1}$ | $\mathbf{W}$ | $\mathbf{f}(\mathbf{W})$ | $\mathbf{U 2}$ | $\mathbf{g}(\mathbf{W})$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.280 |  |  | 0.165 |  |  |  |
| 2 | 0.318 |  |  | 0.684 |  |  |  |
| 3 | 0.270 |  |  | 0.768 |  |  |  |
| 4 | 0.890 |  |  | 0.667 |  |  |  |
| 5 | 0.091 |  |  | 0.257 |  |  |  |
| 6 | 0.238 |  |  | 0.084 |  |  |  |
| 7 | 0.611 |  |  | 0.494 |  |  |  |
| 8 | 0.772 |  |  | 0.849 |  |  |  |

