King Saud University,
College of Sciences
Mathematical Department.

Mid-Term Exam/S2/2023
Full Mark:30. Time 2H
11/01/2022

Question 1. **[5,4]** **a)** Determine and sketch the largest local region of the $xy-$plane for which the initial value problem

$$\begin{cases} (x^2-4x+3)\frac{dy}{dx} = \sqrt{1-\ln(1-y)} \\ y(2)=0, \end{cases}$$

has a unique solution.

b) Find the solution of the differential equation

$$\frac{dy}{dx} = xy + \sqrt{xy} + x\sqrt{y} + y\sqrt{x}, \ x>0, \ y>0$$

Question 2. **[4, 4]**. **a)** Solve the initial value problem

$$\begin{cases} (x^2 - x\sec^2 y)dy + (2xy - \tan y)dx = 0 \\ y(1)=\pi. \end{cases}$$

b) Obtain the solution of the differential equation

$$x^2(x^2+1)\frac{dy}{dx} - [\ln(x^2+1)+1]y^4 - x(x^2+1)y = 0, \quad x>0.$$

Question 3. **[4, 4]**. **a)** Solve the initial value problem

$$\begin{cases} x\sin\left(\frac{y}{x}\right)\frac{dy}{dx} = y\sin\left(\frac{y}{x}\right) - x, \quad x>0 \\ y(1)=\frac{\pi}{2}. \end{cases}$$

b) Verify that $\mu(x,y) = x^{-2}y^{-2}$ is an integrating factor for the differential equation

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0, \quad x>0, \ y>0.$$

Then solve it.

Question 4. **[5]** In a murder investigation a corpse was found by a detective at exactly 8 PM. Being alert, the detective also measured the body temperature and found it to be 70^0F. Two hours later, the detective measured the body temperature again and found it to be 60^0F. If the room temperature is 50^0F, and assuming that the temperature of the person before death was 99^0F, at what time did the murder occur?.