Lecture (5) Matrices

Def. (1)

A matrix is a rectangular array of numbers. The numbers in the array are called the entries in the matrix. We denotes it by Capital letter A, B or C, ...

- **Examples**

(1) $A = \begin{bmatrix} -3 & 0 \\ 1 & 2 \\ 4 & -1 \end{bmatrix}$ ⇒ A is a matrix of size 3 by 2 i.e its size is 3X2

(2) $B = \begin{bmatrix} 1 & 2 & -3 & 0 \end{bmatrix}$ ⇒ B is a row matrix its size is 1X4

(3) $C = \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix}$ ⇒ C is a column matrix, its size is 3X1

(4) $D = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$ ⇒ D is a 2 by 2 square matrix

(5) $E = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ ⇒ E is a diagonal matrix of size 3X3

(6) $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a unit matrix of size 3X3

(7) $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a Zero matrix of size 2X2

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• Addition and subtraction for matrices

Example

Consider the matrices

$$A = \begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 4 & -2 & 7 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix}$$

$$, C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Then

$$A + B = \begin{bmatrix} -2 & 4 & 5 & 4 \\ 1 & 2 & 2 & 3 \\ 7 & 0 & 3 & 5 \end{bmatrix} \text{ and } A - B = \begin{bmatrix} 6 & -2 & -5 & 2 \\ -3 & -2 & 2 & 5 \\ 1 & -4 & 11 & -5 \end{bmatrix}$$

The expressions $A+C$, $B+C$, $A-C$ and $B-C$ are undefined.

Note that: Matrices of the same sizes only can be added or subtracted.

• Scalar Multiples

Example

For the matrices

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & -7 \\ -1 & 3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 12 & -6 & 36 \\ 6 & 0 & 12 \end{bmatrix}$$

we have

$$2A = \begin{bmatrix} 4 & 6 & 8 \\ 2 & -6 & 0 \end{bmatrix}, \quad (-1)B = -B = \begin{bmatrix} 0 & -2 & 7 \\ 1 & -3 & -5 \end{bmatrix},$$

$$\frac{1}{6}C = \begin{bmatrix} 2 & -1 & 6 \\ 1 & 0 & 2 \end{bmatrix}$$

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3. Multiplying Matrices

Consider $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$

we have $AB = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}_{2\times 3} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}_{3\times 4}$

$$= \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}_{2\times 4}$$

Note:

1. The def$\underline{n}$ of matrix multiplication requires that the number of columns of the first matrix A be the same as the number of rows of the second matrix B in order to form the product AB. If this condition is not satisfied, the product is undefined.

2. $[1 \ 2 \ 4] \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = 4+0+8 = 12$, $[1 \ 2 \ 4] \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix} = 1-2+28 = 27$

, $[1 \ 2 \ 4] \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} = 4+6+20 = 30$, $[1 \ 2 \ 4] \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = 3+2+8 = 13$

, $[2 \ 6 \ 0] \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = 8+0+0 = 8$, $[2 \ 6 \ 0] \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix} = 2-6+0 = -4$

, $[2 \ 6 \ 0] \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} = 8+18+0 = 26$, $[2 \ 6 \ 0] \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = 6+6+0 = 12$

3. $AB \neq BA$

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