King Saud University,
College of Sciences
Mathematical Department.

Mid-Term Exam/S1/2022
Full Mark:30. Time 2H
05/10/2022

Question 1. [5,4] **a)** Determine and sketch the largest local region of the $xy-$plane for which the initial value problem

$$\begin{cases} (x^2-1)dy + \left(3+y+\sqrt{y-4x}\right)dx = 0 \\ y(0) = 2, \end{cases}$$

has a unique solution.

b) Find the general solution of the differential equation

$$(xy+x)dx = (x^2y^2+x^2+y^2+1)dy = 0.$$

Question 2. [4, 4]. **a)** Show that $\mu(x,y) = xy$ is an integrating factor for the differential equation

$$\left(\frac{y}{x^2}+\frac{2\ln y}{y}\right)dx + \left(\frac{x}{y^2}+\frac{2\ln x}{x}\right) = 0, \ x>0, \ y>0,$$

and hence solve the differential equation.

b) obtain the general solution of the differential equation

$$(2x+y)\frac{dy}{dx} - 1 - (2x+y)^2 = 0, \ 2x+y \neq 0.$$

Question 3. [4, 4]. **a)** Solve the initial value problem

$$\begin{cases} (6xy+2y^2-5)dx + (3x^2+4xy-6)dy = 0 \\ y(1) = 1 \end{cases}$$

b) Solve the differential equation

$$5xy^2y' + y^3 = 32(1+\ln x)y^{-2}, \quad x>0, \ y \neq 0.$$

Question 4. [5] Assume that the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 290 K (here K stands for Kelvin) and the substance cools from 370 K to 330 K in 10 minutes, find when the temperature will be 295 K.