

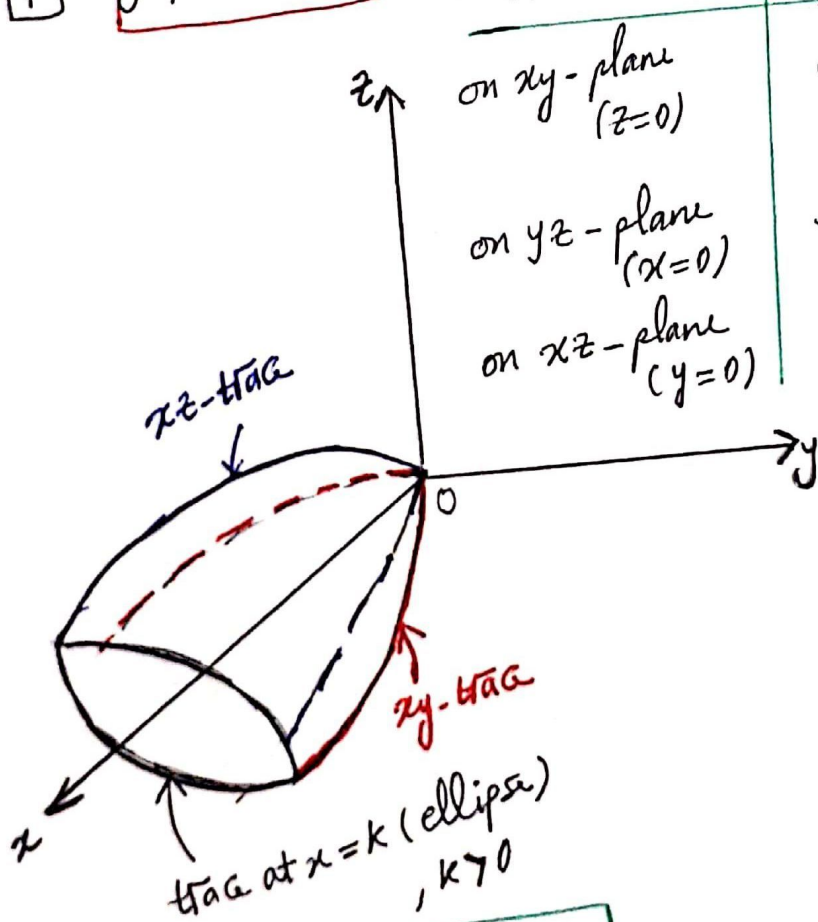
III Surfaces \mathbb{R}^3 & \mathbb{R}^2

We studied before,

- ✓ [1] $\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$ is an equation of paraboloid, its axis is z-axis
- ✓ [2] $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is an eqn of Ellipsoid
- ✓ [3] $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ is an eqn of Hyperboloid of one sheet, its axis is z-axis

H.W Revision

[1] $y^2 + 4z^2 = x$



Trace	Eqn of trace	Description of trace
on xy-plane (z=0)	$y^2 = x$ ($x = y^2$)	parabola
on yz-plane (x=0)	$y^2 + 4z^2 = 0$	origin
on xz-plane (y=0)	$4z^2 = x$ ($x = 4z^2$)	parabola

