Implicit partial Differentiation

Theorem

If an Equation $F(x,y)=0$ determines a differential f_n f, implicitly s.t $y=f(x)$ then

$$\frac{dy}{dx}=\frac{-F_x(x,y)}{F_y(x,y)}$$

Ex Find $\frac{dy}{dx}$ if $2x^3+x^2y+y^3=1$

Ans: let $F(x,y)=2x^3+x^2y+y^3-1=0$

$$F(x,y)=0 \Rightarrow \frac{dy}{dx}=\frac{-F_x(x,y)}{F_y(x,y)}$$

$$\frac{dy}{dx}=-\frac{6x^2+2xy}{x^2+3y^2}$$

Theorem

If an Eqn $F(x,y,z)=0$ determines a diff. f_n f implicitly s.t $z=f(x,y)$ then

$$\frac{\partial z}{\partial x}=-\frac{F_x(x,y,z)}{F_z(x,y,z)} \quad \text{and} \quad \frac{\partial z}{\partial y}=-\frac{F_y(x,y,z)}{F_z(x,y,z)}$$

Ex Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$F(x,y,z)=2xz^3-3yz^2+x^2y^2+4z=0$$

Ans: $$\frac{\partial z}{\partial x}=-\frac{F_x(x,y,z)}{F_z(x,y,z)} \Rightarrow \frac{\partial z}{\partial x}=-\frac{2z^3+2xy^2}{6xz^2-6yz+4}$$

and $$\frac{\partial z}{\partial y}=-\frac{F_y(x,y,z)}{F_z(x,y,z)} \Rightarrow \frac{\partial z}{\partial y}=-\left(\frac{-3z^2+2x^2y}{6xz^2-6yz+4}\right)$$

$$=\frac{3z^2-2x^2y}{6xz^2-6yz+4}$$

#

2

Vector calculus

* Directional Derivative (Gradient form)

let f be a differentiable $f\underline{n}$ of three variables at $P(x, y, z)$

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then (1) The direction derivative of f in direction of a unit vector $\vec{u} = \langle a, b, c \rangle$, $D_u f$ is given by

$$D_u f = \nabla f \cdot \vec{u}$$

اقرأ

where $\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$ (gradient of f)

تقرأ grad f أو نبلا f

$$\Rightarrow$$

$$D_u f = a f_x + b f_y + c f_z$$

(2) The direction of maximum rate of increase of f is given by ∇f

and the max. value of $D_u f$ is $\|\nabla f\|$

(3) the direction of min. rate of increase of f is given by $-\nabla f$

and the min. value of $D_u f$ is $-\|\nabla f\|$

Ex: If $f(x, y, z) = x^2 + y^2 - 4z$ find the following:

(1) gradient of f (2) the direction of max. rate of increase of f at the point $(2, -1, 1)$ and max. of $D_u f$

(3) $D_u f$ in direction $\vec{a} = \langle 1, 3, -2 \rangle$ at $P(2, -1, 1)$

Ans:

(1) $\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$

$\nabla f = 2x\vec{i} + 2y\vec{j} - 4\vec{k} = \langle 2x, 2y, -4 \rangle$ (1)

~~3~~

(2) The direction of max. rate of increase of f at $P(2,-1,1)$ is

1 $\Rightarrow$ $\nabla f(2,-1,1) = \langle 4,-2,-4 \rangle$ 2

(3) $D_u f = \nabla f \cdot \vec{u}$

$$\vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{\langle 1,3,-2 \rangle}{\sqrt{1+9+4}}$$

unit vector

$$= \left\langle \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}} \right\rangle$$

$$D_u f = \langle 4,-2,-4 \rangle \cdot \left\langle \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}} \right\rangle$$

$$D_u f = \frac{4}{\sqrt{14}} - \frac{6}{\sqrt{14}} + \frac{8}{\sqrt{14}} = \frac{6}{\sqrt{14}}$$

2, $\Rightarrow$ and max. of $D_u f$ at $P(2,-1,1)$ $= \|\nabla f\| = \sqrt{16+4+16} = 6$