Second Semester	**Final Exam**	**King Saud University**
(without calculators)	**Time allowed: 3 hours**	**College of Science**
Sunday 6-8-1444	**240 Math**	**Math. Department**

Q1: Solve the following system:

$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ x_2 - 3x_3 &= 1 \\ 2x_3 &= -4 \end{aligned}$$

(2 marks)

Q2: If A,B∈M_{22}, det(B)=2 and det(A)=3, then find det($2A^TB^{-1}$). (2 marks)

<u>Q3</u>: Let V be the subspace of $\mathbb{R}^4$ **<u>spanned</u>** by the set S={v_1=(1,1,1,0), v_2=(−2,0,0,2), v_3=(−1,3,3,4), v_4=(−5,−1,−1,5)}.

(i) Find a **<u>subset</u>** of S that forms a basis of V. (3 marks)

(ii) **<u>Find</u>** dim(V). (1 mark)

(iii) **<u>Express</u>** each vector that is not in the basis as a linear combination of the basis vectors. (2 marks)

<u>Q4</u>: Let W={(2a+1,0)∈ $\mathbb{R}^2$: a ∈ $\mathbb{R}$}. Show that W is a **<u>subspace</u>** of $\mathbb{R}^2$. (3 marks)

<u>Q5</u>: Let B={(1,0),(1,1)} and B'={(1,3),(2,0)} be two bases of $\mathbb{R}^2$. Find the transition matrix from B' to B. (2 marks).

Q6: (i) Show that the Eigenvalues of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ is 0, 1 and 2. (3 marks)

(ii) Show that A is diagonalizable and find the matrix P that diagonalizes A. (3 marks)

(iii) Find A^{1444} . (2 marks)

<u>Q7</u>: Let $\mathbb{R}^3$ be the Euclidean inner product space. Apply the Gram-Schmidt process to transform the following basis $\{u_1=(1,0,0), u_2=(0,1,-1), u_3=(0,4,2)\}$ into an **<u>orthonormal basis</u>**. (5 marks)

<u>Q8</u>: Let M_{22} be the vector space of square matrices of order 2, and let T: $M_{22} \rightarrow M_{22}$ be the map defined by $T(A)=A^T$ for all matrices A in M_{22}. Show that:

(i) T is a linear operator. (2 marks)

(ii) Find ker(T). (2 marks)

(iii) Find $[T]_B$ where B is the standard basis of M_{22}. (2 marks)

(iv) Find rank(T). (2 marks)

Q9: (i) If B={u,v,w} is a basis of a vector space V, then find the coordinate vector $(u)_B$.
(1 mark)

(ii) If *u* and *v* are orthogonal vectors in an inner product space such that $\|u\|=4$ and $\|v\|=3$, then find $\|u+v\|$. (1 mark)

(iii) If B is a 5×9 matrix with nullity(B)=4, then find rank(B^T). (1 mark)

(iv) Show that if u and v are orthogonal in an inner product space V, then au and bv are orthogonal for every a and b in $\mathbb{R}$. (1 mark)