# Discrete Mathematics
# Chapter 01
# Number Systems

إعداد وتقديم


دكتور أحمد السيد

كلية العلوم– قسم الرياضيات –جامعة الملك سعود

aalsayedmm@gmail.com

٠٥٥٩٥٩٦٧٢٠



بسم الله الرحمن الرحيم

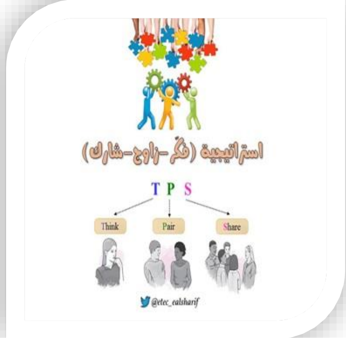
# استراتيجيات التعليم

المناقشة

العرض التقدمي


العصف الذهني

التطبيق العملي




فكر– زاوج –شارك

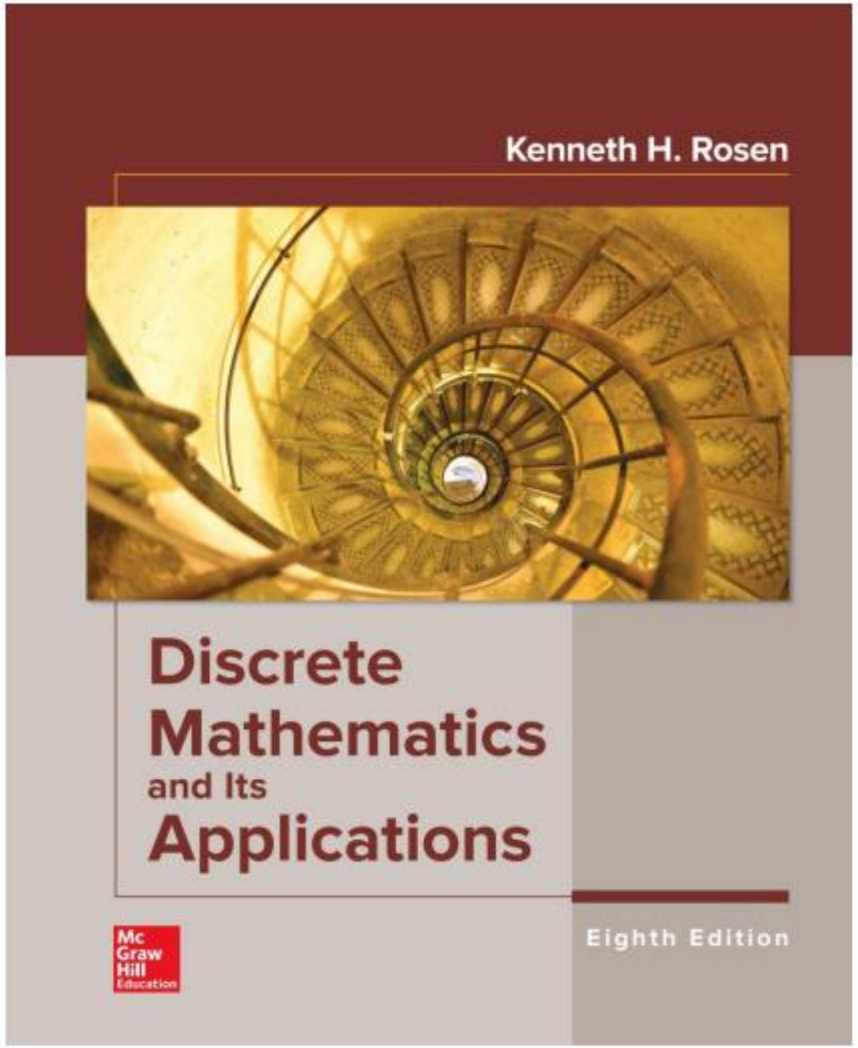
المحاضرة

الاكتشاف

البحث والاستقصاء

- Course code: 153 Math
- Course name: Discrete Mathematics
- Level: 1
- third Semester 2st Year / B.Sc.
- Course Credit: 3 +2 credits

