MID TERM II EXAM. SEMESTER II, 1445

Dept. Math., College of Science
KING SAUD UNIVERSITY

Math: 107 Full Mark: 25 Time: 90 Minutes

Q1. [Marks: 4+2+3=9]

(a) Let a constant force $\mathbf{F} = \langle 2,2,0\rangle$ is applied on a particle displacing it from point $(0,0,0)$ to $(0,3,0)$. Then, find: (i) work done by the force $\mathbf{F}$, and (ii) angle θ between the force $\mathbf{F}$ and the displacement $\mathbf{d}$.

(b) Let $\mathbf{u} = \langle 3,-1,-4\rangle$, $\mathbf{v} = \langle 2,5,-2\rangle$, and $\mathbf{w} = \langle -1,0,6\rangle$.
Compute $comp_{\mathbf{u}}(\mathbf{v}\times\mathbf{w})$.

(c) Find the area of the triangle $\triangle ABC$, where $A(2,-1,1)$, $B(-3,2,0)$, and $C(4,-5,3)$.

Q2. [Marks: 2+3+3=8]

(a) Find an equation of the plane through $P(2,5,-6)$ and parallel to the plane $3x-y+2z=10$.

(b) Let l_1 be the line passing through $A(1,3,0)$ and $B(0,4,5)$, and l_2 be the line passing through $C(-2,-1,2)$ and $D(5,1,0)$. Determine whether l_1 and l_2 are skew lines, that is, neither parallel nor intersecting.

(c) Identify the surface $y = 6x^2+z^2$. Give its traces, and sketch it.

Q3. [Marks: 2+3+3=8]

(a) Find the domain of the vector-valued function $\mathbf{r}(t) = \ln(1-t)\mathbf{i} + \sin t\mathbf{j} + t^3\mathbf{k}$.

(b) If $\mathbf{r}(t) = (1+t)\mathbf{i} + 2t\mathbf{j} + (2+3t)\mathbf{k}$ is the position vector of a moving point P, find its velocity, acceleration, and speed at $t=2$.

(c) Find $\mathbf{r}(t)$ subject to the given conditions: $\mathbf{r}'(t) = 2\mathbf{i} - 4t^3\mathbf{j} + 6\sqrt{t}\mathbf{k}$, $\mathbf{r}(0) = \mathbf{i}+5\mathbf{j}+3\mathbf{k}$.

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Model Answer For MDT II Exam
S2/1445

Q1: Marks [4+2+3=9]

(a)

(i) Work done $= \vec{F} \cdot \vec{d}$

$= \langle 2,2,0\rangle \cdot \langle 0,3,0\rangle$

$= 6$ units of work done. (2)

(ii) $\theta = \cos^{-1}\left(\dfrac{6}{\sqrt{8}\,\sqrt{9}}\right)$ (2)

$= \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45^0$

(b) $\text{Comp}_{\vec{u}}(\vec{V}\times\vec{W})$

$= \vec{V}\times\vec{W} \cdot \dfrac{\vec{u}}{\|\vec{u}\|}$

$= \langle 30,-10,5\rangle \cdot \dfrac{\langle 3,-1,-4\rangle}{\sqrt{26}}$ (2)

$= \dfrac{80}{\sqrt{26}} \simeq 15.69$

(c)

$\because$ Area of $\triangle ABC = \dfrac{1}{2}\|\vec{AB}\times\vec{AC}\|$,

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 3 & -1 \\ 2 & -4 & 2 \end{vmatrix}$$

$= 2\vec{i} + 8\vec{j} + 14\vec{k}$ (3)

$\therefore$ Area of $\triangle ABC = \dfrac{1}{2}(2\sqrt{66})$

$= \sqrt{66} \simeq 8.12$

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Q2: Marks [2+3+3=8]

(a)

$\because P(2,5,-6)$, $\vec{n} = \langle 3,-1,2\rangle$ are given, so the eqn of the plane is

$3(x-2)-(y-5)+2(z+6)=0$ (2)

i.e. $\boxed{3x - y + 2z = -11}$

(b)

