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جامعة
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MID TERM EXAMINATION, SEMESTER III, 2023
DEPT. MATH., COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH: 107 FULL MARK: 30 TIME: 2 HOURS

Q1.[5] Solve the following system of linear equations by Gauss-Jordan elimination method:

$$2x + 2y - 2z = 4$$
$$3x + 5y + z = -8$$
$$-4x - 7y - 2z = 13.$$

Q2. [4] Find the value of λ such that the matrix $A - \lambda I$ is not invertible, where I is the 3×3 identity matrix and

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

Q3. [4+4=8] (a) Let

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 0 & -3 & 2 \\ -1 & 0 & 2 \end{bmatrix}$$

Find $adj(A)$, the adjoint of the matrix A, and then use it to find A^{-1}.

(b) Use row reduction to evaluate the determinant of the matrix given below

$$\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$$

Q4. [4+3+3+3=13] Let $\mathbf{a} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

(a) Find (i) $\text{Comp}_{\mathbf{a}}\mathbf{b}$, and (ii) the angle between the vectors **a** and **b**.

(b) Find the work done that exerted by a constant force $\mathbf{F} = -\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ to move a particle from a point $P(4, 0, -7)$ to another point $Q(2, 4, 0)$.

(c) Find the volume of a box having adjacent sides AB, AC and AD where $A(2, 1, -1)$, $B(3, 0, 2)$, $C(4, -2, 1)$ and $D(5, -3, 0)$.

(d) If a line l has parametric equations $x = 5 - 3t$, $y = -2 + t$, $z = 1 + 9t$, find parametric equations for the line through $P(-6, 4, -3)$ that is parallel to the line l.

Model answer of MDT Exam
S3/1444

Q1 [5 Marks]

$$\begin{pmatrix} 2 & 2 & -2 & 4 \\ 3 & 5 & 1 & -8 \\ -4 & -7 & -2 & 13 \end{pmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 3 & 5 & 1 & -8 \\ -4 & -7 & -2 & 13 \end{pmatrix}$$

$$\xrightarrow{-3R_1+R_2;\ 4R_1+R_3} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 2 & 4 & -14 \\ 0 & -3 & -6 & 21 \end{pmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & -7 \\ 0 & -3 & -6 & 21 \end{pmatrix}$$

$$\xrightarrow{3R_2+R_3} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & -7 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{-R_2+R_1} \begin{pmatrix} 1 & 0 & -3 & 9 \\ 0 & 1 & 2 & -7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

So, we have infinite many solutions.

To get the form of solutions, let $z = t,\ t \in \mathbb{R}$

$\Rightarrow \because y + 2z = -7 \quad \therefore y = -7 - 2t$

$\because x - 3z = 9 \quad \therefore x = 9 + 3t$

i.e $x = 9+3t,\ y = -7-2t,\ z = t,\ t \in \mathbb{R}$

Q2 [4 Marks]

$$|A - \lambda I| = \begin{vmatrix} -\lambda & -1 & -2 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)[-\lambda(2-\lambda)+1]$$
$$= (3-\lambda)(\lambda^2 - 2\lambda + 1)$$
$$= (3-\lambda)(\lambda - 1)^2$$

$\therefore (A - \lambda I)$ is not invertible iff $|A - \lambda I| = 0$

i.e the values of λ in this case are

$$\lambda = 3 \text{ and } \lambda = 1$$

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Q3 [4+4=8 Marks]

(a)

The Cofactor Matrix is given by

$$C = \begin{pmatrix} -6 & -2 & -3 \\ 0 & 1 & 0 \\ -9 & -4 & -6 \end{pmatrix}$$

$$adj(A) = C^T = \begin{pmatrix} -6 & 0 & -9 \\ -2 & 1 & -4 \\ -3 & 0 & -6 \end{pmatrix}$$

$$det(A) = \begin{vmatrix} 2 & 0 & -3 \\ 0 & -3 & 2 \\ -1 & 0 & 2 \end{vmatrix} = 2(-6) - 3(-3) = -3$$

$$A^{-1} = \frac{1}{det(A)} adj(A)$$

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$$\therefore A^{-1} = \frac{-1}{3}\begin{pmatrix} 6 & 0 & -9 \\ -2 & 1 & -4 \\ -3 & 0 & -6 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 3 \\ 2/3 & -1/3 & 4/3 \\ 1 & 0 & 2 \end{pmatrix}$$

(b)

$$\text{let } A = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}$$

$$\because det(A) = det(A^T)$$

$$\therefore det(A) = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{matrix} -R_1 + R_2 \\ -R_2 + R_3 \end{matrix}$$

$$\Rightarrow det(A) = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-b & c^2-b^2 \end{vmatrix}$$

$$\Rightarrow$$

$$det(A) = (b-a)(c-b)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+b \end{vmatrix}$$

$-R_2 + R_3$

$$\Rightarrow det(A) = (b-a)(c-b)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-a \end{vmatrix}$$

$$= (b-a)(c-b)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & 1 \end{vmatrix}$$

$$\therefore det(A) = (b-a)(c-b)(c-a).$$

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Q4 [4+3+3+3 = 13 Marks]

(a)

(i) $\text{Comp}_{a} b = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|} = \dfrac{\langle 6,-3,2\rangle \cdot \langle 5,1,4\rangle}{\sqrt{36+9+4}} = \dfrac{35}{7} = 5$ 2

(ii) $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|} = \dfrac{35}{7\sqrt{42}} = \dfrac{5}{\sqrt{42}}$ 2

$\therefore \theta = \cos^{-1}\left(\dfrac{5}{\sqrt{42}}\right) \approx 39.5^{0}$

(b)

$W = \vec{F}\cdot\vec{d}$, where $\vec{d} = \vec{PQ}$

$d = \langle 2,4,0\rangle - \langle 4,0,-7\rangle = \langle -2,4,7\rangle$ 3

$\therefore W = \langle -1,5,-3\rangle \cdot \langle -2,4,7\rangle = 1$ unit of work.

(c)

$\because \vec{a} = \vec{AB} = \langle 1,-1,3\rangle$, $\vec{b} = \vec{AC} = \langle 2,-3,2\rangle$ and $\vec{c} = \vec{AD} = \langle 3,-4,1\rangle$

$\therefore V = \left|(\vec{a}\times\vec{b})\cdot\vec{c}\right| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 3 \\ 2 & -3 & 2 \\ 3 & -4 & 1 \end{vmatrix}$ 3

$= 4$ unit of volume.

(d)

$\because$ The direction vector $\vec{a} = \langle -3,1,9\rangle$ for the line ℓ that passes through $P(-6,4,-3)$

$\therefore$ The parametric equations for this line is given by 3

$x = -6-3t,\ y = 4+t,\ z = -3+9t,\ t \in \mathbb{R}.$

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