# Lectures Reference

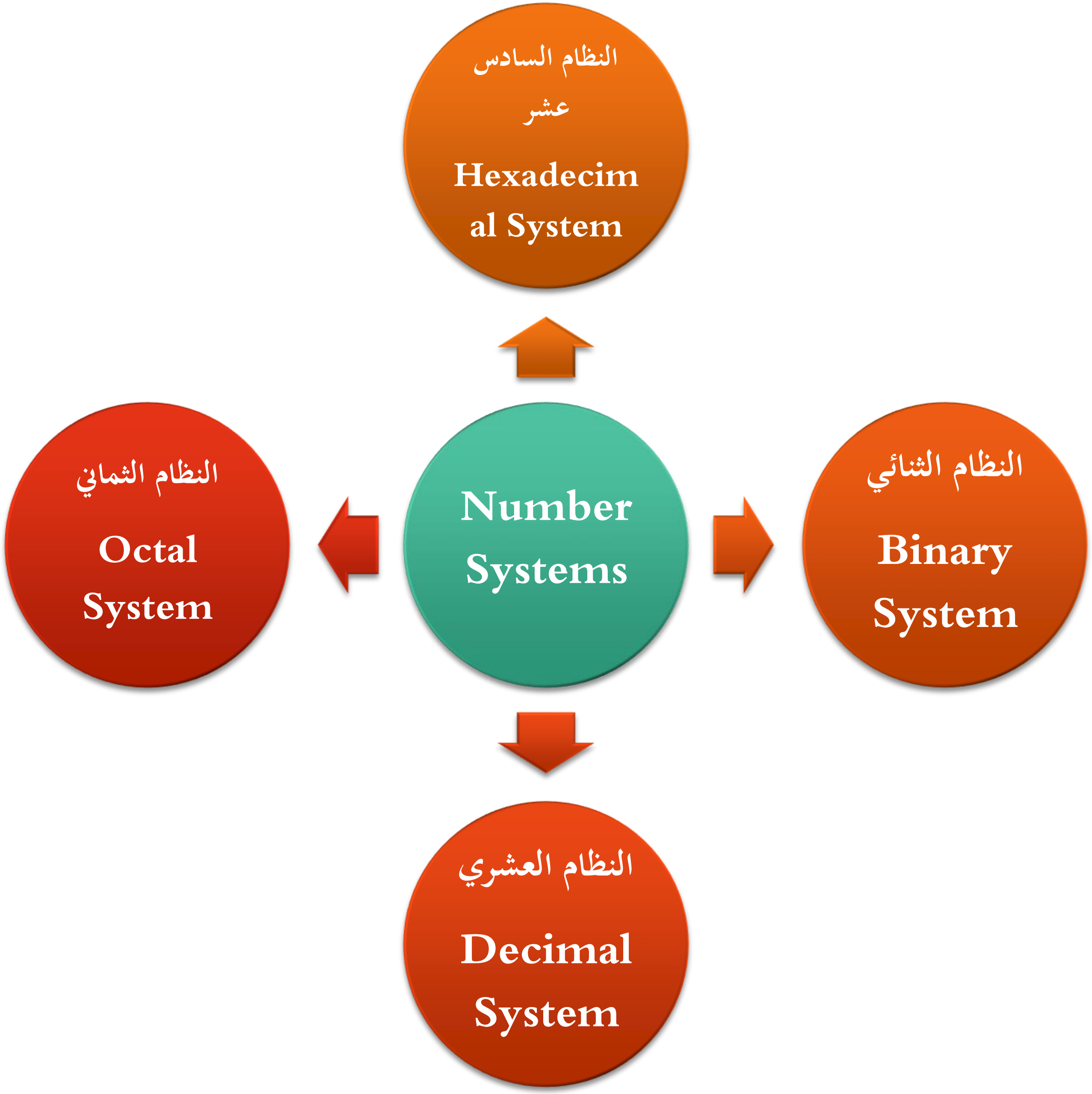



**Textbook**

2019

# Course Outcomes

- Learn how to think mathematically.
- Grasp the basic logical and reasoning mechanisms of mathematical thought.
- Acquire logic and proof as the basics for abstract thinking.
- Improve problem-solving skills.
- Grasp the basic elements of induction, recursion, combination and discrete structures.

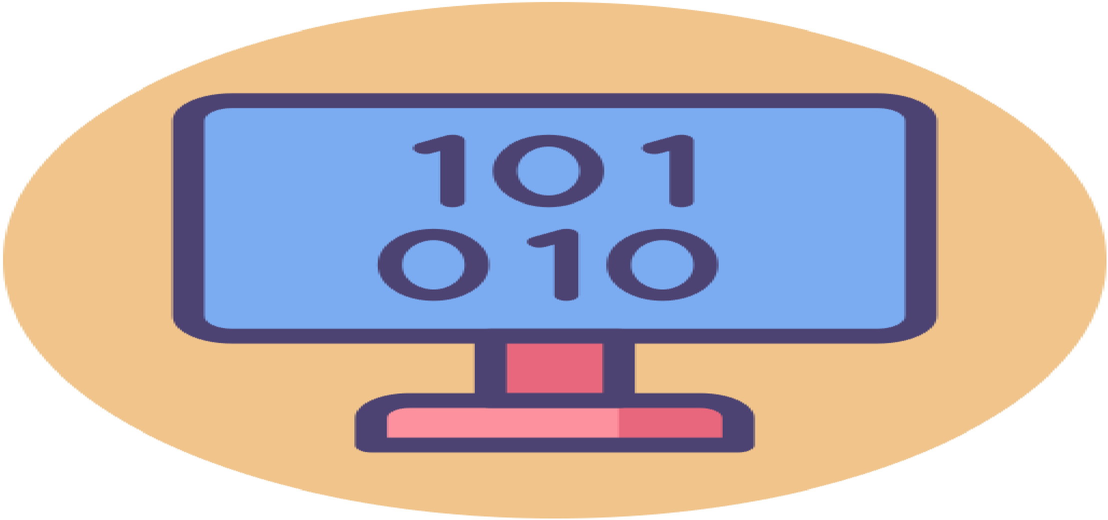

Number Systems
النظام السادس عشر
Hexadecimal System
النظام الثمانى
Octal System
النظام الثنائي
Binary System
النظام العشري
Decimal System

