King Saud University: Mathematics Department Math-254
Second Semester 1444 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:———————— **I.D. No.** ————————

Name of the Teacher:———————— **Section No.** ————————

Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15
a,b,c,d					

Quest. No.	Marks Obtained	Marks for Questions
Q. 1 to Q. 15		30
Q. 16		5
Q. 17		5
Total		40

Question 1: The value of k which insures rapid convergence of $x_{n+1} = x_n - k(3 - x_n^2)$ to $\alpha = \sqrt{3}$ is:

(a) $\frac{1}{2\sqrt{3}}$ (b) $\frac{-1}{2\sqrt{3}}$ (c) $\frac{-1}{4\sqrt{3}}$ (d) None of these

Question 2: If $x_{n+1} = g(x_n) = \ln(x_n + 2), x_0 = 1.5$ and $k = max|g'(x)| = \dfrac{1}{3}$, then the number of iterations to achieve accuracy 10^{-2} is:

(a) 4 (b) 3 (c) 2 (d) None of these

Question 3: The first approximation using Newton's method of the x-value of the intersection point of the curves $f(x) = x^3$ and $g(x) = 2x + 1$ with $x_0 = 1.5$ is:

(a) 3.6316 (b) 2.6316 (c) 1.6316 (d) None of these

Question 4: The next iterative value of the root of $x^2 - 4x + 4 = 0$ using the secant method, if the initial guesses are $x_0 = 3$ and $x_1 = 2.5$ is:

(a) 3.3333 (b) 2.3333 (c) 1.3333 (d) None of these

Question 5: Let $\alpha = 0$ be a root for the equation $\ln(x + 1) = x$. This root is:

(a) multiple root, m=2 (b) multiple root, m=3 (c) simple root (d) None of these

Question 6: The matrix obtained by forward elimination using simple Gaussian method on the linear system $A\mathbf{x} = [5, 7, 3]^T$ with $A = \begin{pmatrix} 1 & 1 & -1 \\ 5 & -3 & 2 \\ 2 & -1 & 1 \end{pmatrix}$ is:

(a) $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 8 & -7 \\ 0 & 0 & \frac{3}{8} \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 8 & 7 \\ 0 & 0 & -\frac{3}{8} \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & -1 \\ 0 & -8 & 7 \\ 0 & 0 & \frac{3}{8} \end{pmatrix}$ (d) None of these

Question 7: In the LU factorization with Doolittles method of the matrix $A = \begin{pmatrix} 1 & -1 \\ \alpha & 1 \end{pmatrix}$, the matrix U is singular if α is equal to:

(a) 1 **(b)** -1 quad **(c)** ± 1 **(d)** None of these

Question 8: If $\|T_J\|_\infty = \dfrac{1}{3}$ and $\|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\|_\infty = \dfrac{2}{3}$, then the number of the Jacobi iterations needed to achieve accuracy 10^{-2} in solving linear system $A\mathbf{x} = [1, 2]^T$ by Jacobi iterative method with $A = \begin{pmatrix} 4 & -1 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{x}^{(0)} = [0, 0]^T$ is:

(a) 3 **(b)** 4 **(c)** 5 **(d)** None of these

Question 9: The first approximation for solving linear system $A\mathbf{x} = [4, 5]^T$ using Gauss-Seidel iterative method with $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{x}^{(0)} = [0.5, 0.5]^T$ is:

(a) $[1.7500, 1.6250]^T$ **(b)** $[1.0625, 1.9688]^T$ **(c)** $[1.2500, 1.8750]^T$ **(d)** None of these

Question 10: The error bound for $|f(\frac{\pi}{10}) - p_1(\frac{\pi}{10})|$ in approximating $f(\frac{\pi}{10})$ by the linear Lagrange polynomial passing through $x_0 = 0$ and $x_1 = \frac{\pi}{6}$, where $f(x) = \sin x$ is:

(a) 0.2510 **(b)** 0.1500 **(c)** 0.0164 **(d)** None of these

Question 11: Absolute error $|f(0.3) - p_1(0.3)|$ in approximating $f(0.3)$ by Newton polynomial of degree one passing through $x_0 = 0$ and $x_1 = 1$, where $f(x) = x^2 - 2x - 1$ is:

(a) 0.1500 (b) 0.2100 (c) 0.0500 (d) None of these

Question 12: Let $f(x) = x^2 + \cos x$ (x in radian) and $h = 0.1$. Then, using the best 3-point formula for the approximation of $f'(1)$, then the absolute error is:

(a) 0.0014 (b) 0.0134 (c) 0.0125 (d) None of these

Question 13: The number of subintervals n needed to approximate the integral $\int_0^1 \frac{1}{x+1}\,dx$ to an accuracy of at least 10^{-3} using composite Simpson's rule is:

(a) 2 (b) 6 (c) 4 (d) None of these

Question 14: If the actual solution of the initial value problem, $\frac{1}{x}y' - y^2 = 0, y(1) = 1,\ n = 1,$ is $y(x) = \frac{2}{(3 - x^2)}$, then the absolute error by using Euler's method of $y(1.2)$ is:

(a) 0.0821 **(b)** 0.0723 **(c)** 0.0712 **(d)** None of these

Question 15: Using the Taylor's method of order 2 to find the approximate value of $y(0.1)$ for the initial-value problem, $y' = e^{-2x} - 2y, \quad y(0) = 0.1, \quad n = 1$, is:

(a) 0.1884 **(b)** 0.1983 **(c)** 0.1620 **(d)** None of these

Question 16: Let $x_0 = 1, x_1 = 1, x_2 = 1, x_3 = 2$, $f(x) = \dfrac{2}{x}$, and the third divided difference is $f[1,1,1,2] = -1$. Compute the absolute error and an error bound for the approximation of $f(1.5)$ using cubic Newton's polynomial.

Question 17: Find the approximation of $f''(0.8)$ by using the following set of data points using three-point central difference rule:

x	0.0	0.11	0.24	0.3	0.4	0.5	0.6	0.72	0.8	0.9	1.05	1.11	1.2
$f(x)$	1.00	1.10	1.2	1.26	1.32	1.38	1.43	1.47	1.50	1.52	1.55	1.55	1.56

The function tabulated is $f(x) = x + \cos x$ (x in radian), how many subintervals approximate the given derivative to within accuracy of 10^{-6} using the differentiation rule of $f''(x)$?