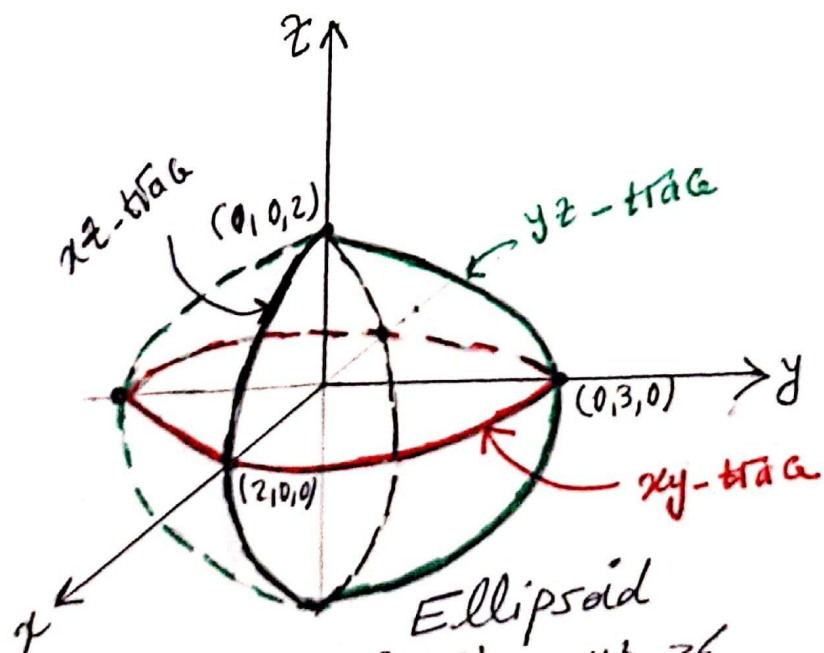
paraboloid $y^2 + 4z^2 = x$

2

$$9(x^2 + z^2) + 4y^2 = 36 \quad (\div 36)$$

$$\frac{x^2}{4} + \frac{z^2}{4} + \frac{y^2}{9} = 1$$

Trace	Eqn of trace	Description
on xy -plane ($z=0$)	$\frac{x^2}{4} + \frac{y^2}{9} = 1$	ellipse
on yz -plane ($x=0$)	$\frac{y^2}{9} + \frac{z^2}{4} = 1$	ellipse
on xz -plane ($y=0$)	$\frac{x^2}{4} + \frac{z^2}{4} = 1$ $\Rightarrow x^2 + z^2 = 4$	circle



Ellipsoid

$$9(x^2 + z^2) + 4y^2 = 36$$

$$\Rightarrow \frac{x^2}{4} + \frac{z^2}{4} + \frac{y^2}{9} = 1$$

4

4 Hyperboloid of two sheets

It's of the form $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

whose its axis is z-axis

Ex Sketch the graph of $3x^2 - 4y^2 - z^2 = 12$, name the surface and describe the traces

Ans $3x^2 - 4y^2 - z^2 = 12$ ($\div 12$)
 $\rightarrow \frac{x^2}{4} - \frac{y^2}{3} - \frac{z^2}{12} = 1$

It's a Hyperboloid of two sheets, its axis is x-axis.

Trace

on xy-plane ($z=0$)

on yz-plane ($x=0$)

on xz-plane ($y=0$)