# Content

| Week | Basic and support material to be covered |
|---|---|
| (1) | Introduction to Number Systems: Binary System (Binary to Decimal Conversion |
| (2) | Introduction to Number Systems: Decimal to Binary Conversion – Arithmetic: addition, subtraction, multiplication),) |
| (3) | Introduction to Number Systems:, Octal Number System (Conversions and Arithmetic), Hexadecimal Number System (Conversions and Arithmetic) |

٨

# Binary System



# Binary System

- The binary system is a different number system.
- The coefficients of the binary numbers system have only two possible values: 0 or 1.
- Each coefficient d is multiplied by $2^n$.
- For example, the decimal equivalent of the binary number 11010.11 is 26.75, as shown from the multiplication of the coefficients by powers of 2:
- $1x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 0x2^0 + 1x2^{-1} + 1x2^{-2} = 26.75$
- the digits in a binary number are called bits.

# Binary to Decimal Conversion

A binary number can be converted to decimal by forming the sum of powers of 2 of those coefficients whose value is 1.

## Example 2

Convert the binary number $(1101001)_2$ to decimal.

## Solution:

$(1101001)_2 = 1x2^6 + 1x2^5 + 0x2^4 + 1x2^3 + 0x2^2 + 0x2^1 + 1x2^0$

$= 64 + 32 + 0 + 8 + 0 + 0 + 1$

$= 105$

# Binary Fractions

- decimal number system, each digit of a number represents an increasing power of ten. This is true for all digits to the left of the decimal point ... for numbers to the right; each digit represents a decreasing power of ten.
- In binary, the concept is the same, except that digits to the right of the "binary point" represent a decreasing power of two.

**Example3:**

Convert the $(110.001)_2$ to decimal.

**Solution:**

$(110.001)_2 = 1x2^2 + 1x2^1 + 0x2^0 + 0x2^{-1} + 0x2^{-2} + 1x2^{-3}$

$= 4+2+0+0+0+0.125$

$= 6.125$

**Example 4**

Convert $(0.11101)_2$ to decimal.

**Solution:**

$(0.11101)_2 = 1x2^{-1} + 1x2^{-2} + 1x2^{-3} + 0x2^{-4} + 1x2^{-5}$

$= 0.5 + 0.25 + 0.125 + 0 + 0.03125$

$= 0.90625$

# Decimal to Binary Conversion

*Algorithm 1*

To convert from a base-10 integer numeral to its base-2 (binary) equivalent,

* the number is divided by two, and the remainder is the least-significant bit.
* The (integer) result is again divided by two, its remainder is the next least significant bit.
* This process repeats until the quotient becomes zero.

# Decimal to Binary Conversion



**Example 5**

*Convert* $23_{10}$ to binary number.

**Solution:**

| | | Quotient | | Remainder |
|---|---|---|---|---|
| 1. | 23 ÷ 2 = | 11 | | 1 |
| 2. | 11 ÷ 2 = | 5 | | 1 |
| 3. | 5 ÷ 2 = | 2 | | 1 |
| 4. | 2 ÷ 2 = | 1 | | 0 |
| 5. | 1 ÷ 2 = | 0 | | 1 |
| | | | | |
| | | | | |
| | sign bit | | | |

The answer is **found by reading "up" from the bottom.**

Therefore, $23_{10} = 1\ 0111_2$

# Decimal to Binary Conversion

King Saud University

**Example 6:**

Convert **46** $_{10}$ to base 2.

**Solution:**

| | | Quotient | | Remainder |
|---|---|---|---|---|
| 1. | 46 ÷ 2 = | 23 | | 0 |
| 2. | 23 ÷ 2 = | 11 | | 1 |
| 3. | 11 ÷ 2 = | 5 | | 1 |
| 4. | 5 ÷ 2 = | 2 | | 1 |
| 5. | 2 ÷ 2 = | 1 | | 0 |
| 6. | 1 ÷ 2 = | 0 | | 1 |
| | | | | |
| | sign bit | | | |

Therefore, $46_{10}$ = $10\ 1110_2$

# Decimal Fractions to Binary Fractions Conversions

To convert the fractional part successive multiplications are done instead of divisions. In each case the remaining fractional part is used in the succeeding multiplication

# Decimal Fractions to Binary Fractions Conversions

## Example 7

Convert the decimal fraction **$0.59375_{10}$** to binary fraction.

