5

• pb(6.1.1) p.284

$P_n(t) = pr\{X(t)=n\}$

For Yule process

$$P_n(t) = e^{-\beta t}(1-e^{-\beta t})^{n-1}, \quad n \geq 1$$

$pr\{X(U)=k\}$?? يطلب

$k = 1, 2, \ldots$, where U is a random time uniformly distributed from 0 to 1.

Ans:

$$pr\{X(U)=k\} = \int_0^1 e^{-\beta u}(1-e^{-\beta u})^{k-1}\,du$$

$$= \frac{1}{\beta}\int_0^1 (1-e^{-\beta u})^{k-1}\cdot \beta e^{-\beta u}\,du$$

$$= \frac{1}{\beta}\left[\frac{(1-e^{-\beta u})^k}{k}\right]_0^1$$

$$= \frac{1}{\beta k}\left[(1-e^{-\beta})^k - 0\right]$$

$$\therefore pr\{X(U)=k\} = \frac{\rho^k}{\beta k}, \quad k = 1, 2, \ldots$$

where $\rho = (1-e^{-\beta})$

//