Here $\vec{a} = \langle -1,1,5\rangle$, $\vec{b} = \langle 7,2,-2\rangle$

$l_1: x = 1-t,\ y = 3+t,\ z = 5t,\ t \in \mathbb{R}$

$l_2: x = -2+7u,\ y = -1+2u,\ z = 2-2u,\ u \in \mathbb{R}$

l_1 and l_2 are not parallel because

$\dfrac{-1}{7} \neq \dfrac{1}{2} \neq -\dfrac{5}{2}$

if they are intersected, we should have $1-t = -2+7u$

i.e. $\boxed{t+7u=3}$ ①

& $3+t = -1+2u$ (3)

i.e. $\boxed{t-2u=-4}$ ②

Solving ①, ② we get $t = \dfrac{-22}{9}$, $u = \dfrac{7}{9}$

but this leads to

$z = -\dfrac{110}{9}$ and $z = \dfrac{4}{9}$ (contradiction)

This means that l_1 and l_2 are never intersecting. #

• Note that, the shortest distance between l_1 and l_2 is

$$d = \frac{|\vec{AB}\times\vec{CD}\cdot\vec{AC}|}{\|\vec{AB}\times\vec{CD}\|},$$ where

$$\vec{AB}\times\vec{CD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 5 \\ 7 & 2 & -2 \end{vmatrix}$$

$= -12\vec{i} + 33\vec{j} - 9\vec{k}$,

$\vec{AC} = \langle -3,-4,2\rangle$

$\therefore d = \dfrac{114}{3\sqrt{146}} = \dfrac{38}{\sqrt{146}} \simeq 3.14$

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(c)

The graph of $y = 6x^2 + z^2$ is a paraboloid

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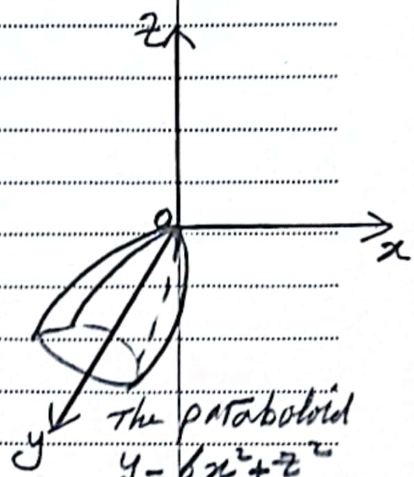
Trace	Eq. of trace	Description
on xy-plane $(z=0)$	$y = 6x^2$	parabola
on yz-plane $(x=0)$	$y = z^2$	parabola
on xz-plane $(y=0)$	$6x^2 + z^2 = 0$	origin
on $y = k$, $k > 0$	$6x^2 + z^2 = k$	ellipse



The paraboloid $y = 6x^2 + z^2$

Q3 Marks [2+3+3=8]

(a)

$\vec{r}(t) = \ln(1-t)\vec{i} + \sin t\,\vec{j} + t^3\vec{k}$

The domain of $\vec{r}$ is

$D = (-\infty, 1) \cap (-\infty, \infty) \cap (-\infty, \infty)$

$\therefore D = (-\infty, 1)$ 2

i.e. $D = \{t : t < 1\}$

$1 - t > 0$
$t < 1$

(b)

$\vec{r}(t) = (1+t)\vec{i} + 2t\vec{j} + (2+3t)\vec{k}$

The velocity is

$\vec{v}(t) = \vec{r}\,'(t) = \langle 1, 2, 3 \rangle$,

The acceleration is

$\vec{a}(t) = \vec{r}\,''(t) = \langle 0, 0, 0 \rangle = \vec{0}$

and the speed is

$\|\vec{v}(t)\| = \sqrt{14} \simeq 3.74$

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(c)

$\vec{r}\,'(t) = 2\vec{i} - 4t^3\vec{j} + 6\sqrt{t}\,\vec{k}$

$\therefore \vec{r}(t) = 2t\,\vec{i} - t^4\vec{j} + 4t^{3/2}\vec{k} + \vec{c}$

at $t = 0$, $\vec{r}(0) = \langle 1, 5, 3 \rangle$

$\therefore \vec{c} = \langle 1, 5, 3 \rangle$

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thus,

$\vec{r}(t) = (2t+1)\vec{i} + (5 - t^4)\vec{j} + (4t^{3/2} + 3)\vec{k}$.