## Solution:

To convert the fractional part $(0.59375)_{10}$, successive multiplications are done instead of divisions. In each case the remaining fractional part is used in the succeeding multiplication

| | | Integer | | Fraction |
|---|---|---|---|---|
| 1. | 0.59375 x 2 = | 1 | ↓ | 0.1875 |
| 2. | 0.1875 x 2 = | 0 | | 0.375 |
| 3. | 0.375 x 2 = | 0 | | 0.75 |
| 4. | 0.75 x 2 = | 1 | | 0.5 |
| 5. | 0.5 x 2 = | 1 | | .0 |

Therefore $0.59375_{10} = 0.10011_{2}$

# Decimal to Binary Conversion

**Example 8**

**Convert $46.59375_{10}$ to base 2.**

**Solution:**

* **First, convert the whole number (46) using the previous method.**

$46_{10} = 10\ 1110_2$

* **Next, convert the fractional part (0.59375), also use the previous method.**

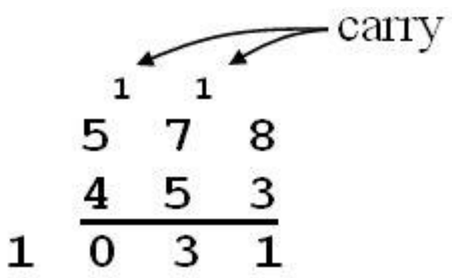
$0.59375_{10} = 0.10011_2$

* **Therefore, $46.59375_{10} = 10\ 1110.10011_2$**

# ***Arithmetic in the Binary System***

# Binary Addition

* The process for adding binary numbers is the same in any number system, except that you must be aware of when (and what) to ***"carry".***
* In the decimal system, a carry occurs when the sum of 2 digits is 10 or more. For example,




* In binary, a carry occurs when the sum of 2 binary digits is 2 or more. This leaves only four possibilities:

$0 + 0 = 0_2$

$0 + 1 = 1_2$

$1 + 1 = 10_2$ (therefore, 0 with a carry)

$1 + 1 + 1 = 11_2$ (therefore, 1 with a carry)

# Binary Addition

**Addition table**

| + | 0 | 1 |
|---|---|---|
| **0** | 0 | 1 |
| **1** | 1 | 10 |

## Example9:

Add the binary numbers

**$0011\ 0010_2 + 0011\ 0111_2$:**

## Solution:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 1 | | 1 | 1 | | | | |
| | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | = | 50 |
| + | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | = | 55 |
| | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | = | 105 |

# Binary Addition



## Example10:

Add the binary numbers

$1011.01_2 + 11.011_2$

**Addition table**

| + | 0 | 1 |
|---|---|---|
| **0** | 0 | 1 |
| **1** | 1 | 10 |

Solutior

```
  1 1    1
 1011.01
+  11.011
---------
 1110.101
```

# Binary Subtraction

For binary subtraction, there are *four* facts instead of one hundred:

