Revise example p. 98

example, we will suppose

$$\mathbf{P} = \begin{array}{c|cccc|} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0.4 & 0.3 & 0.2 & 0.1 \\ 2 & 0.1 & 0.3 & 0.3 & 0.3 \\ 3 & 0 & 0 & 0 & 1 \end{array}. \tag{3.23}$$

The first step analysis equations (3.21) and (3.22) for u_1 and u_2 are

$$u_1 = 0.4 + 0.3u_1 + 0.2u_2,$$
$$u_2 = 0.1 + 0.3u_1 + 0.3u_2,$$

or

$$0.7u_1 - 0.2u_2 = 0.4,$$
$$-0.3u_1 + 0.7u_2 = 0.1.$$

The solution is $u_1 = \frac{30}{43}$ and $u_2 = \frac{19}{43}$. Note that one cannot, in general, solve for u_1 without bringing in u_2, and vice versa. The result $u_2 = \frac{19}{43}$ tells us that once begun in state $X_0 = 2$, the Markov chain $\{X_n\}$ described by (3.23) will ultimately end up in state 0 with probability $u_2 = \frac{19}{43}$, and alternatively, will be absorbed in state 3 with probability $1 - u_2 = \frac{24}{43}$.

The mean time to absorption also depends on the starting state. The first step analysis equations for $v_i = E[T|X_0 = i]$ are

$$\begin{aligned} v_1 &= 1 + P_{11}v_1 + P_{12}v_2, \\ v_2 &= 1 + P_{21}v_1 + P_{22}v_2. \end{aligned} \tag{3.24}$$

The right side of (3.24) asserts that at least one step is always taken. If the first move is to either $X_1 = 1$ or $X_1 = 2$, then additional steps are needed, and on the average, these are v_1 and v_2, respectively. Weighting the contingencies $X_1 = 1$ and $X_1 = 2$ by their respective probabilities and summing according to the law of total probability results in (3.24).

For the transition matrix given in (3.23), the equations are

$$v_1 = 1 + 0.3v_1 + 0.2v_2,$$
$$v_2 = 1 + 0.3v_1 + 0.3v_2,$$

and their solutions are $v_1 = \frac{90}{43}$ and $v_2 = \frac{100}{43}$. Again, v_1 cannot be obtained without also considering v_2, and vice versa. For a process that begins in state $X_0 = 2$, on the average $v_2 = \frac{100}{43} = 2.33$ steps will transpire prior to absorption.