CEN352
Digital Signal Processing

By

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Lecture No. 9

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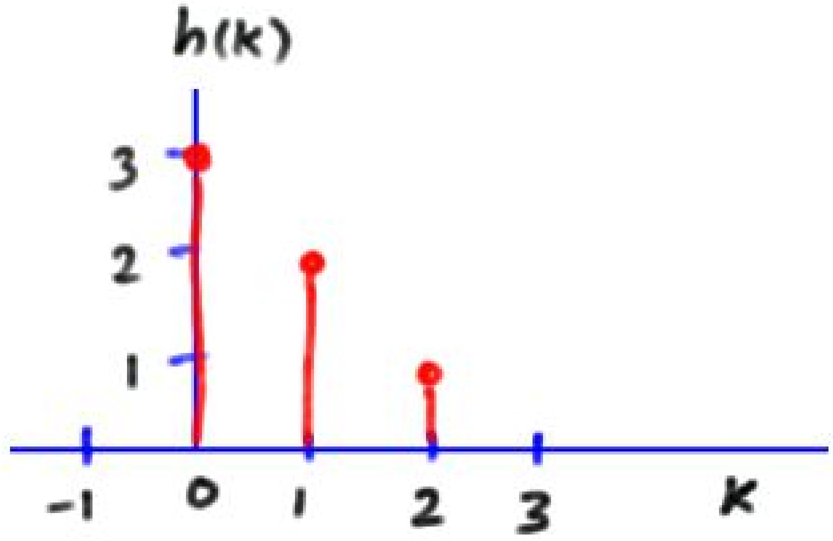
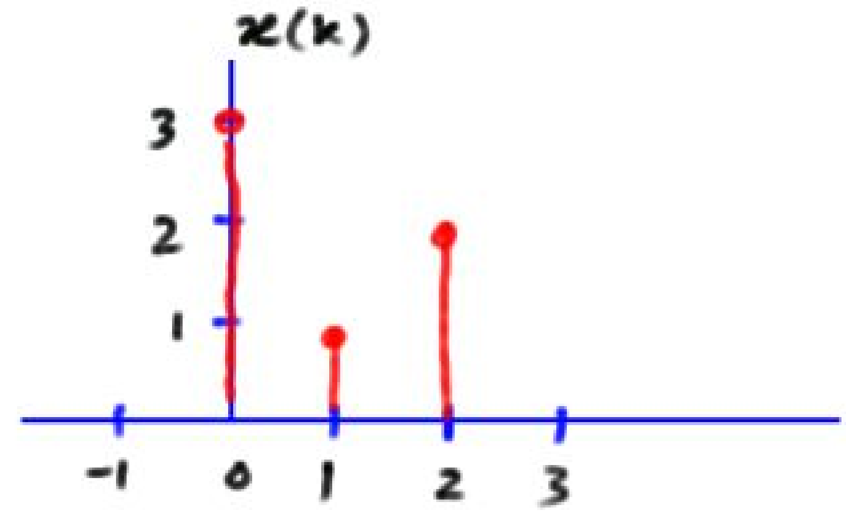
Example 3-11





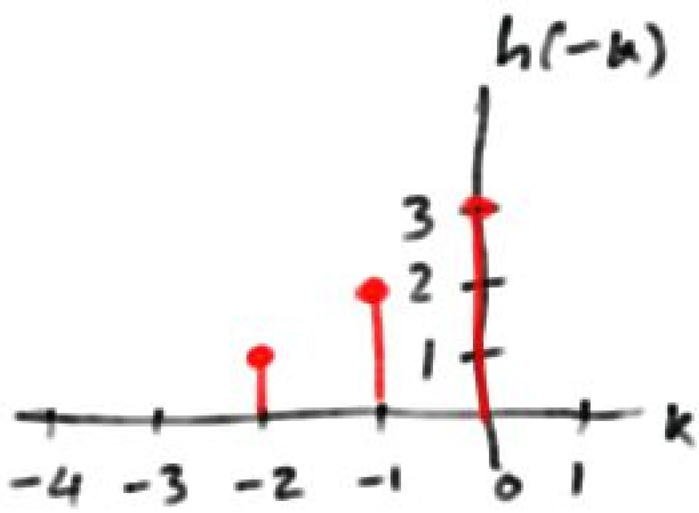
Using the sequences defined in the above figure, evaluate the digital convolution