- 0 - 0 = 0
- 1 - 0 = 1
- 1 - 1 = 0
- 10 - 1 = 1

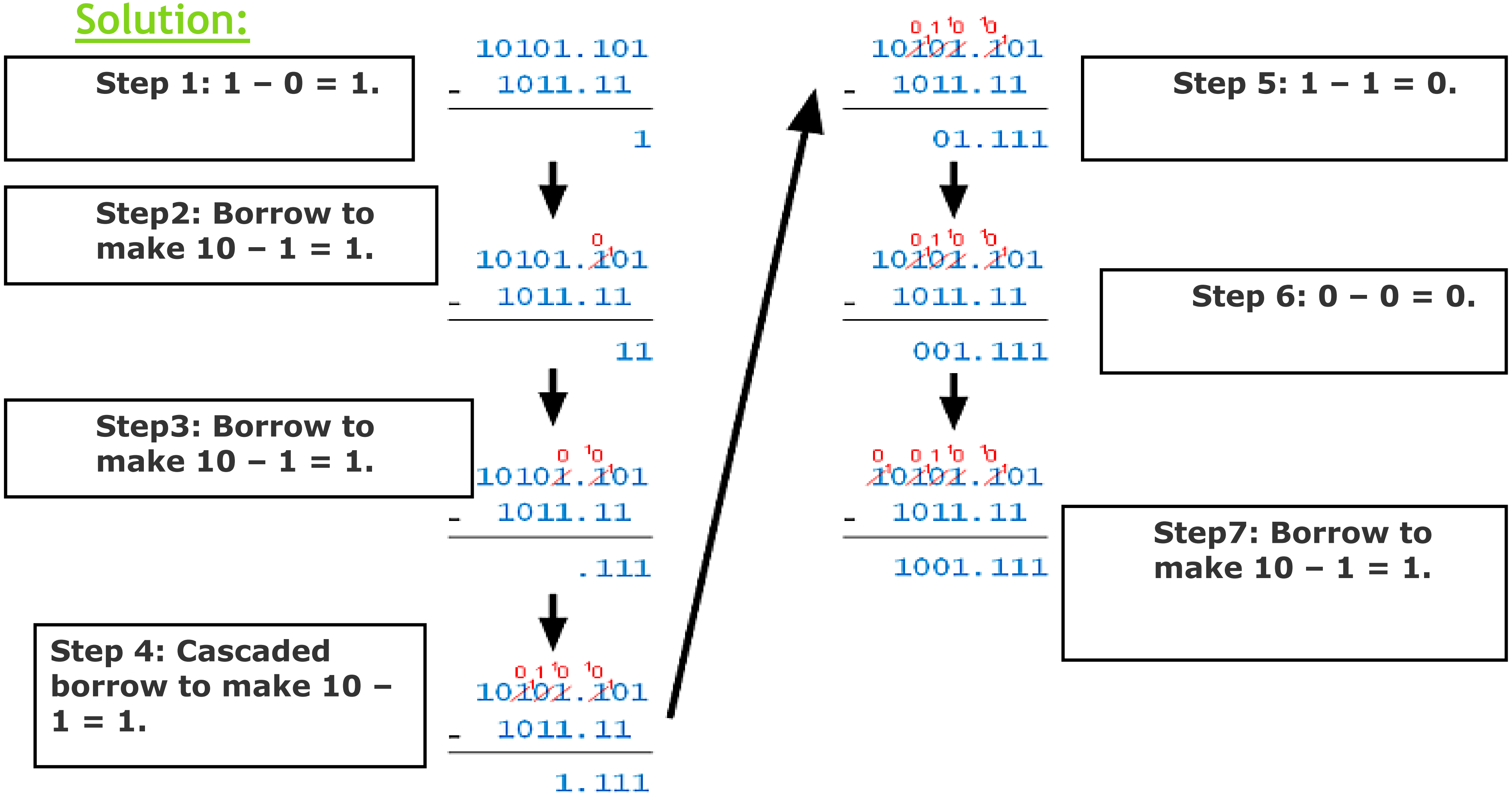
# Binary Subtraction

- 0 – 0 = 0
- 1 – 0 = 1
- 1 – 1 = 0
- 10 – 1 = 1

## Example 11:

Subtract: **10101.101–1011.11**

## Solution:



## Checking the Answer

You can check the answer in a few ways. One way is to add the result (1001.111) to the subtrahend (1011.11), and check that that answer matches the minuend (10101.101):

```
 1   1 1 1   1
   1001.111
+  1011.11
-----------
  10101.101
```

# Binary Multiplication

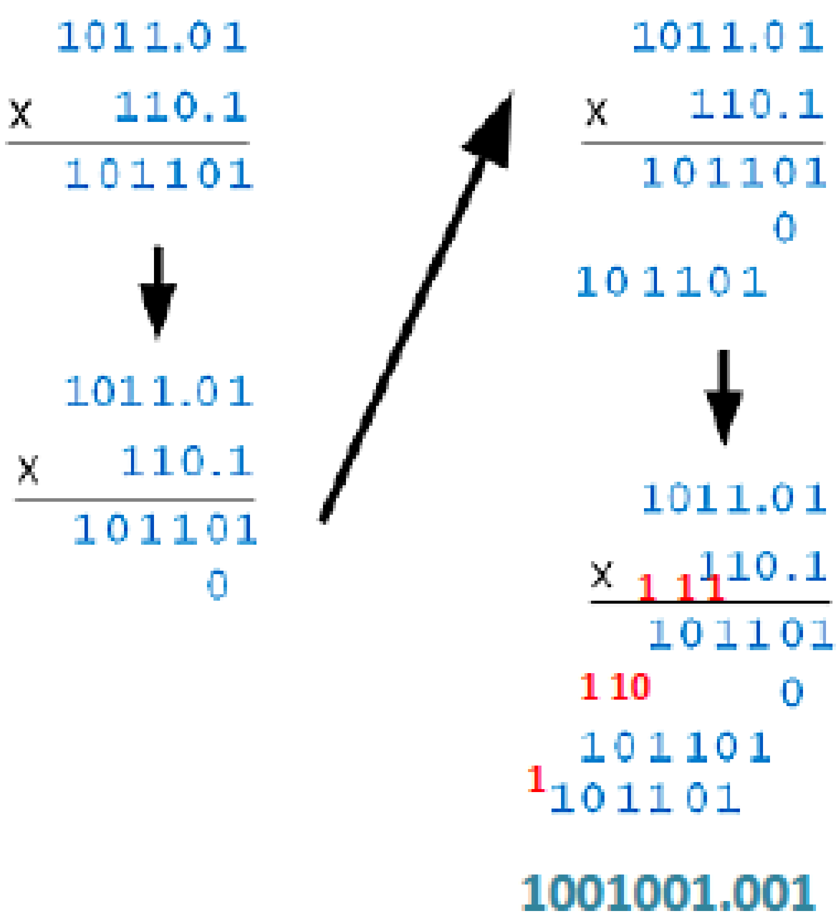
Binary multiplication uses the same algorithm as in decimal, but uses just three order-independent facts:

- *0 x 0 = 0,
- *1 x 0 = 0,
- *1 x 1 = 1

Example 12:

Multiply 1011.01 x 110.1

**Solution:**



# *Octal System*

# Octal System

* The octal, or base 8, number system is a common system used with computers.
* Because of its relationship with the binary system, it is useful in programming some types of computers.
* Octal is fancy for Base Eight meaning eight symbols are used to represent all the quantities. They are 0, 1, 2, 3, 4, 5, 6, and 7.

| Octal | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 10 | 11 | 12 … | 17 | 20 … | 30 … | 77 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Deci mal | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 … | 15 | 16 … | 24 … | 63 | 64 |

# Octal to decimal conversion



An octal number can be converted to decimal by forming the ***sum of powers of 8*** of the coefficients.

