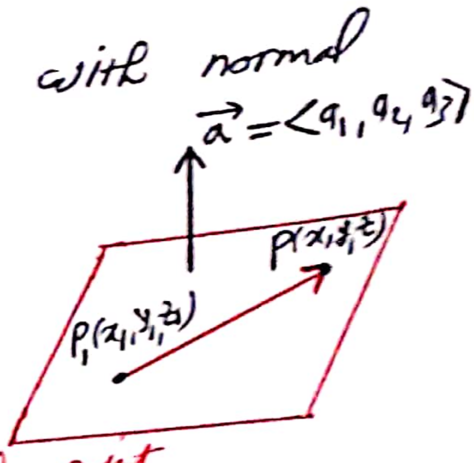
line and plane q.v lecture (19)

Theorem

An equation of the plane through $P_1(x_1, y_1, z_1)$ with normal vector $\vec{a} = \langle a_1, a_2, a_3 \rangle$ is



$$a_1(x - x_1) + a_2(y - y_1) + a_3(z - z_1) = 0$$

or $\boxed{a_1 x + a_2 y + a_3 z + d = 0}$, d is Const.

Ex Find an equation of the plane through the point $(5, -2, 4)$ with normal vector $\vec{a} = \langle 1, 2, 3 \rangle$

Ans:

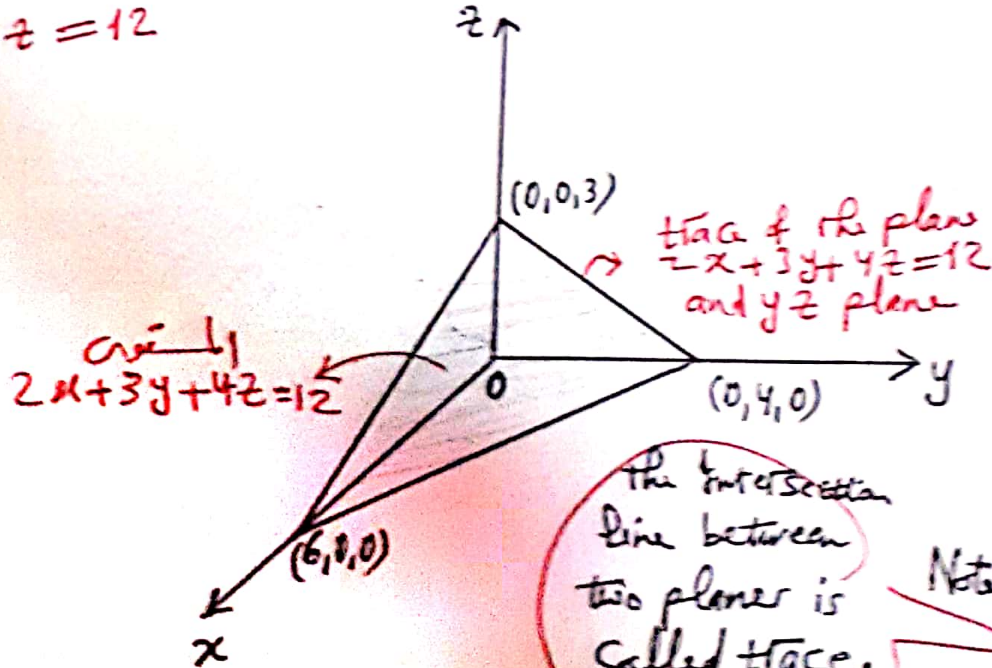
$$1(x - 5) + 2(y + 2) + 3(z - 4) = 0$$

$$\Rightarrow x + 2y + 3z - 13 = 0$$

is the eqn of the plane.