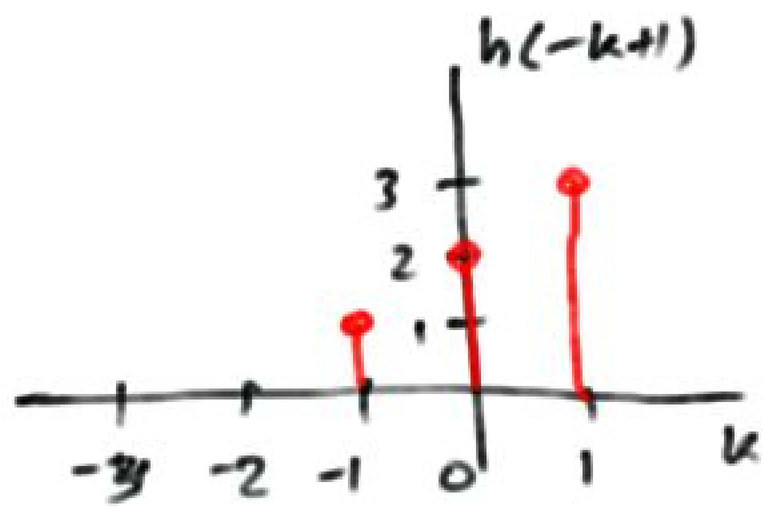
$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

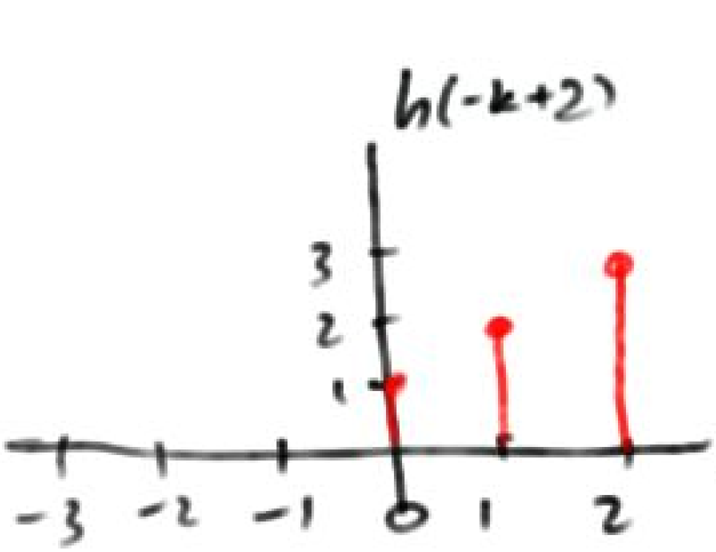
a. By the graphical method;

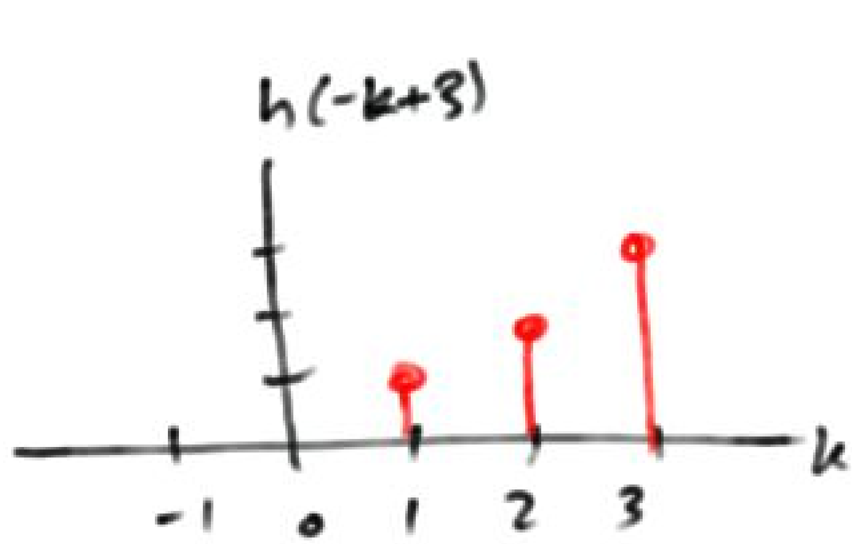
b. By applying the formula directly.