**Example1**

Convert $764_8$ to decimal:

**Solution**

$764 = 7x8^2 + 6x8^1 + 4x8^0 = 448+48+4 = 500_{10}$

**Example**

$65_8 = 6x8 + 5 = 53_{10}$

**Example 2**

Convert 0.2358 to decimal:

**Solution:**

$0.235_8 = 2x8^{-1} + 3x8^{-2} + 5x8^{-3}$

$= 2x0.125 + 3x8^{-2} + 5x8^{-3}$

= ٠,٣٠٦٦٤

# ***Decimal to octal conversion***

To convert a decimal fraction to octal, **multiply by 8; the integer part of the result is the first digit** of the octal fraction. Repeat the process with the fractional part of the result, until it is null or within acceptable error bounds.

# *Conversion of decimal fraction to octal fraction*

**Example3:**

Convert 0.1640625 to octal:

0.1640625x8= 1.3125 =0.3125 + **1**

0.3125 x8= 2.5=0.5+ **2**

0.5x8 = 4= **4**

Therefore, $0.1640625_{10} = 0.\mathbf{124}_8$.

**Example4:**

convert $(0.523)_{10}$ to octal equivalent up to 3 decimal places.

**Solution**

0.523 x 8 = 4.184, its integer part is **4**

0.184 x 8 = 1.472, its integer part is **1**

0.472 x 8 = 3.776, its integer part is **3**

So the answer is $(0.\mathbf{413}..)_8$

## Conversion of decimal to octal ( base 10 to base 8)

To convert from a base-10 integer numeral to its base-2 (binary) equivalent, the number is divided by two, and the remainder is the least-significant bit. The (integer) result is again divided by two, its remainder is the next least significant bit. This process repeats until the quotient becomes zero.

**Example5:**

**convert $(177)_{10}$ to octal equivalent**

177 / 8 = 22 remainder is **1**

22 / 8 = 2 remainder is **6**

2 / 8 = 0 remainder is **2**

Answer = **261**

Note: the answer is read **from bottom to top** as $(261)_8$, the same as with the binary case.

# Octal to binary conversion

To convert octal to binary, **replace each octal digit by its binary representation in 3 bits**, so **add zeros to the left if necessary.**

## Example 6:

Convert $51_8$ to binary:

## Solution

$٥_{٨}$= 101

$1_8$=001

Therefore, $51_8$ = 101 $001_2$.

| Decimal Base-10 | Binary Base-2 | Octal Base-8 | Hexadecimal Base-16 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 2 | 10 | 2 | 2 |
| 3 | 11 | 3 | 3 |
| 4 | 100 | 4 | 4 |
| 5 | 101 | 5 | 5 |
| 6 | 110 | 6 | 6 |
| 7 | 111 | 7 | 7 |
| 8 | 1000 | 10 | 8 |
| 9 | 1001 | 11 | 9 |

# Binary to Octal conversion

In order to convert the Binary number into its equivalent octal numbers, split the given binary number into groups and each group should contain ***three binary bits (because $2^3$=8), add zeros to the left if necessary, and then converting each group into its equivalent octal number.***

**Example 7:**

convert binary **1010111100** to octal.

**Solution:**

$1010111100_2 = 1274_8$.

| | | | |
|---|---|---|---|
| 001 | 010 | 111 | 100 |
| 1 | 2 | 7 | 4 |

**Example 7:**

Convert binary **11100.01001** to octal:

**Solution:**

$11100.01001_2 = 34.22_8$.

| | | | | |
|---|---|---|---|---|
| 011 | 100 | . | 010 | 010 |
| 3 | 4 | . | 2 | 2 |

# Arithmetic in octal system

# Octal Addition

**Addition of the octal number is carried out in the same way as the decimal addition is performed. The steps are given below:**

1. First, add the two digits of the unit column of the octal number in decimal.
2. This process is repeated for each larger significant digit of the octal number.
3. During the process of addition, if the sum is less than or equal to 7, then it can be directly written as an octal digit.
4. If the sum is greater than 7, then subtract 8 from the digit and carry 1 to the next digit position.
5. Note that in this addition the largest octal digit is 7.