Ex 2 sketch the graph of the Eqn of the plane

$$2x + 3y + 4z = 12$$

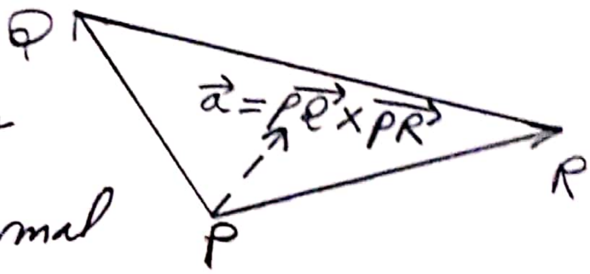


The intersection line between two planes is called trace.

Note

Ex 3 Find an equation of the plane determined by the points $P(4,-3,1)$, $Q(6,-4,7)$ and $R(1,2,2)$

Ans: The Eqn of the plane is determined by a given point and the normal vector $\vec{a}$ to it.



Let $P(4,-3,1)$ is the given point and $\vec{a}=\vec{PQ}\times\vec{PR}$ is the normal vector to the plane, where

$$\vec{PQ}=\langle 6-4,-4+3,7-1\rangle=\langle 2,-1,6\rangle$$

$$\vec{PR}=\langle 1-4,2+3,2-1\rangle=\langle -3,5,1\rangle$$

$$\therefore \vec{a}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 6 \\ -3 & 5 & 1 \end{vmatrix}$$

$$\vec{a}=-31\vec{i}-20\vec{j}+7\vec{k}=\langle -31,-20,7\rangle$$

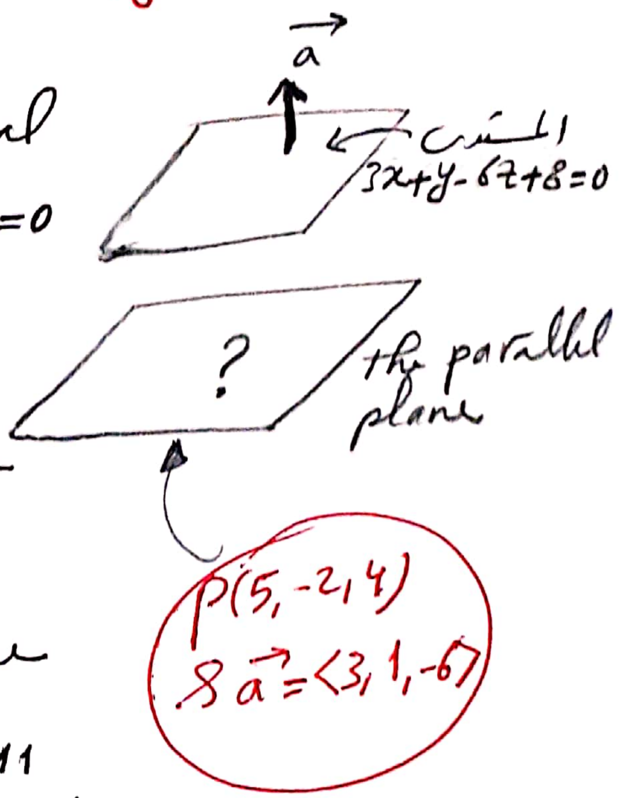
$\therefore$ the eqn of the required plane is

$$-31(x-4)-20(y+3)+7(z-1)=0$$

$$31(x-4)+20(y+3)-7(z-1)=0$$

$$\Rightarrow \therefore \boxed{31x+20y-7z-57=0}$$

EX 4 Find an Equation of the plane through $P(5,-2,4)$ that is parallel to the plane $3x+y-6z+8=0$



Ans:

$\because$ $\vec{a} = \langle 3, 1, -6 \rangle$ is normal vector to the plane $3x+y-6z+8=0$

Also, $\vec{a}$ is normal vector to the parallel plane

$\Rightarrow$ The Eqn of the parallel plane is $3x+y-6z+d=0$

$\because$ $P(5,-2,4)$ lie on this plane

$\therefore$ $15-2-24+d=0 \Rightarrow d=11$

$\therefore$ the Eqn of required parallel plane is

$$\boxed{3x+y-6z+11=0}$$

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