assignment (2)

H.W 2 Math 380

\# first Question : $P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.6 & 0.4 \\ 1 & 0.2 & 0.8 \end{array}$, what's the Prob that today is dry and the comming two days are rainy ?

$0 \rightarrow$ dry

$1 \rightarrow$ rainy

$$P(X_2 = 1 \mid X_0 = 0) = P^2_{01}$$

So $P^2 = P \cdot P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$

$$= \begin{bmatrix} 0.44 & 0.56 \\ 0.28 & 0.72 \end{bmatrix} = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.44 & 0.56 \\ 1 & 0.28 & 0.72 \end{array}$$

So $P^2_{01} = 0.56$

\# Second Question : pb 3.2.3 p.86 Textbook

Let X_n denote the quality of the nth item produced by a production system with $X_n = 0$ meaning "good" and $X_n = 1$ meaning "defective." Suppose that X_n evolves as a Markov chain whose transition probability matrix is

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.99 & 0.01 \\ 1 & 0.12 & 0.88 \end{array}.$$

, what's the Prob that the fourth item is defective given the first item is good?

$$P(X_3 = 1 \mid X_0 = 0) = P^3_{01}$$

$P^3 = P \cdot P^2$ and given that $P^2 = \begin{bmatrix} 0.9813 & 0.0187 \\ 0.2244 & 0.7756 \end{bmatrix}$

So $P^3 = P \cdot P^2 = \begin{bmatrix} 0.99 & 0.01 \\ 0.12 & 0.88 \end{bmatrix} \cdot \begin{bmatrix} 0.9813 & 0.0187 \\ 0.2244 & 0.7756 \end{bmatrix}$

$$= \begin{bmatrix} 0.9737 & 0.0262 \\ 0.3152 & 0.6847 \end{bmatrix}$$

So $P^3_{01} = 0.0262$

assignment ③

Question 1

To increase Availability add a duplicate repair facility so. That both computers can be repaired simultaneously

$$
\begin{array}{c|cccc}
 & (2,0) & (1,0) & (1,1) & (0,1) \\
\hline
(2,0) & q & p & 0 & 0 \\
(1,0) & 0 & 0 & q & p \\
(1,1) & q & p & 0 & 0 \\
(0,1) & 0 & 0 & 1 & 0 \\
\hline
 & \downarrow & \downarrow & \downarrow & \downarrow \\
 & \pi_0 & \pi_1 & \pi_2 & \pi_3
\end{array}
$$