## Example 8:

**Evaluate:**

(i) $(162)_8 + (537)_8$

## Solution:

| | |
|---|---|
| 1 1 | <---- carry |
| 1 6 2 | |
| 5 3 7 | |
| 7 2 1 | |

Therefore, sum = $721_8$

(ii) $(136)_8 + (636)_8$

**Solution:**

1 <---- carry

1 3 6

6 3 6

7 7 4 ←---- 6+6=12>8 in decimal, so in octal 6+6= 12-8=14 (4 and carry 1)

**Therefore, sum = $774_8$**

جامعة الملك سعود
King Saud University

(iii) $(25.27)_8 + (13.2)_8$

**Solution:**

```
1              <---- carry

25.27

13.2
-----
40.47
```

5+3= 8>7 in decimal, so in octal 5+3=8-8=10 (0 and carry 1)

**Therefore, sum = $(40.47)_8$**

(iv) $(67.5)_8 + (45.6)_8$

**Solution:**

```
 1 1          <---- carry
 6 7 . 5
 4 5 . 6
1 3 5 . 3
```

5+6=11>8 in decimal, so in octal 5+6=11-8= 13

1+7+5=13>8, so in octal 13=13-8=15 (5 and carry 1)

1+6+4=11>8, so in octal 11= 11-8=13 (3 and carry 1)

**Therefore, sum = $(135.3)_8$**

# Subtraction of Octal Numbers

The subtraction of octal numbers follows the same rules as the subtraction of numbers in any other number system. The only variation is in the quantity of ***the borrow***.

In the decimal system, you had to borrow a group of $10_{10}$.

In the binary system, you borrowed a group of $2_{10}$.

In the octal system you will borrow a group of $8_{10}$.

| DECIMAL | BINARY | OCTAL |
| --- | --- | --- |
| $10_{10}$ | $10_{2}$ | $10_{8}$ |
| $-\ 1_{10}$ | $-\ 1_{2}$ | $-\ 1_{8}$ |
| $9_{10}$ | $1_{2}$ | $7_{8}$ |

| | | | |
| --- | --- | --- | --- |
| 10 | 2 | 8 | Borrow |
| $\cancel{1}0_{10}$ | $\cancel{1}0_{2}$ | $\cancel{1}0_{8}$ | |
| $-\ 1_{10}$ | $-\ 1_{2}$ | $-\ 1_{8}$ | |
| $9_{10}$ | $1_{2}$ | $7_{8}$ | |

# Example 9

Subtract $532_8$-1

**Solution:**

```
    +8
     2  +8
  4  5 3 2_8
 -   1 7 4_8
 ___________
     3 3 6
```

**Steps:**

1. Since 2<4 then borrow 1 from 3 and add 8 to 2.
2. 2+8=10 in decimal so 10-4=6.
3. In the second column we have 2 after borrowing but 2<7, so we need to borrow 1 from 5 and add 8 to 2.
4. 2+8 = 10, 10-7=3
5. We have 4 after borrowing so we have 4-1=3.

# ***Hexadecimal System***

# *Hexadecimal System*

Hexadecimal is the name given to a special number system which uses "16" as a base.

In a hexadecimal (base 16) system, we need 16 single digits.

We could use 0-9, then invent six more.

More conveniently, we use the letters A - F for the remaining digits (where A = 10, B = 11, C = 12, D = 13, E = 14 and F = 15).

# Hexadecimal to Decimal Conversion

The principle of converting a base 16 number to decimal is the same as previously discussed, except that each column now represents an increasing power of 16 .

**Example 1**

Convert D30C16 to decimal.

**Solution**

D30C16 = $13 \times 16^3 + 3 \times 16^2 + 0 \times 16^1 + 12 \times 16^0$

= 13x4096 + 3x256 +0 + 12

=53248 + 768 + 12

= 54028

# Decimal To Hexadecimal Conversion

Similarly, any decimal number can be converted to hexadecimal by successive divisions by 16, keeping track of the remainder.

**Example 2**

Convert **$2{,}563_{10}$ to base 16,**

**Solution**

| | | Quotient | | Remainder |
|---|---|---|---|---|
| 1. | 2,563 ÷ 16 = | 160 | | 3 |
| 2. | 160 ÷ 16 = | 10 | | 0 |
| 3. | 10 ÷ 16 = | 0 | | 10 |

The process stops when the quotient becomes zero. The answer is found by reading "up" from the bottom.
Therefore, $\mathbf{2{,}563_{10}} = A03_{16}$

# Binary To Hexadecimal Conversion

Hexadecimal has another important property.

Since there are *exactly 16 hexadecimal digits*, *it **requires exactly 4 bits** to represent every hexadecimal digit (since $2^4 = 16$).*

* ***In order to convert the Binary number into its equivalent octal numbers, split the given binary number into groups and each group should contain four binary bits*** *(because $2^4=16$),*
* ***add zeros to the left if necessary****,*
* *and **then converting each group into its equivalent octal number.***

*This can be shown by the following table on the right:*

# Binary To Hexadecimal Conversion

**Example 3:**

Convert $(100000111001110)_2$ to hexadecimal.

**Solution:**

| Binary | Hex |
|---|---|
| 0000 | 0 |
| 0001 | 1 |
| 0010 | 2 |
| 0011 | 3 |
| 0100 | 4 |
| 0101 | 5 |
| 0110 | 6 |
| 0111 | 7 |
| 1000 | 8 |
| 1001 | 9 |
| 1010 | A |
| 1011 | B |
| 1100 | C |
| 1101 | D |
| 1110 | E |
| 1111 | F |

# Hexadecimal To Binary Conversion

To convert Hexadecimal to binary, replace each Hexadecimal digit by its binary representation in 3 bits, so add zeros to the left if necessary.

| Binary | Hex |
|---|---|
| 0000 | 0 |
| 0001 | 1 |
| 0010 | 2 |
| 0011 | 3 |
| 0100 | 4 |
| 0101 | 5 |
| 0110 | 6 |
| 0111 | 7 |
| 1000 | 8 |
| 1001 | 9 |
| 1010 | A |
| 1011 | B |
| 1100 | C |
| 1101 | D |
| 1110 | E |
| 1111 | F |

**Example 4:**

Convert **$F2D3_{16}$ to binary.**

**Solution:**

# Octal to Hexadecimal Conversion

When converting from octal to hexadecimal, it is often easier to first convert the octal number into binary and then from binary into hexadecimal.