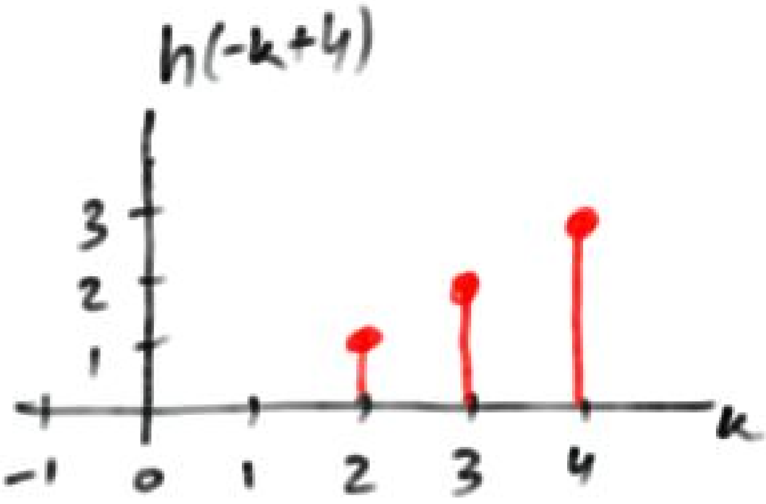
Solution

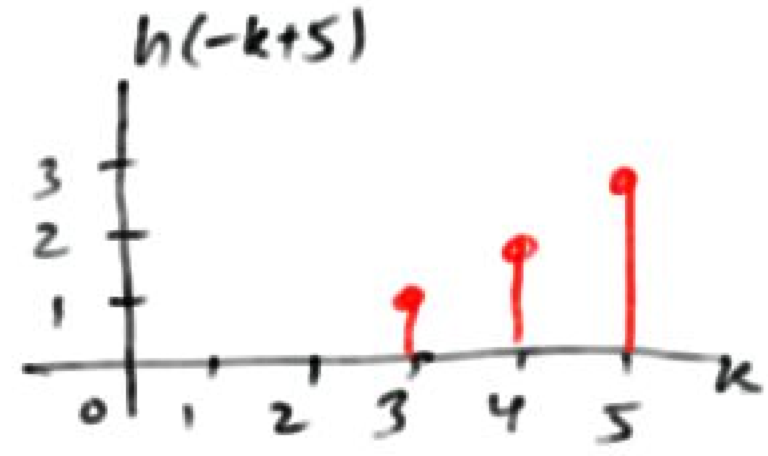












We find out the convolution by multiplying $x(n)$ with each of the reversed and shifted impulse response as shown in the figures above.

sum of product of $x(k)$ and $h(-k)$:

$$y(0) = 3 \times 3 = 9$$

sum of product of $x(k)$ and $h(-k+1)$

$$y(1) = 1 \times 3 + 3 \times 2 = 9$$

sum of product of $x(k)$ and $h(-k+2)$

$$y(2) = 2 \times 3 + 1 \times 2 + 3 \times 1 = 11$$

sum of product of $x(k)$ and $h(-k+3)$

$$y(3) = 2 \times 2 + 1 \times 1 = 5$$

sum of product of $x(k)$ and $h(-k+4)$

$$y(4) = 2 \times 1 = 2$$

sum of product of $x(k)$ and $h(-k+5)$

$$y(5) = 0$$

Also for $n \geq 5$, $y(n) = 0$, as the sequences do not overlap.

