Second Semester	**Second Quiz**	**King Saud University**
(without calculators)	**Time: 30 mins**	**College of Science**
Thursday 11-7-1444	**240 Math**	**Math. Department**
Name:	**ID No.:**	

Q1: Let B={(1,1,1),(1,1,0),(1,0,0)} be a basis of $\mathbb{R}^3$. Find the transition matrix from B to S, where S is the standard basis of $\mathbb{R}^3$. (2 marks)

Q2: Find a basis for the column space of the matrix: (3 marks)

$$A=\begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 5 & 9 & -4 & 8 \end{bmatrix}$$

Q3: If A is a matrix of size 4×5 and nullity(A)=2, then find rank(A^T). (2 marks)

Q4: Show that the function f: $\mathbb{R}^3\to\mathbb{R}^3$ defined by $f((x_1,y_1,z_1),(x_2,y_2,z_2))=x_1x_2$ for all $(x_1,y_1,z_1),(x_2,y_2,z_2)\in\mathbb{R}^3$ is not an inner product on $\mathbb{R}^3$. (1 mark)

Q5: Let $\mathbb{R}^3$ be the Euclidean inner product space. Find $\|u\|$ and $\langle u,v\rangle$, where u=(1,1,1) and v=(1,−1,1). (2 marks)

Answers:

Q1:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\Rightarrow P_{B\to S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Q2:

$$A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 5 & 9 & -4 & 8 \end{bmatrix} \xrightarrow[(-5)R_{24}]{\substack{(-3)R_{21} \\ (-2)R_{23}}} \begin{bmatrix} 0 & -1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -1 & 1 & 3 \end{bmatrix} \xrightarrow[(\frac{1}{3})R_3]{(-1)R_{14}} \begin{bmatrix} 0 & -1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_{12}} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{(-1)R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So the basis is $\{(3,1,2,5)^T,(5,2,4,9)^T,(-2,-1,1,-4)^T)\}$.

Q3: $\text{rank}(A^T)= \text{rank}(A)=n-\text{nullity}(A)=5-2=3$.

Q4: $f((0,1,1),(0,1,1))=0$ whereas $(0,1,1)\neq(0,0,0)$.

Q5:

$$\|u\| = \|(1,1,1)\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$
$$\langle u,v \rangle = \langle (1,1,1),(1,-1,1) \rangle = 1-1+1=1$$