SEMESTER II FINAL EXAMINATION, 1443
Dept. Math., College of Science, KSU
Math: 107 Full Mark: 40 Time: 3 Hours

Q1. [Marks: 4+4+3=11]

(a) For which values of m will the following linear system have no solutions? Exactly one solution? Infinitely many solutions?

$$\begin{aligned} x + my + 2z &= 3 \\ 4x + (6+m)y - mz &= 13 - m \\ x + 2(m-1)y + (m+4)z &= m + 2 \end{aligned}$$

(b) Let R and S be 3×3 matrices such that $RS + R - 2I = 0$. Find R^{-1} if

$$S = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 2 & 5 \end{bmatrix}$$

(c) Find the value of x so that $|A| = 6$, where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3x-6 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Q2. [Marks: 3+4+3=10]

(a) Find the volume of the parallelepiped (box) having adjacent sides **AB**, **AC**, and **AD**, where $A(1,0,-1)$, $B(1,0,3)$, $C(4,3,2)$, $D(7,1,0)$.

(b) Determine whether the following lines l_1 and l_2 are parallel or they intersect. If they intersect, find the point of intersection:

$$l_1 : x = 3 + t,\ y = 5 - t,\ z = -2 + 2t; \quad l_2 : x = 2 + s,\ y = 3 - 2s,\ z = -1 + 3s.$$

(c) Sketch the graph of $z^2 - 4y^2 - 9x^2 = 4$, and identify the surface.

Q3. [Marks: 2+4=6]

(a) Find the position vector $\mathbf{r}(t)$ if $\mathbf{r}'(t) = 2\mathbf{i} - 4t^3\mathbf{j} + 6\sqrt{t}\,\mathbf{k}$ subject to the condition $\mathbf{r}(0) = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$.

(b) Find general formula for the tangential and normal components of acceleration and for the curvature of the curve C given by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \frac{t^3}{3}\mathbf{k}$

Q4. [Marks: 2+3+5+3=13]

(a) Use partial derivative to find $\frac{dy}{dx}$ if $x^2 + 3xy + 2x - y - 10 = 0$

(b) Find an equation of the tangent plane and parametric equations of normal line to the surface $z = e^x(1 + \cos y)$ at the point $P(0,0,2)$.

(c) Find the local extrema and saddle points if any of the function $f(x,y) = \frac{4}{3}x^3 - xy^2 + 5y^2 + 24y$

(d) Use Lagrange multipliers to find extrema of $f(x,y,z) = 4x^2 + y^2 + 5z^2$ subject to the constraint $2x + 3y + 4z = 12$.