

Question 1[4]. Find and sketch the largest local region of the xy -plane for which the initial value problem

$$\begin{cases} \sqrt{2 - \ln(y - 3)}dx - (x^2 - 5x - 6)dy = 0 \\ y(5) = 4, \end{cases}$$

has a unique solution.

Question 2[4+4]. a) Solve the initial value problem

$$\begin{cases} x(1 + y)dx + (x^2 - 1 + y^2)dy = 0, & y > -1, x \neq 0, \\ y(1) = 1. \end{cases}$$

b) Reduce the following equation to a Bernoulli equation and obtain its general solution

$$(1 + x^2)^2 y \frac{dy}{dx} + 2x(1 + x^2)y^2 - 1 = 0, \quad y \neq 0.$$

Question 3[4+4]. a) Solve the differential equation

$$\frac{dy}{dx} - e^{-y} \cos(2x) - 2 = -2 \cos^2(x) - e^{-y}, \quad y \neq 0.$$

b) Find the general solution of the differential equation

$$(2x + y + x \ln x)dx - 2xdy = 0, \quad x > 0.$$

Question 4[5]. The sum of 5000 SAR is invested at a rate of 8% per year. Compounded contiguously. What will be the amount after 25 years.‘