MID TERM EXAMINATION, SEMESTER I, 2023-24

Dept. Math., College of Science, KSU

Math: 107 Full Mark: 25 Time: 90 Minutes

Q1.[3] Solve the following system of linear equations by Gaussian elimination method:

$$\begin{aligned} y - 8z &= 9 \\ x - 2y + 3z &= -3 \\ 7y - 5z &= 12. \end{aligned}$$

Q2. [3] Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

and $P(X) = X^2 - X - 6$. Compute the matrix $P(A)$.

Q3. [6] Let

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

and

$$D = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} \end{bmatrix}$$

Compute $(DB^T)B$ and deduce the inverse matrix of B.

Q4. [3] Let

$$A = \begin{bmatrix} -1 & -1 & -2 \\ 1 & \lambda & 1 \\ 0 & 0 & \lambda+1 \end{bmatrix}$$

Find the values of λ such that the matrix A is invertible.

Q5. [5] Solve the following system of linear equations:

$$\begin{aligned} x + y &= 0 \\ x + 2y + 3z &= 5 \\ 2x + 4y + z &= -5. \end{aligned}$$

by finding A^{-1} using cofactors and adjoint.

Q6. [5] Use Cramer's Rule to solve the linear system:

$$\begin{aligned} -x + 2y - 3z &= 1 \\ 2x + z &= 0 \\ 3x - 4y + 4z &= 2. \end{aligned}$$

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Model Answer of 1^{st} MDT Exam
S1/1445

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Q1: 3 Marks

$$\begin{bmatrix} 0 & 1 & -8 & 9 \\ 1 & -2 & 3 & -3 \\ 0 & 7 & -5 & 12 \end{bmatrix}$$

3

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & -2 & 3 & -3 \\ 0 & 1 & -8 & 9 \\ 0 & 7 & -5 & 12 \end{bmatrix}$$

$-7R_2 + R_3$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 & -3 \\ 0 & 1 & -8 & 9 \\ 0 & 0 & 51 & -51 \end{bmatrix}$$

$\frac{1}{51} R_3$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 & -3 \\ 0 & 1 & -8 & 9 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

So, we have

$$x - 2y + 3z = -3$$
$$y - 8z = 9$$
$$z = -1$$

$\therefore z = -1,\ y = 1,\ x = 2$

$\therefore$ the solution is $x = 2$, $y = 1$ and $z = -1$

Q2: 3 Marks

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

3

$$P(A) = A^2 - A - 6I$$

$$A^2 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & 3 & -1 \\ 3 & 6 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

$\therefore P(A)$

$$= \begin{bmatrix} 6 & 3 & -1 \\ 3 & 6 & 1 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} - 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & -2 \\ 1 & -1 & 2 \\ -2 & 2 & -4 \end{bmatrix}$$

Q3: 6 Marks

DB^T

$$= \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & -1/3 & -1/3 \\ 1/6 & -1/6 & 1/3 \end{bmatrix}$$

3

$(DB^T)B$

$$= \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & -1/3 & -1/3 \\ 1/6 & -1/6 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore B^{-1} = DB^T$

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$$\therefore B^{-1} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & -1/3 & -1/3 \\ 1/6 & -1/6 & 1/3 \end{bmatrix}$$

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Q4: 3 Marks

$$A=\begin{bmatrix}-1 & -1 & -2\\ 1 & \lambda & 1\\ 0 & 0 & \lambda+1\end{bmatrix}$$

3

$$\det(A)=\begin{vmatrix}-1 & -1 & -2\\ 1 & \lambda & 1\\ 0 & 0 & \lambda+1\end{vmatrix}$$

$$=(\lambda+1)(-\lambda+1)$$

$\det(A)=0 \iff \lambda=\pm 1$

i.e. A is invertible iff $\lambda \neq \pm 1$

Q5: 5 Marks

$$A=\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 3\\ 2 & 4 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},$$

$$B=\begin{bmatrix}0\\ 5\\ -5\end{bmatrix}$$

$$C=\begin{bmatrix}-10 & 5 & 0\\ -1 & 1 & -2\\ 3 & -3 & 1\end{bmatrix}$$

2

$$adj(A)=C^T=\begin{bmatrix}-10 & -1 & 3\\ 5 & 1 & -3\\ 0 & -2 & 1\end{bmatrix}$$

$$A^{-1}=\frac{1}{\det(A)}\,adj(A)$$

$$\det(A)=\begin{vmatrix}1 & 1 & 0\\ 1 & 2 & 3\\ 2 & 4 & 1\end{vmatrix}=-5$$

$$\therefore A^{-1}=-\frac{1}{5}\begin{bmatrix}-10 & -1 & 3\\ 5 & 1 & -3\\ 0 & -2 & 1\end{bmatrix}=\begin{bmatrix}2 & 1/5 & -3/5\\ -1 & -1/5 & 3/5\\ 0 & 2/5 & -1/5\end{bmatrix}$$

$$\therefore X=A^{-1}B$$

$$=\begin{bmatrix}2 & 1/5 & -3/5\\ -1 & -1/5 & 3/5\\ 0 & 2/5 & -1/5\end{bmatrix}\begin{bmatrix}0\\ 5\\ -5\end{bmatrix}$$

$$=\begin{bmatrix}4\\ -4\\ 3\end{bmatrix}$$

3

The solution of the system is

$x=4,\ y=-4,\ z=3$

Q6: 5 Marks

$$A=\begin{bmatrix}-1 & 2 & -3\\ 2 & 0 & 1\\ 3 & 4 & 4\end{bmatrix}$$

$\det(A)=10$

$$\det(A_1)=\begin{vmatrix}1 & 2 & -3\\ 0 & 0 & 1\\ 2 & 4 & 4\end{vmatrix}=8$$

2

$$\det(A_2)=\begin{vmatrix}-1 & 1 & -3\\ 2 & 0 & 1\\ 3 & 2 & 4\end{vmatrix}=-15$$

$$\det(A_3)=\begin{vmatrix}-1 & 2 & 1\\ 2 & 0 & 0\\ 3 & 4 & 2\end{vmatrix}=-16$$

$$x=\frac{\det(A_1)}{\det(A)}=\frac{8}{10}=\frac{4}{5}$$

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$$y=\frac{\det(A_2)}{\det(A)}=\frac{-15}{10}=-\frac{3}{2}$$

$$z=\frac{\det(A_3)}{\det(A)}=\frac{-16}{10}=-\frac{8}{5}$$

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