

1

Lecture 2

Reduced Row Echelon Form Matrix & Gauss Jordan elimination Method

The Reduced Row Echelon Form M_X (RREF) is a matrix satisfies all the following conditions

- ① the first non-zero number in each row is 1 which is called leading 1. 1 is left side,
- ② Every leading 1 in each row must be on the right of the above leading 1.
- ③ A row containing entirely of zeros must be at the bottom of the matrix.
- ④ Each column that contains leading 1 has zeros everywhere else in that column.

Note that: A matrix which satisfies only the first three conditions is said to be Row Echelon Form (REF)

* Ex: Determine whether the matrix is a row echelon form or reduced row echelon form or neither

a)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$
 REF

b)
$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$
 RREF

c)
$$\begin{bmatrix} 1 & 4 & -3 & 7 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$
 REF

2

d)
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

REF

e)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RREF

f)
$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Neither

g)
$$\begin{bmatrix} 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

RREF

h)
$$\begin{bmatrix} 1 & -2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

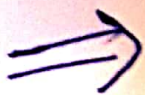
RREF

i)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

RREF

Gauss-Jordan Method

The Augmented Matrix
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 & B_1 \\ 0 & 1 & 0 & B_2 \\ 0 & 0 & 1 & B_3 \end{bmatrix}$$

RREF

Example

Use Gauss-Jordan elimination method to solve

$$x + y + 2z = 9$$

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0$$

3

Ans: The augmented Matrix is

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

$$\begin{array}{l} -2R_1 + R_2 \\ 1 - 3R_1 + R_3 \end{array} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{bmatrix}$$

$$\frac{1}{2}R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 3 & -11 & -27 \end{bmatrix}$$

$$\begin{array}{l} -R_2 + R_1 \\ 1 - 3R_2 + R_3 \end{array} \Rightarrow \begin{bmatrix} 1 & 0 & 11/2 & 35/2 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & -1/2 & -3/2 \end{bmatrix}$$

$$\begin{array}{l} -7R_3 + R_2 \\ 11R_3 + R_1 \end{array} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1/2 & -3/2 \end{bmatrix}$$

$$-2R_3 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

\therefore The solution is $x=1, y=2, z=3$

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