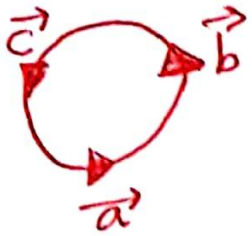
Lecture (17) Vectors

* The triple scalar product

Defn

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

where $\vec{a}=\langle a_1, a_2, a_3\rangle$, $\vec{b}=\langle b_1, b_2, b_3\rangle$ and $\vec{c}=\langle c_1, c_2, c_3\rangle$ are three vectors

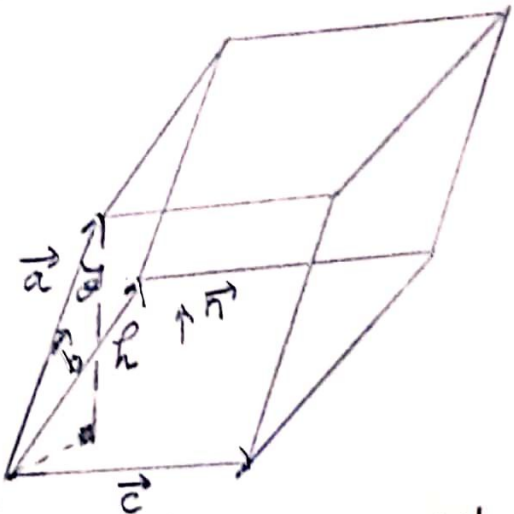


Note: $\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{c}\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot(\vec{c}\times\vec{a})$

* Volume of parallelepiped is

متوازي السطوح

$$V = |\vec{a}\cdot(\vec{b}\times\vec{c})|$$



proof

∵ مساحة القاعدة (متوازي الأضلاع) هي

$$\|\vec{b}\times\vec{c}\|$$

و ارتفاع h هو

$$h = \|\vec{a}\|\cos\theta$$

∴ حجم متوازي السطوح هو

$$V = \|\vec{b}\times\vec{c}\|\,\|\vec{a}\|\cos\theta$$

ومن تعريف الضرب القياسي

$$\therefore\ V = |\vec{a}\cdot(\vec{b}\times\vec{c})|$$

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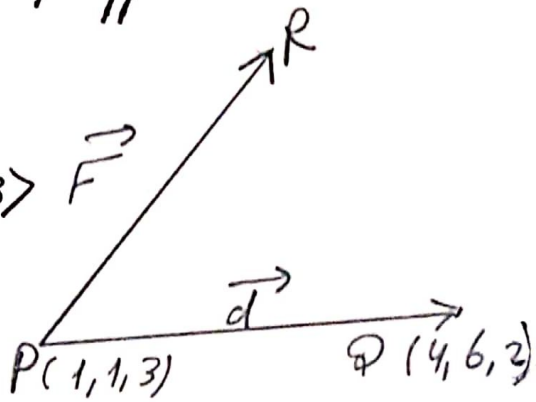
Applications on vectors

Ex 2 Find the Work done by a constant force $\vec{F} = 2\vec{i} + 4\vec{j} + \vec{k}$ if a point of application moves from $P(1,1,3)$ to $Q(4,6,2)$



Ans: $\vec{d} = \overrightarrow{PQ} = \langle 4,6,2\rangle - \langle 1,1,3\rangle$

