Third Semester	**Second Quiz**	**King Saud University**
(without calculators)	**Time: 30 mins**	**College of Science**
Tuesday 10-11-1444	**240 Math**	**Math. Department**
Name:	**ID No.:**	

Q1: Let B={(1,1,1),(1,1,0),(1,0,0)}

(a) Show that B is a basis of $\mathbb{R}^3$. (3 marks)

(b) If

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

is the transition matrix from S to B, where S={u_1,u_2,u_3} is another basis of $\mathbb{R}^3$. Then find u_1. (2 marks)

Q2: Find a basis for the column space of the matrix: (3 marks)

$$A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 5 & 9 & -4 & 8 \end{bmatrix}$$

Q3: If A is a matrix of size 6×5 and nullity(A^T)=2, then find rank(A). (2 marks)

Q1: (a) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = (1)\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = (-1) \neq 0$

So B is a basis of $\mathbb{R}^3$.

(b) $(u_1)_B=(1,0,1)$. So $u_1=1(1,1,1)+0(1,1,0)+1(1,0,0)=(2,1,1)$

Q2:

$$A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 5 & 9 & -4 & 8 \end{bmatrix} \xrightarrow[(-5)R_{24}]{\substack{(-3)R_{21} \\ (-2)R_{23}}} \begin{bmatrix} 0 & -1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -1 & 1 & 3 \end{bmatrix} \xrightarrow[(\frac{1}{3})R_3]{(-1)R_{14}} \begin{bmatrix} 0 & -1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_{12}} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{(-1)R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So the basis is $\{(3,1,2,5)^T,(5,2,4,9)^T,(-2,-1,1,-4)^T)\}$.

Q3: $\text{rank}(A)= \text{rank}(A^T)=m-\text{nullity}(A^T)=6-2=4$.