**Example 5**

Convert 345 octal into hex.

**Solution:**

# Hexadecimal To Octal Conversion

When converting from hexadecimal to octal, it is often easier to first convert the hexadecimal number into binary and then from binary into octal.

**Example 6**

Convert A2DE hex into octal:

**Solution:**

# Hexadecimal Addition:

Use the following steps to perform hexadecimal addition:

1. Add one column at a time.

2. Convert to decimal and add the numbers.

3a. If the result of step two is 16 or larger subtract the result from 16 and carry 1 to the next column.

3b. If the result of step two is less than 16, convert the number to hexadecimal.

## Example 7:

Add: **AC5A9+ED694**

## Solution:

Carry Over:

1. Add one column at a time
2. Convert to decimal & add (**9** + **4** = **13**)
3. Follow less than 16 rule
   Decimal **13** is hexadecimal **D**

| | | | | |
|---|---|---|---|---|
| A | C | 5 | A | 9 |
| E | D | 6 | 9 | 4 |
| | | | | D |

Carry Over:

1. Add next column
2. Convert to decimal & add (**10** + **9** = **19**)
3. Follow 16 or larger than 16 rule
   (**19** – 16 = **3** carry a **1**)

| | | | | |
|---|---|---|---|---|
| | | 1 | | |
| A | C | 5 | A | 9 |
| E | D | 6 | 9 | 4 |
| | | | 3 | D |

Carry Over:

1. Add next column
2. Convert to decimal & add (**1** + 5 + 6 = **12**)
3. Follow less than 16 rule, convert to hex
   Decimal 12 is hexadecimal **C**

| | | | | |
|---|---|---|---|---|
| | | 1 | | |
| A | C | 5 | A | 9 |
| E | D | 6 | 9 | 4 |
| | | C | 3 | D |

Carry Over:

1. Add next column
2. Convert to decimal & add (12 + 13 = 25)
3. Follow 16 or larger than 16 rule
   (25 – 16 = 9 carry a 1)

| | | | | |
|---|---|---|---|---|
| 1 | | | | |
| A | C | 5 | A | 9 |
| E | D | 6 | 9 | 4 |
| | 9 | C | 3 | D |

Carry Over:

1. Add next column
2. Convert and add (1 + 10 + 11 = 22)
3. Follow 16 or larger than 16 rule
   (22 – 16 = 6 carry a 1)

| | | | | |
|---|---|---|---|---|
| 1 | | | | |
| A | C | 5 | A | 9 |
| B | D | 6 | 9 | 4 |
| 6 | 9 | C | 3 | D |

Carry Over:

1. Add next column
2. Convert and add (1 + 0 + 0 = 1)
3. Follow less than 16 rule

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | |
| 0 | A | C | 5 | A | 9 |
| 0 | B | D | 6 | 9 | 4 |
| 1 | 6 | 9 | C | 3 | D |

# Hexadecimal Subtraction

Subtraction in hexadecimal works similar to subtraction in decimal except that we occasionally have a digit larger than 9.

**Example:**

Subtract A8D2 - 3EAC (hexadecimal)

**Solution:**

- We'll align our numbers:

```
  A 8 D 2
- 3 E A C
 -------------
```

- Now in the ones place, we can't subtract C (12) from 2 so we borrow 1 from the sixteens place.

```
      12  18
  A 8 D̸ 2
- 3 E A C
 -------------
        6
```

(D = 13 minus the 1 we borrowed) and gives us 18 ones (2 plus the 16 we got from the borrow), then subtract 18-12 = 6.

- Now we don't need to borrow because we can subtract

10 (A) from 12:

```
      12 18
 A  8  D  2
-3  E  A  C
------------
       2  6
```

- In the 256's place, we again need to borrow. We'll borrow 1 from the 4096's place and exchange it for sixteen 256's (one 4096 equals sixteen 256's). This leaves us 9 in the 4096's place (A = 10 minus the 1 that we borrowed), and gives us 24 in the 256's place (8 plus the 16 from the borrow). We then can subtract 24-14 = 10 = A. So we have:

```
 9  24 12  18
 A  8  D  2
-3  E  A  C
------------
    A  2  6
```

- **Finally, we subtract 9-3 = 6 in the 4096's place:**

```
9    24 12    18
  A  8  D  2
- 3  E  A  C
-------------
  6  A  2  6
```

جامعة
الملك سعود
King Saud University

شكرًا لحسن استماعكم

# Thank you