$\vec{d} = \langle 3,5,-1\rangle$

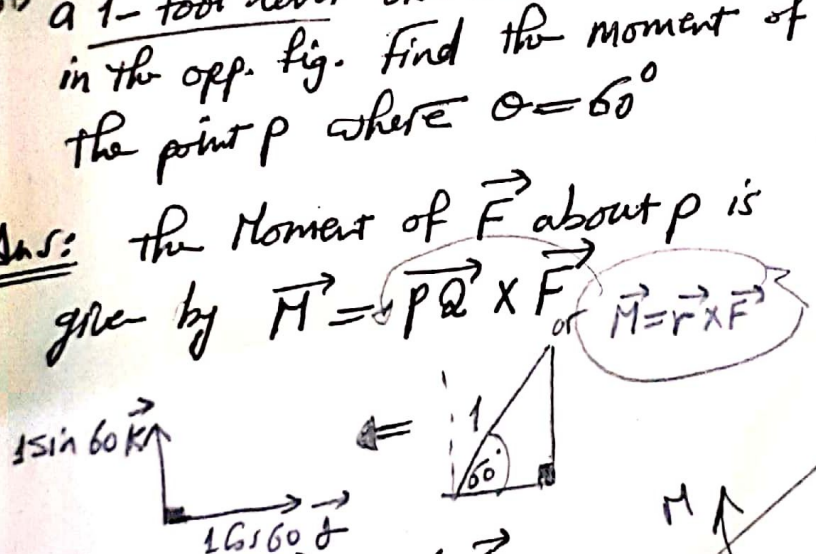
$W = \vec{F} \cdot \vec{d}$

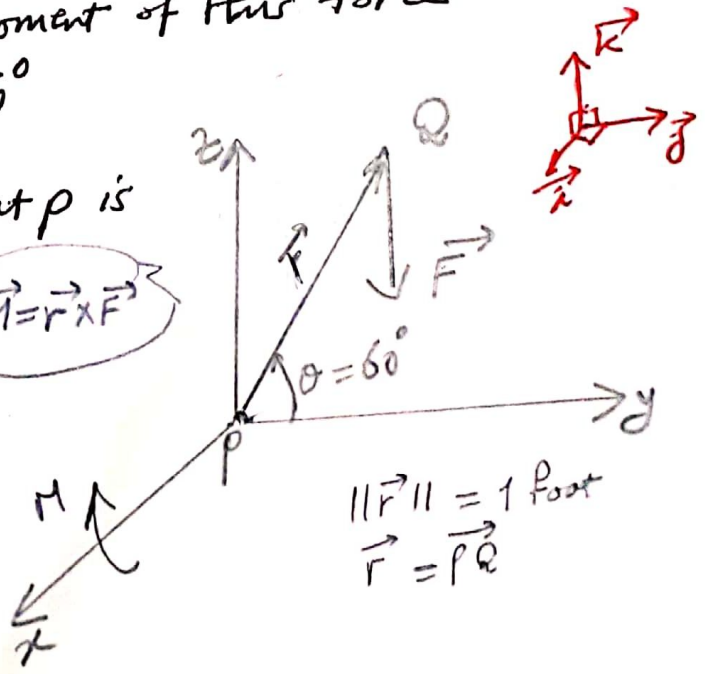
$W = \langle 2,4,1\rangle \cdot \langle 3,5,-1\rangle$

$W = 6 + 20 - 1 = \boxed{25}$ unit of work

Ex Solved problem 1 A vertical force of 50 pounds is applied to the end of a 1-foot lever that attached to an axis at point P, as shown in the opp. fig. Find the moment of this force about the point P where $\theta = 60^\circ$

Ans: the Moment of $\vec{F}$ about P is given by $\vec{M} = \overrightarrow{PQ} \times \vec{F}$ or $\vec{M} = \vec{r} \times \vec{F}$





$\vec{r} = \overrightarrow{PQ} = \cos 60\,\vec{j} + \sin 60\,\vec{k}$

$= \frac{1}{2}\vec{j} + \frac{\sqrt{3}}{2}\vec{k}$

$\vec{F} = -50\,\vec{k}$

$\therefore \vec{M} = \vec{r} \times \vec{F} = \overrightarrow{PQ} \times \vec{F}$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & -50 \end{vmatrix} = -25\,\vec{i}$$

$\|\vec{F}\| = 1$ foot

$\vec{r} = \overrightarrow{PQ}$

$\therefore$ the moment of $\vec{F}$ about P is $-25\,\vec{i}$ and its magnitude is 25 foot-pounds.

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