(b) With zero initial condition, the convolution equation becomes

$$y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2)$$

Therefore,

$n=0,\ y(0) = x(0)h(0) + x(1)h(0-1) + x(2)h(0-2)$

$= 3\times3 + 1\times0 + 2\times0 = 9$

$n=1,\ y(1) = x(0)h(1) + x(1)h(1-1) + x(2)h(1-2)$

$= 3\times2 + 1\times3 + 2\times0 = 9$

$n=2,\ y(2) = x(0)h(2) + x(1)h(2-1) + x(2)h(2-2)$

$= 3\times1 + 1\times2 + 2\times3 = 11$

$n=3,\ y(3) = x(0)h(3) + x(1)h(3-1) + x(2)h(3-2)$

$= 3\times0 + 1\times1 + 2\times2 = 5$

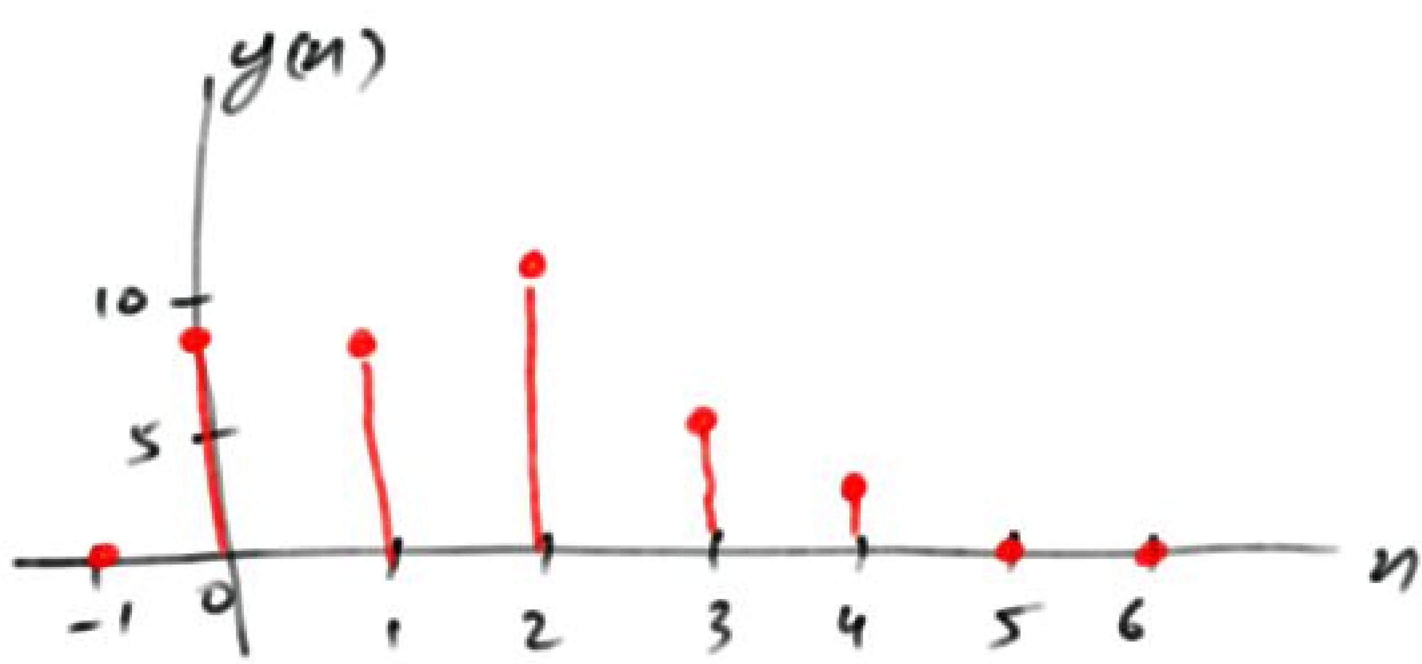
$n=4,\ y(4) = x(0)h(4) + x(1)h(4-1) + x(2)h(4-2)$

$= 3\times0 + 1\times0 + 2\times1 = 2$

$n=5,\ y(5) = x(0)h(5) + x(1)h(5-1) + x(2)h(5-2)$

$= 3\times0 + 1\times0 + 2\times0 = 0$

In fact, for $n \geqslant 5$, $y(n) = 0$.



