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Lecture ②

Reduced Row Echelon Form Matrix & Gauss Jordan elimination Method

The Reduced Row Echelon Form Mx (RREF) is a matrix satisfies all the following conditions

① the first non-zero number in each row is 1 which is called leading 1. 1 رئيسي

② Every leading 1 in each row must be on the right of the above leading 1.

③ A row containing entirely of zeros must be at the bottom of the matrix.

④ Each column that contains leading 1 has zeros everywhere else in that column.

Not that: A matrix which satisfies only the first three conditions is said to be Row Echelon Form (REF)

* Ex: Determine whether the matrix is a row echelon form or reduced row echelon form or neither

a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ REF ,

b) $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ RREF ,

c) $\begin{bmatrix} 1 & 4 & -3 & 7 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ REF

d) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ REF

e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ RREF

f) $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Neither

g) $\begin{bmatrix} 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ RREF

h) $\begin{bmatrix} 1 & -2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ RREF

i) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ RREF

• Gauss - Jordan Method

The Augmented $M\underline{x}$ $\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & B_1 \\ 0 & 1 & 0 & B_2 \\ 0 & 0 & 1 & B_3 \end{bmatrix}$$

RREF

• Example

Use Gauss - Jordan elimination method to solve

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

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Ans: the augmented Matrix is

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

$-2R_1 + R_2$, $-3R_1 + R_3$ $\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{bmatrix}$

$\frac{1}{2} R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 3 & -11 & -27 \end{bmatrix}$

$-R_2 + R_1$, $-3R_2 + R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 11/2 & 35/2 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & -1/2 & -3/2 \end{bmatrix}$

$-7R_3 + R_2$, $11R_3 + R_1$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1/2 & -3/2 \end{bmatrix}$

$-2R_3 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\therefore$ The solution is $x = 1, y = 2, z = 3$

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