Eqn of trace

$\frac{x^2}{4} - \frac{y^2}{3} = 1 \rightarrow$

Description

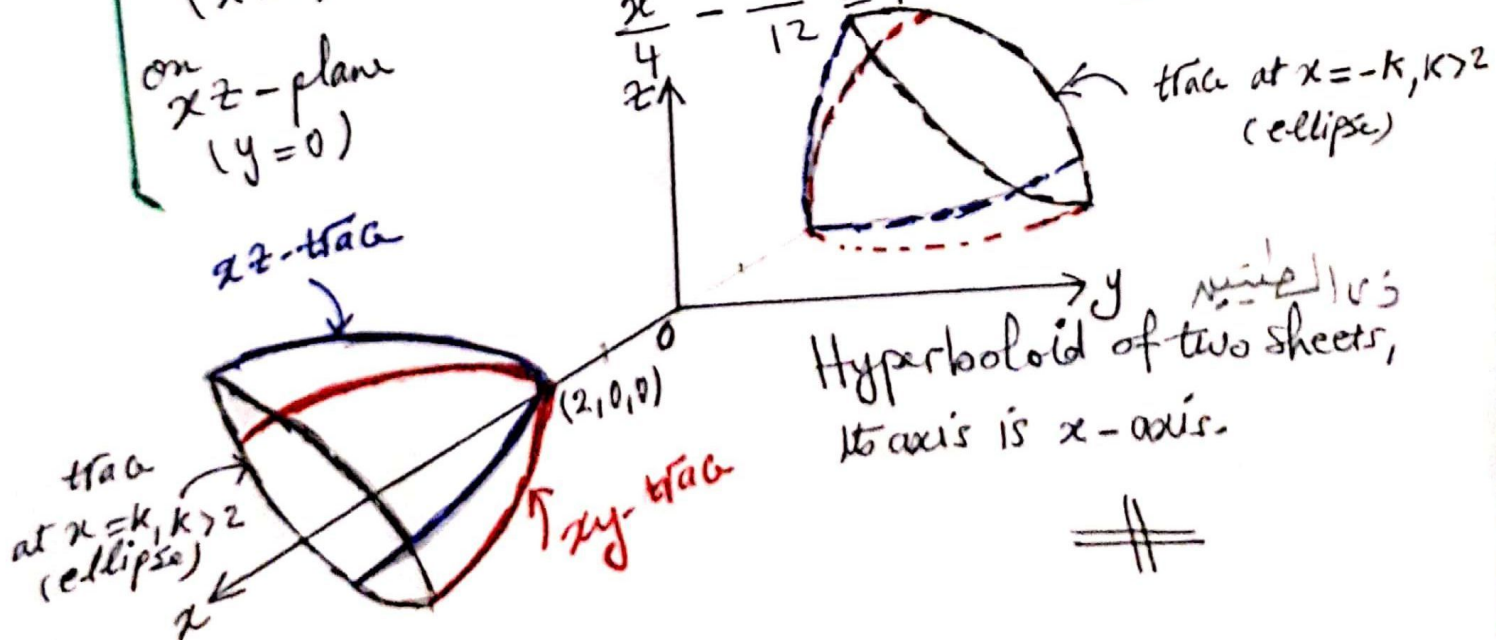
Hyperbola

$-\frac{y^2}{3} - \frac{z^2}{12} = 1 \rightarrow$

None

$\frac{x^2}{4} - \frac{z^2}{12} = 1 \rightarrow$

Hyperbola



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5] Cone

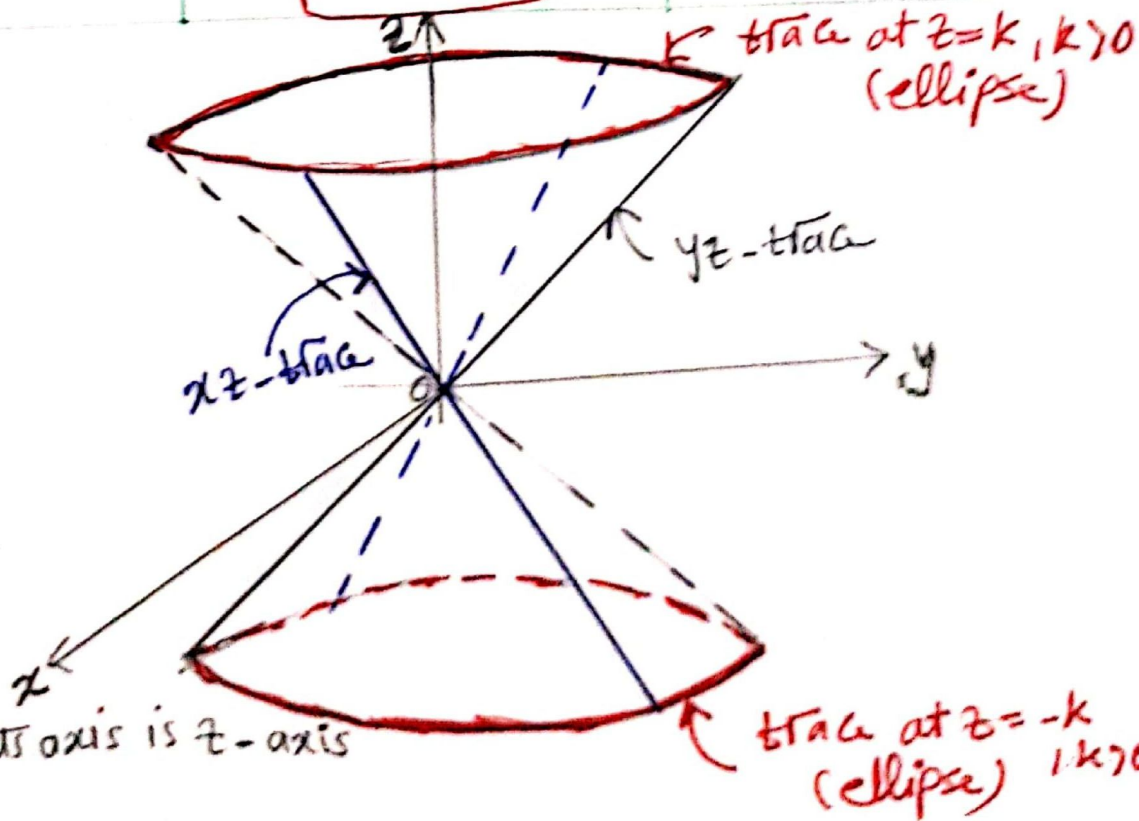
It's of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

whose its axis is z-axis

Ex: Sketch the graph of the surface $\frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{4} = 0$

Ans:	Trace	Eqn of trace	Description
	on xy-plane (z=0)	$\frac{x^2}{9} + \frac{y^2}{4} = 0$	point (origin)
	on yz-plane (x=0)	$\frac{y^2}{4} - \frac{z^2}{4} = 0$ $\Rightarrow y = \pm z$	intersecting lines
	on xz-plane (y=0)	$\frac{x^2}{9} - \frac{z^2}{4} = 0$ $\Rightarrow z^2 = \frac{4}{9}x^2$ $z = \pm \frac{2}{3}x$ $x = \pm \frac{3}{2}z$	intersecting lines



* Cone, its axis is z-axis