Example 3.12

Given the following two rectangular sequences,

$$x(n) = \begin{cases} 1 & n = 0,1,2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1,2 \\ 0 & \text{otherwise} \end{cases}$$

Convolve them using the table method.

Solution

k	-2	-1	0	1	2	3	4	5	
$x(k)$			3	1	2				
$h(-k)$	1	2	3						$y(0) = 3\times3 = 9$
$h(1-k)$		1	2	3					$y(1) = 2\times3 + 1\times2 = 9$
$h(2-k)$			1	2	3				$y(2) = 3\times2 + 2\times1 + 1\times3 = 11$
$h(3-k)$				1	2	3			$y(3) = 2\times2 + 1\times1 = 5$
$h(4-k)$					1	2	3		$y(4) = 1\times2 = 2$
$h(5-k)$						1	2	3	$y(5) = 0$

Chapter 4

Discrete Fourier Transform and Signal Spectrum

Review: (Continuous-Time Case)

Fourier Series for periodic signals

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T_0)t}$$

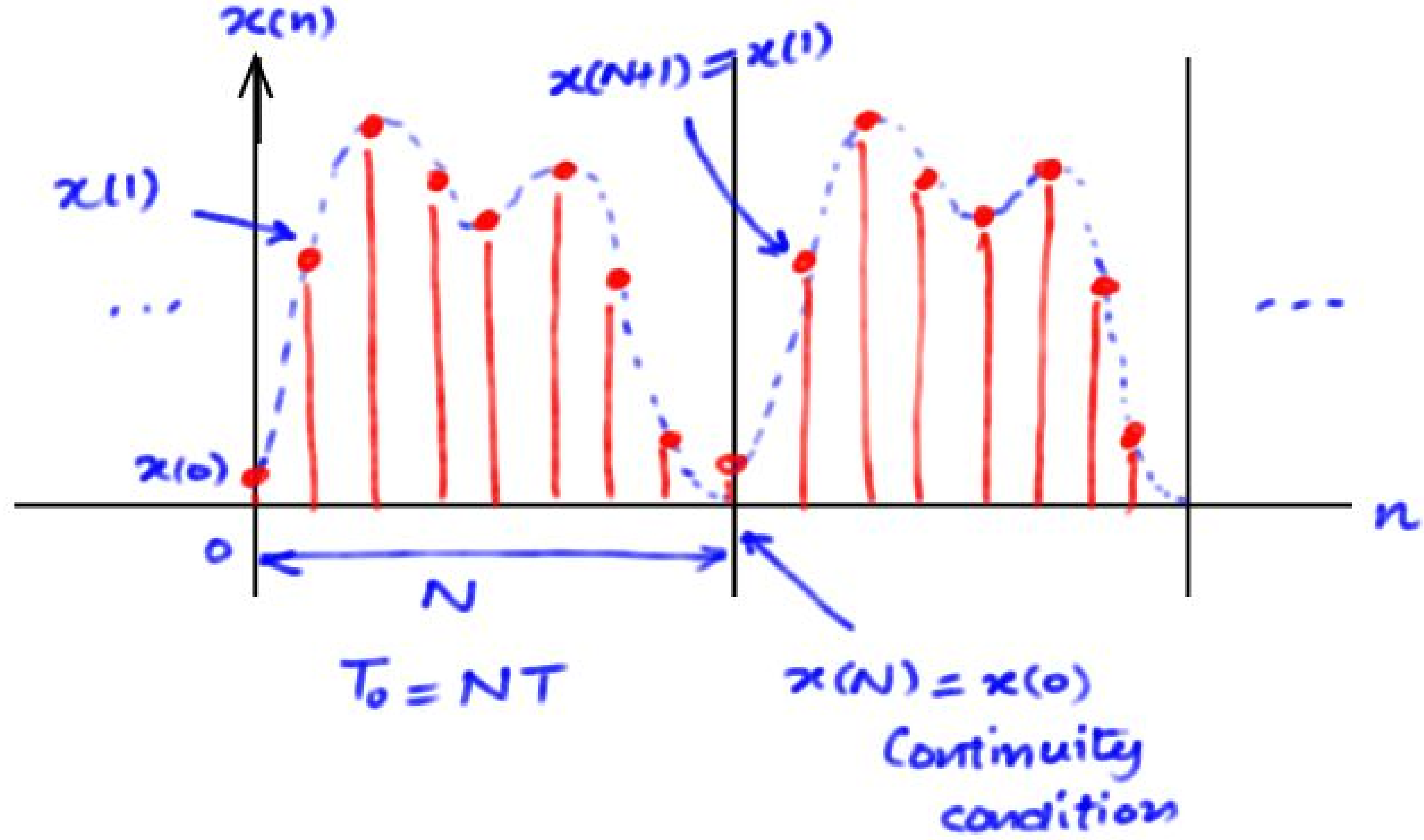
$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\left(\frac{2\pi}{T_0}\right)t}\, dt$$

