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Lecture (4)

Solving Homogeneous System of Linear Eqns

Defn (1)

A system of homogeneous system of Eqns is of the form $\boxed{AX = 0}$

i.e.
$$\Rightarrow \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{array}$$

Defn (2) The homogeneous system $AX=0$ has at least one solution, $x_1 = x_2 = \cdots = x_n = 0$ which is called trivial solution. الحل التافة

Defn (3) the system of homogeneous eqns $\boxed{AX=0}$ has a trivial solution or it has infinitely many non-trivial solutions in addition to the trivial solution.

Note that: the system $AX = B$ is called non-homogeneous system.

Ex Solve the homogeneous system of linear Eqns

$$2x + 2y + 4z = 0$$
$$w \quad - y - 3z = 0$$
$$2w + 3x + y + z = 0$$
$$-2w + x + 3y - 2z = 0$$

Solution: the augmented matrix is

$$\begin{array}{ccccc} w & x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccccc} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{array}\right] \Rightarrow R_1 \leftrightarrow R_2 \left[\begin{array}{ccccc} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{array}\right]$$

$$\begin{array}{l} \frac{1}{2}R_2 \\ -2R_1 + R_3 \\ 2R_1 + R_4 \end{array} \Rightarrow \left[\begin{array}{ccccc} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array}\right] \Rightarrow \begin{array}{l} -3R_2 + R_3 \\ -R_2 + R_4 \end{array} \left[\begin{array}{ccccc} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -10 & 0 \end{array}\right]$$

$$\begin{array}{l} 3R_3 + R_1 \\ -2R_3 + R_2 \\ 10R_3 + R_4 \end{array} \Rightarrow \begin{array}{c} \begin{array}{ccccc} w & x & y & z & \end{array} \\ \left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \text{ RREF}$$

$$\Rightarrow \begin{array}{r} w - y = 0 \\ x + y = 0 \\ z = 0 \end{array}$$

where w, x and z are leading variables, but y is a free variable

So, let $y = t$, $t \in \mathbb{R} \Rightarrow w = t$, $x = -t$ and $z = 0$

i.e. the soln is $x = -t$, $y = w = t$ and $z = 0$, $t \in \mathbb{R}$

$\therefore$ the system has infinitely many solutions besides the trivial soln.

EX Use Gauss–Jordan elimination method to solve the homogeneous linear system

$$x_1 + 3x_2 - 2x_3 + \quad + 2x_5 = 0$$
$$2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = 0$$
$$5x_3 + 10x_4 \quad + 15x_6 = 0$$
$$2x_1 + 6x_2 \quad + 8x_4 + 4x_5 + 18x_6 = 0$$

Ans: the augmented Mx is

$$\begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \\ \end{array}$$
$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 2 & 6 & 0 & 8 & 4 & 18 & 0 \end{bmatrix}$$

$-2R_1 + R_2$, $-2R_1 + R_4$ $\Rightarrow$
$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 0 & 0 & 4 & 8 & 0 & 18 & 0 \end{bmatrix}$$

$-2R_2 + R_1$, $5R_2 + R_3$, $4R_2 + R_4$ $\Rightarrow$
$$\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 6 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \end{bmatrix}$$

$-R_2$, $\frac{1}{6}R_4$, $R_3 \leftrightarrow R_4$ $\Rightarrow$
$$\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 6 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3R_3 + R_2$, $-6R_3 + R_1$ $\Rightarrow$
$$\begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \\ \end{array}$$
$$\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ **RREF**

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We have $x_1 + 3x_2 + 4x_4 + 2x_5 = 0$

$x_3 + 2x_4 = 0$

$x_6 = 0$

where x_1, x_3, x_6 are leading variables

and x_2, x_4, x_5 are free variables

, So let $x_2 = r$, $x_4 = s$, $x_5 = t$ where $r, s, t \in \mathbb{R}$

$\Rightarrow$ $x_3 = -2s$, $x_6 = 0$

$x_1 = -3r - 4s - 2t$

$\therefore$ the solution is $x_1 = -3r - 4s - 2t$, $x_2 = r$, $x_3 = -2s$,

$x_4 = s$, $x_5 = t$, $x_6 = 0$

where $r, s, t \in \mathbb{R}$

i.e the system has infinitely many solutions

including also, the trivial solution.

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