The limiting distn $= (\pi_0, \pi_1, \pi_2, \pi_3)$

* $\pi_0 = q\pi_0 + q\pi_2$

$q\pi_2 = \pi_0 - q\pi_0$

$q\pi_2 = \pi_0(1-q)$

$\boxed{\pi_2 = \frac{p\pi_0}{q}}$ 1

* $\pi_1 = p\pi_0 + p\pi_2$

using 1

$\pi_1 = p\pi_0 + p\left(\frac{p}{q}\right)\pi_0$

$\pi_1 = \frac{pq\pi_0 + p^2\pi_0}{q}$

$\pi_1 = \frac{p(\overbrace{q+p}^{1})}{q}\pi_0$

$\boxed{\pi_1 = \frac{p}{q}\pi_0}$ 2

* $\pi_3 = p\pi_1$

using 2 $\pi_3 = p\left(\frac{p}{q}\pi_0\right)$

$\boxed{\pi_3 = \frac{p^2}{q}\pi_0}$ 3

* $\boxed{\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1}$ 4

* Sub 1, 2, 3 in 4

$\pi_0 + \frac{p}{q}\pi_0 + \frac{p}{q}\pi_0 + \frac{p^2}{q}\pi_0 = 1$

$\pi_0\left(1 + \frac{p}{q} + \frac{p}{q} + \frac{p^2}{q}\right) = 1$

$\pi_0\left(\frac{\overbrace{q+p}^{1} + p + p^2}{q}\right) = 1$

$\pi_0\left(\frac{1+p+p^2}{q}\right) = 1$

$\boxed{\pi_0 = \frac{q}{1+p+p^2}}$ 5

* sub 5 in 1, 2, 3

1 $\pi_1 = \pi_2 = \frac{p}{q} \cdot \frac{q}{1+p+p^2} = \frac{p}{1+p+p^2}$

2 $\pi_3 = \frac{p^2}{q} \cdot \frac{q}{1+p+p^2} = \frac{p^2}{1+p+p^2}$

* Availability is the prob that at least one computer is operating

$R_2 = 1 - \pi_3 = \pi_0 + \pi_1 + \pi_2 = 1 - \frac{p^2}{1+p+p^2} = \frac{1+p}{1+p+p^2}$

Question 2

4.2.4 Section 4.2.2 determined the availability R of a certain computer system to be

$$R_1 = \frac{1}{1+p^2} \quad \text{for one repair facility,}$$

$$R_2 = \frac{1+p}{1+p+p^2} \quad \text{for two repair facilities,}$$

where p is the computer failure probability on a single day. Compute and compare R_1 and R_2 for $p = 0.01, 0.02, 0.05$, and 0.10.

P(failure)	$R_1 = \frac{1}{1+p^2}$	$R_2 = \frac{1+P}{1+P+P^2}$
0,01	0,9999	0,999901
0,02	0,9996	0,99961
0,05	0,9975	0,9976
0,1	0,99	0,991

So The availability increase in Two epair facilities

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Question 3

4.2.7 Consider a machine whose condition at any time can be observed and classified as being in one of the following three states:

State 1: Good operating order
State 2: Deteriorated operating order
State 3: In repair

We observe the condition of the machine at the end of each period in a sequence of periods. Let X_n denote the condition of the machine at the end of period n for $n = 1, 2, \ldots$. Let X_0 be the condition of the machine at the start. We assume that the sequence of machine conditions is a Markov chain with transition probabilities

$$\begin{array}{lll} P_{11} = 0.9, & P_{12} = 0.1, & P_{13} = 0, \\ P_{21} = 0, & P_{22} = 0.9, & P_{23} = 0.1, \\ P_{31} = 1, & P_{32} = 0, & P_{33} = 0, \end{array}$$

and that the process starts in state $X_0 = 1$.
(a) Find $\Pr\{X_4 = 1\}$.
(b) Calculate the limiting distribution.
(c) What is the long run rate of repairs per unit time?

$$P = \begin{array}{c} \\ O \\ D \\ R \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} O & D & R \end{array} \\ \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \\ 1 & 0 & 0 \\ \downarrow & \downarrow & \downarrow \\ \pi_1 & \pi_2 & \pi_3 \end{bmatrix} \end{array}$$

$\boxed{a}$

$$= P_r[X_4 = 1 \mid X_0 = 1] = P_{11}^4$$

$$P^2 = P \cdot P = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0.1 & 0.81 & 0.09 \\ 0.9 & 0.1 & 0 \end{bmatrix}$$

$$P^4 = P^2 \cdot P^2 = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0.1 & 0.81 & 0.09 \\ 0.9 & 0.1 & 0 \end{bmatrix} \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0.1 & 0.81 & 0.09 \\ 0.9 & 0.1 & 0 \end{bmatrix} = \begin{bmatrix} 0.6831 & 0.2926 & 0.0243 \\ 0.243 & 0.6831 & 0.0759 \\ 0.739 & 0.243 & 0.018 \end{bmatrix}$$

so, $P_{11}^4 = 0.6831$

$\boxed{b}$

* $\pi_1 = 0.9\pi_1 + \pi_3$

$\pi_1 = 10\,\pi_3$

$\boxed{\pi_3 = \frac{1}{10}\pi_1}$ $\boxed{1}$

* $\pi_2 = 0.1\pi_1 + 0.9\pi_3$

$\boxed{\pi_2 = \pi_1}$ $\boxed{2}$

* $\boxed{\pi_1 + \pi_2 + \pi_3 = 1}$ $\boxed{3}$

subs $\boxed{1}$, $\boxed{2}$ in $\boxed{3}$

$\pi_1 + \pi_1 + \frac{1}{10}\pi_1 = 1$

$\frac{21}{10}\pi_1 = 1$

$\pi_1 = \frac{10}{21}$

$\boxed{* \; \pi_1 = \frac{10}{21} \qquad * \; \pi_2 = \frac{10}{21} \qquad * \; \pi_3 = \frac{1}{10}\left(\frac{10}{21}\right) = \frac{1}{21}}$