Fourier Transform pair is given by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

Fourier Series Coefficient of Periodic Digital Signals



For the continuous-time periodic signal, the coefficients of the Fourier series expansion (in complex form) are given by:

$$c_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt, \quad -\infty < k < \infty \qquad \text{—(4.1)}$$

where k is the number of harmonics corresponding to the harmonic frequency of kf_0.

For the discrete time case, we set

$$T_0 = NT, \quad \omega_0 = \frac{2\pi}{T_0}$$

and approximate the integration over one period using a summation by putting $dt = T$, and $t = nT$. We get

$$c_k = \frac{1}{NT}\sum_{n=0}^{N-1} x(n)\, e^{-jk\frac{2\pi}{NT}\cdot nT} \cdot T$$

$$\Rightarrow c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}, \quad -\infty < k < \infty \qquad \text{—(4.2)}$$

It should be noted that Fourier series coefficients c_k are periodic with a period N. It can be proved very easily as:

$$C_{k+N} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi(k+N)n}{n}} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-jk\frac{2\pi kn}{N}} \cdot e^{-j2\pi n} \qquad \text{—(4.3)}$$

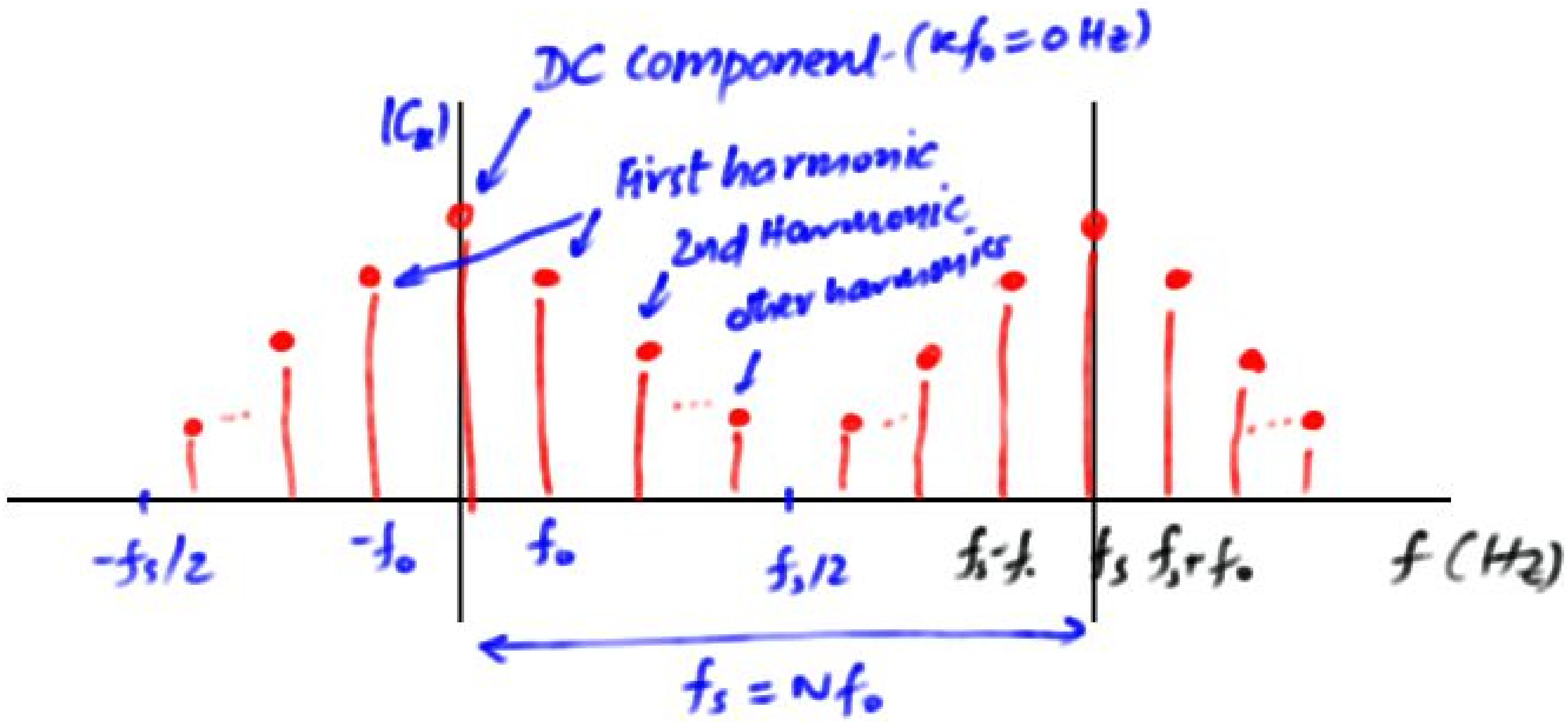
Since $e^{-j2\pi m} = \cos(2\pi m) - j\sin(2\pi m) = 1$, Thus

