King Saud University, Department of Mathematics
Math 204 (2H), 30/100, Mid term Exam S1. 42/43

Question 1 [4,3] a) Find and sketch the largest local region in $\mathbb{R}^2$, for wich
the following initial value problem admits a unique solution

$$\begin{cases} (x-3)\frac{dy}{dx} + y\ln x = 2x \\ \qquad\qquad y(1) = 2. \end{cases}$$

b) Solve the differential equation

$$\frac{dy}{dx} = \frac{\sin x + x\cos x}{y(2\ln y + 1)}, \quad y > 0.$$

Question 2 [3,3] a) Find the general solution of the differential equation

$$\left(x + y^2 + \sin^{-1} y\right) dx + \left(2xy + \frac{x}{\sqrt{1-y^2}}\right)dy = 0.$$

b) Solve the initial value problem

$$\begin{cases} (x-y)dx + (3x+y)dy = 0 \\ \qquad\qquad y(3) = -2 \end{cases}$$

Question 3. [3,3]. a) Find the largest interval for which the following
initial value problem has a unique solution

$$\begin{cases} (x-1)(x+4)y'' + \frac{\ln(x-2)}{x^2+1}y' + e^x y = 4x^2 + 1 \\ \qquad\qquad y(4) = -1, \ y'(4) = 2 \end{cases}$$

b) determine whether the functions $f_1 = 1 + x$, $f_2 = x$, $y_3 = 2x + 3$ are
linearly dependent or linearly independent on $\mathbb{R}$.

Question 4 [5]. A culture has initially y_0 number of bacteria. After one
hour the number of bacteria is measured to be $\frac{5}{2}y_0$. If the rate of growth is
proportional to the number of bacteria $y(t)$, present at time t, then determine
the time necessary for the number of bacteria to be quadruple.