* The limiting dist $\pi = \left(\frac{10}{21}, \frac{10}{21}, \frac{1}{21}\right)$

$\boxed{c}$ $\pi_3 = \frac{1}{21} = \pi_R$

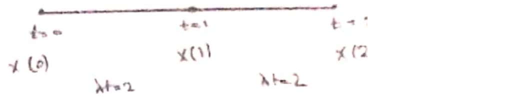
assignment 3 تاج

Question 4

5.1.1 Defects occur along the length of a filament at a rate of $\lambda = 2$ per foot.

(a) Calculate the probability that there are no defects in the first foot of the filament.

(b) Calculate the conditional probability that there are no defects in the second foot of the filament, given that the first foot contained a single defect.



a

$$P_r[X(1)=0] = p[X(1)-X(0)=0] = \frac{e^{-2}\,2^{0}}{0!} = e^{-2} \approx 0{,}13534$$

b

since X(2), X(1) are independant

$$P_r[X(2)=0 \mid X(1)=1] = P_r(X(2)=0) = P_r(X(2)-X(1)=0) = \frac{e^{-2(2-1)}\,2(2-1)^{0}}{0!} = 0{,}13534$$

assignment (4)

Q2

$$\boxed{P(X(s+t) - X(s) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}}$$

$$\boxed{\lambda = 2}$$

a) $pr\{X(6) = 9\}$

$$= P(X(6) - X(0) = 9) = \frac{(2 \cdot 6)^9 e^{-(2 \cdot 6)}}{9!} = \frac{12^9 e^{-12}}{9!} = \boxed{0.08736}$$

b) $pr\{X(20) = 13 | X(6) = 9\}$

$$P(X(20) - X(6) = 4 \mid X(6) = 9) = P(X(20) - X(6) = 4) = \frac{(2 \cdot 14)^4 e^{-(2 \cdot 14)}}{4!} = \frac{28^4 e^{-28}}{4!} = \boxed{1.7705 \times 10^{-8}}$$

indep r.v

c) $pr\{X(6) = 9 | X(20) = 13\}$

$$X \sim \text{Binomial}(\overset{n}{13}, \overset{p}{0.6923})$$

$$P(X(6) = 9 \mid X(20) = 13) = \binom{n}{x} p^x q^{n-x}$$

$$= \binom{13}{9} (0.6923)^9 (0.30769)^4$$

$p = \frac{9}{13} = 0.6923$

$q = \frac{4}{13} = 0.30769$

$$\boxed{= 0.2341}$$

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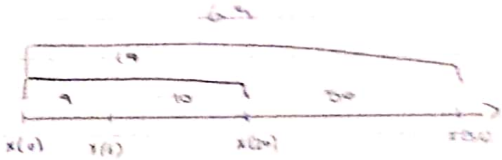

d) $pr\{X(6) = 9, X(20) = 19, X(56) = 69\}$

$$P(X(6) = 9, X(20) = 19, X(56) = 69) = P(X(6) - X(0) = 9, X(20) - X(6) = 10, X(56) - X(20) = 50)$$

$$P(X(6) - X(0) = 9) \cdot P(X(20) - X(6) = 10) \cdot P(X(56) - X(20) = 50) \leftarrow \text{indep r.v}$$

$$= \frac{12^9 e^{-12}}{9!} \cdot \frac{28^{10} e^{-28}}{10!} \cdot \frac{72^{50} e^{-72}}{50!} = 0.08736 \cdot (5.6438 \times 10^{-3}) \cdot (1.3012 \times 10^{-3})$$

$$\boxed{= 6.4154 \times 10^{-7}}$$

1)

$$
\begin{array}{cc}
 & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\
P = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{Vmatrix} 0.30 & 0.50 & 0.20 \\ 0.10 & 0.70 & 0.20 \\ 0.05 & 0.60 & 0.35 \end{Vmatrix}
\end{array}
$$