$$C_{k+N} = C_k \qquad \text{—(4.4)}$$

Plot of the magnitude of the Fourier coefficients $|C_k|$ as a function of frequency $f = kf_0$ is called 'Amplitude spectrum of the periodic signal'.

We note the following points

(a) Only the line spectrum portion between the frequency $-f_s/2$ and frequency $f_s/2$ (folding frequency) represents the frequency information of the periodic signal.



(b) The spectral portion from $f_s/2$ to f_s is a copy of the spectrum in the negative frequency range $-f_s/2$ to 0 Hz due to periodicity.

(c) For the kth harmonic, the frequency is

$$f = kf_0 \text{ Hz} \qquad \text{—(4.6)}$$

Example 4.1

Given the periodic signal

$$x(t) = \sin(2\pi t)$$

It is sampled at the rate $f_s = 4$ Hz.

a. Compute the spectrum c_k using the samples in one period.

b. Plot the two-sided amplitude spectrum $|c_k|$ over the range from -2 to 2 Hz.

Solution

Here $x(t) = \sin(2\pi t)$

$$\Rightarrow x(n) = x(nT) = \sin(2\pi nT)$$

$$= \sin(2\pi n/4) = \sin(0.5\pi n)$$

Therefore, for $n = 0, 1, 2, 3$ $(N = 4)$,

$x(0) = 0$; $x(1) = 1$; $x(2) = 0$; $x(3) = -1$

Using

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}}$$

$$c_0 = \frac{1}{4}\sum_{n=0}^{3} x(n) e^{0} = \frac{1}{4}(x(0)+x(1)+x(3)+x(4)) = 0$$

$$c_1 = \frac{1}{4}\sum_{n=0}^{3} x(n) e^{-j\frac{2\pi n}{4}} = \frac{1}{4}\left(x(0)e^{0} + x(1)e^{-j\frac{\pi}{2}} + x(2)e^{-j\pi} + x(3)e^{-j\frac{3\pi}{2}}\right)$$

