

**King Saud University/College of Sciences/Department of Mathematics**  
**Semester 442 / MATH-244 / Quiz-2**

**Max. Marks: 10**

**Max. Time: 35 Min.**

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**Note: Choose the correct answers to all the 6 questions. Calculators are not allowed!**

**Question 1** [Marks: 1.5]: Let  $B = \{(2, 1, 0), (-3, 0, 1)\}$  be a given basis for the null space  $N(A)$ , where  $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \end{bmatrix}$ . Find the vector  $u \in N(A)$  that has the coordinate vector  $[u]_B = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ .

- (a)  $u = (-3, 0, 1)$       (b)  $u = (-9, -3, 1)$       (c)  $u = (-3, 1, 0)$       (d)  $u = (-6, -3, 0)$ .

**Question 2** [Marks: 2]: Let  $B = \{b_1, b_2, b_3\}$  and  $C = \{c_1, c_2, c_3\}$  be two ordered bases for the Euclidean space  $\mathbb{R}^3$ . If  ${}_C P_B = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$  is the transition matrix from  $B$  to  $C$ , then find the coordinate vector  $[b_1 - b_3 + b_2]_C$ .

- (a)  $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$       (b)  $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$       (c)  $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$       (d)  $\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ .

**Question 3** [Marks: 1.5]: If  $\text{nullity}(A^T) = 3$  where  $A$  is a matrix of size  $9 \times 5$ , then find  $\text{rank}(A)$ .

- (a) 1                                      (b) 2                                      (c) 3                                      (d) 6.

**Question 4** [Marks: 1.5]: Let  $M_{2 \times 2}(\mathbb{R})$  be the vector space of all  $2 \times 2$  real matrices with the standard inner product. Which of the following statements is correct if  $A = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$ ?

- (a) Angle between  $A$  and  $B$  is  $60^\circ$                                       (b)  $A$  and  $B$  are linearly dependent  
(c)  $A$  and  $B$  are orthogonal                                      (d)  $\{A, B\}$  is a basis for  $M_{2 \times 2}(\mathbb{R})$ .

**Question 5** [Marks: 2]: If  $E = \{a, b, c\}$  is an orthogonal set of non-zero vectors in the Euclidean space  $\mathbb{R}^4$ , then which of the following statements is true?

- (a)  $E$  is an orthonormal basis for  $\mathbb{R}^3$ .  
(b)  $\text{span}(E)$  is a proper subspace of  $\mathbb{R}^3$ .  
(c)  $E$  is an orthogonal basis of  $\text{span}(E)$ .  
(d)  $E$  is linearly dependent in  $\mathbb{R}^4$ .

**Question 6** [Marks: 1.5]: Let  $V \neq \{0\}$  be a finite-dimensional inner product space. Which of the following statements is correct?

- (a)  $V$  is a basis of itself.                                      (b)  $V$  may not have any basis.  
(c)  $V$  is a finite set.                                      (d)  $V$  must have an orthonormal basis.