Lecture (4)'

Application on solving linear system of Eqns

Consider the following linear system where λ is a real number

$$2x + 3y + (\lambda+2)z = 5$$
$$x + y + z = 2$$
$$4\lambda x + 3\lambda y + 3z = 8\lambda - 3$$

Determine for which values of λ this system has (i) no solution, (ii) a unique solution and (iii) infinitely many solutions

Ans

The augmented Matrix is given as

$$\begin{bmatrix} 2 & 3 & \lambda+2 & 5 \\ 1 & 1 & 1 & 2 \\ 4\lambda & 3\lambda & 3 & 8\lambda-3 \end{bmatrix}$$

$$R_1 \longleftrightarrow R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & \lambda+2 & 5 \\ 4\lambda & 3\lambda & 3 & 8\lambda-3 \end{bmatrix}$$

$$-2R_1 + R_2, \; -4\lambda R_1 + R_3 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & \lambda & 1 \\ 0 & -\lambda & -4\lambda+3 & -3 \end{bmatrix}$$

$$\lambda R_2 + R_3 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & \lambda & 1 \\ 0 & 0 & \lambda^2-4\lambda+3 & \lambda-3 \end{bmatrix}$$

* From the bottom line of the matrix we have:

$$(\lambda^2-4\lambda+3)z=\lambda-3$$

$$\therefore (\lambda-3)(\lambda-1)z=\lambda-3$$

$\therefore$ We have the following three cases

Case ① If $\lambda=3$, then the system has infinitely many solutions.

The 3rd row is (the bottom row) $0 \ 0 \ 0 \ 0$

Case ② If $\lambda=1$, then the system has no solution

the 3rd row gives $0z=-2$ which is impossible

Case ③ If $\lambda\neq 3$ and $\lambda\neq 1$, then the system has a unique solution.

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