$$= \frac{1}{4}(x(0) - jx(1) - x(2) + jx(3))$$

$$= \frac{1}{4}(0 - j(1) - 0 + j(-1)) = -0.5j$$

Similarly we get

$$c_2 = \frac{1}{4}\sum_{k=0}^{3} x(n)e^{-j2\pi\cdot 2n/4} = 0$$

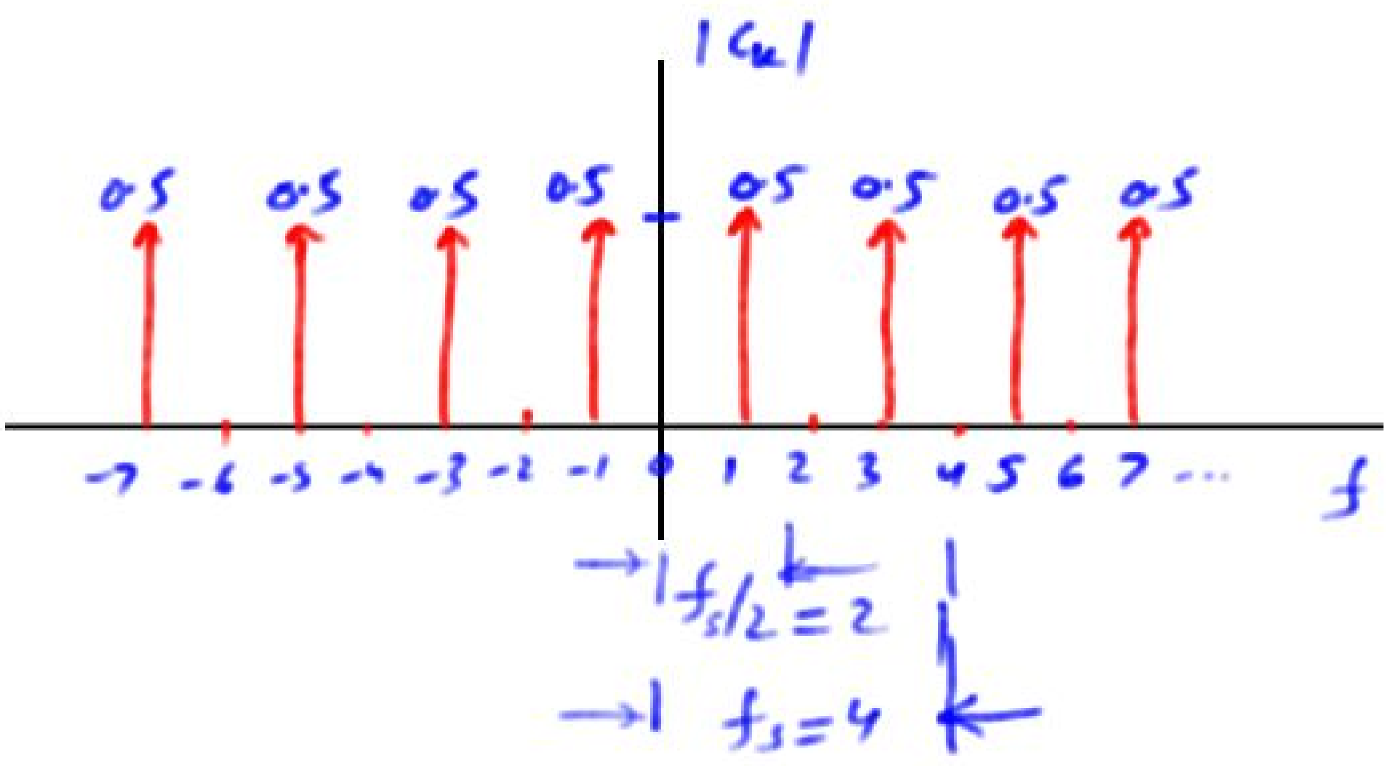
and

$$c_4 = \frac{1}{4}\sum_{k=0}^{3} x(n)e^{-j2\pi\cdot 3n/4} = j\,0.5$$

Using periodicity, it follows that

$$c_{-1} = c_3 = j0.5 \quad \text{and} \quad c_{-2} = c_2 = 0.$$

(b) The amplitude spectrum for the digital signal is plotted below.



The spectrum in the range of -2 to 2 Hz presents the information of the sinusoid with a frequency of 1 Hz and a peak value of $2|c_1| = 1$.