- $\pi_0 = \pi_0 P_{00} + \pi_1 P_{10} + \pi_2 P_{20}$

  $\pi_0 = 0.3\pi_0 + 0.1\pi_1 + 0.05\pi_2$

  $\boxed{0.7\pi_0 - 0.1\pi_1 - 0.05\pi_2 = 0}$ ① $\times 100$

- $\pi_1 = \pi_0 P_{01} + \pi_1 P_{11} + \pi_2 P_{21}$

  $\pi_1 = 0.5\pi_0 + 0.7\pi_1 + 0.6\pi_2$

  $\boxed{0.5\pi_0 - 0.3\pi_1 + 0.6\pi_2 = 0}$ ② $\times 10$

- $\boxed{\pi_0 + \pi_1 + \pi_2 = 1}$ ③

Solving ①, ②, ③ using cramer's Rule:

$\pi_0 = \frac{\Delta_0}{\Delta}$; $\pi_1 = \frac{\Delta_1}{\Delta}$, $\pi_2 = \frac{\Delta_2}{\Delta}$

1) $\Delta = \begin{vmatrix} 70 & -10 & -5 \\ 5 & -3 & 6 \\ 1 & 1 & 1 \end{vmatrix} = -680$

2) $\Delta_0 = \begin{vmatrix} 0 & -10 & -5 \\ 0 & -3 & 6 \\ 1 & 1 & 1 \end{vmatrix} = -75$

3) $\Delta_1 = \begin{vmatrix} 70 & 0 & -5 \\ 5 & 0 & 6 \\ 1 & 1 & 1 \end{vmatrix} = -445$

4) $\Delta_2 = \begin{vmatrix} 70 & -10 & 0 \\ 5 & -3 & 0 \\ 1 & 1 & 1 \end{vmatrix} = -160$

$\therefore \pi_0 = \frac{-75}{-680}$ $\quad \pi_1 = \frac{-445}{-680}$ $\quad \pi_2 = \frac{-160}{-680}$

The limiting dist:

$\pi = (0.11029, 0.6544, 0.23529)$

lower middle high

$\pi_1 = 0.6544$

assignment (5)

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Ex: 6.1.1

$$P_3(t) = \lambda_0 \lambda_1 \lambda_2 \left[B_{0,3}\, e^{-\lambda_0 t} + B_{1,3}\, e^{-\lambda_1 t} + B_{2,3}\, e^{-\lambda_2 t} + B_{3,3}\, e^{-\lambda_3 t} \right] ?$$

$$B_{0,3} = \prod_{i=1}^{3} \frac{1}{\lambda_i - \lambda_0} = \frac{1}{(3-1)(2-1)(5-1)} = \frac{1}{8}$$

$$B_{1,3} = \prod_{i=0}^{3} \frac{1}{\lambda_i - \lambda_1} = \frac{1}{(1-3)(2-3)(5-3)} = \frac{1}{4} \qquad i \neq 1$$

$$B_{2,3} = \prod_{i=0}^{3} \frac{1}{\lambda_i - \lambda_2} = \frac{1}{(1-2)(3-2)(5-2)} = -\frac{1}{3} \qquad i \neq 2$$

$$B_{3,3} = \prod_{i=0}^{2} \frac{1}{\lambda_i - \lambda_3} = \frac{1}{(1-5)(3-5)(2-5)} = -\frac{1}{24}$$

So $P_3(t) = 1(3)(2) \left[\frac{1}{8} e^{-t} + \frac{1}{4} e^{-3t} - \frac{1}{3} e^{-2t} - \frac{1}{24} e^{-5t} \right]$