Lecture (4) ++

For what values of λ does the following system of linear Equations have (a) unique solution (b) infinite many solutions, and (c) no solution.

EX (1)

$$x+y+2z=\lambda$$
$$x\quad -3z=\lambda^2$$
$$2x+y-z=0$$

Ans:

The augmented Mx is

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & \lambda \\ 1 & 0 & -3 & \lambda^2 \\ 2 & 1 & -1 & 0 \end{array}\right]$$

$$\begin{matrix} -R_1+R_2 \\ -2R_1+R_3 \end{matrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & \lambda \\ 0 & -1 & -5 & \lambda^2-\lambda \\ 0 & -1 & -5 & -2\lambda \end{array}\right]$$

$$R_2 \leftrightarrow R_3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & \lambda \\ 0 & -1 & -5 & -2\lambda \\ 0 & -1 & -5 & \lambda^2-\lambda \end{array}\right]$$

$$-R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & \lambda \\ 0 & 1 & 5 & 2\lambda \\ 0 & -1 & -5 & \lambda^2-\lambda \end{array}\right]$$

$$R_2+R_3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & \lambda \\ 0 & 1 & 5 & 2\lambda \\ 0 & 0 & 0 & \lambda^2+\lambda \end{array}\right]$$

For $\lambda^2+\lambda=0$ i.e $\lambda(\lambda+1)=0$

$\lambda=0, \lambda=-1$

So, we have infinite many solutions at $\lambda=0$, $\lambda=-1$, no solution at $\lambda\neq -1$, $\lambda\neq 0$ and no unique solution

Ans: (a) No unique solution
(b) $\lambda=0$ or $\lambda=-1$
(c) $\lambda\neq -1, \lambda\neq 0$

EX (2)

$$x+y-z=2$$
$$x+y+z=3$$
$$x+y+(\lambda^2-5)z=\lambda$$

Ans:

The augmented Mx is

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & \lambda^2-5 & \lambda \end{array}\right]$$

$$\begin{matrix} -R_1+R_2 \\ -R_1+R_3 \end{matrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & \lambda^2-4 & \lambda-2 \end{array}\right]$$

$$\Rightarrow (\lambda^2-4)z=\lambda-2$$
$$(\lambda-2)(\lambda+2)z=\lambda-2$$

So, we have the following three cases

1. If $\lambda=2$, the system has infinitely many solutions $\Rightarrow$ the 3rd row is $0\ 0\ 0\ 0$
2. If $\lambda=-2$, the system has no solution $\Rightarrow$ the 3rd row is $0\ 0\ 0\ -4$
3. If $\lambda\neq\pm 2$ the system has a unique solution

Ans: (a) $\lambda\neq\pm 2$
(b) $\lambda=2$
(c) $\lambda=-2$

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