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Lecture (14)

Ch 10 : vectors and the Geometry of space

Textbook: Calculus by Swokowski, 6 th Ed.

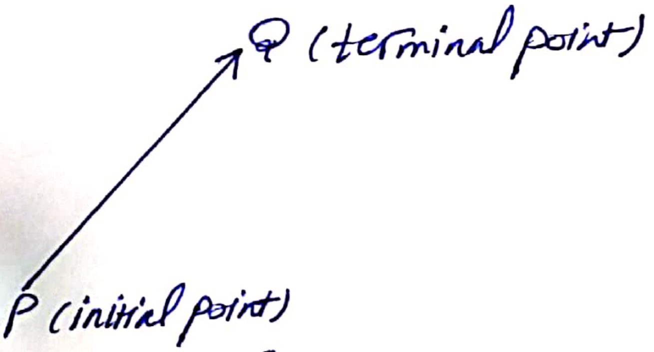
- Quantities

Scalars
that have magnitude only such as, length, Temperature, area, volume, mass, ...

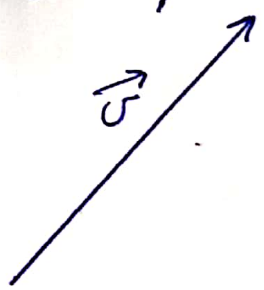
Vectors
that have magnitude and direction such as Displacement, velocity, acceleration, force, moment, momentum, ...

- Vector representation

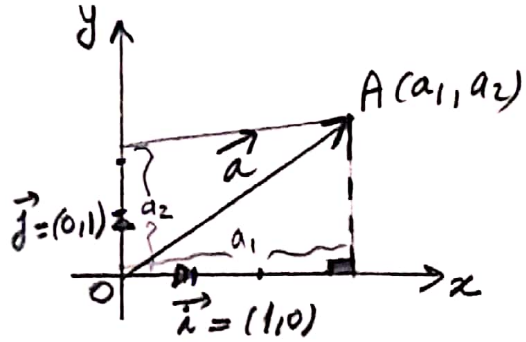
vectors in the plane



or



- position vector



$\vec{OA}$ is the position vector for the point $A(a_1, a_2)$ w.r.t O (referred to O),
We denote it by $\vec{OA}$ or $\vec{a}$
where $\vec{OA} = \vec{a} = \langle a_1, a_2 \rangle$ and its magnitude is

$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2}$$

(also called norm of the vector $\vec{a}$)

The position vector $\vec{a}$ can be written as

$$\overrightarrow{OA} = \vec{a} = a_1 \vec{i} + a_2 \vec{j}$$

where a_1 and a_2 are the Components of the vector $\vec{a} = \langle a_1, a_2 \rangle$, $\vec{i}$ and $\vec{j}$ are called fundamental unit vectors defined as $\vec{i} = (1,0)$ and $\vec{j} = (0,1)$

- Unit vector $\vec{u}$

It's a vector has magnitude 1, $u = \dfrac{\vec{a}}{\|\vec{a}\|}$, $\vec{a} \neq \vec{0}$

EX $\vec{a} = \langle 1, 2, -2 \rangle \Rightarrow \vec{u} = \frac{1}{3} \langle 1, 2, -2 \rangle$

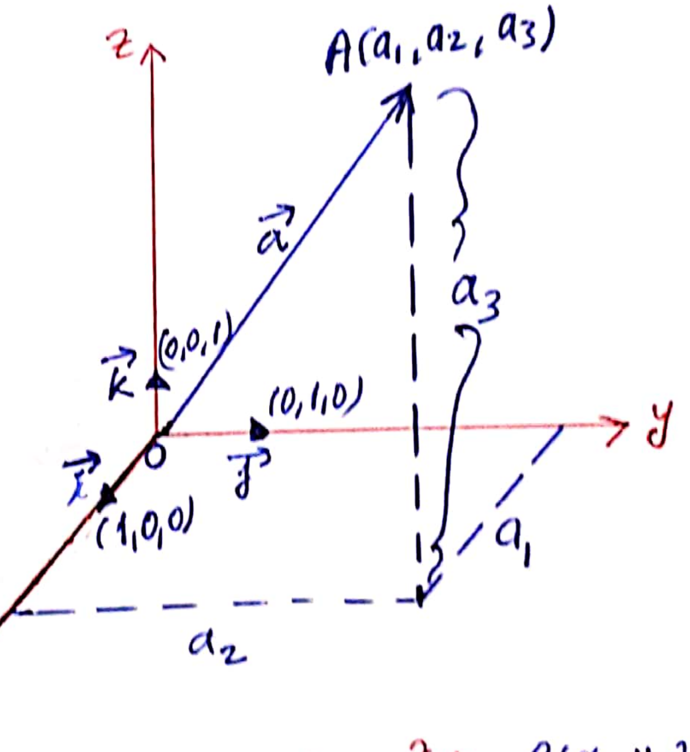
- vectors in space

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

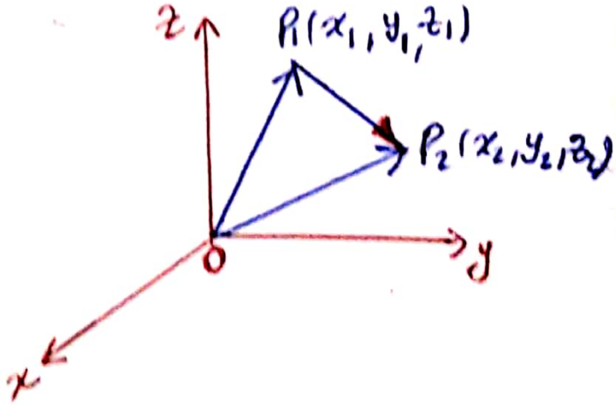
The norm of the vector $\vec{a}$ (its magnitude) is

$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the fundamental unit vectors.



- In three dimensional space, we can Consider the vector as a directed line Segment as follows:



$$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$$

terminal initial

$$= \langle x_2, y_2, z_2 \rangle - \langle x_1, y_1, z_1 \rangle$$

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

EX) if $\vec{a} = \langle -2, 6, 1\rangle$ and $b = \langle 3, -3, -1\rangle$

find ① $\vec{a}+\vec{b}$ ② $\vec{a}-\vec{b}$ ③ $5\vec{a}-4\vec{b}$

④ $\|\vec{a}\|$ ⑤ $\|3\vec{a}\|$

Ans:

① $\vec{a}+\vec{b} = \langle -2, 6, 1\rangle + \langle 3, -3, -1\rangle$

$\therefore \vec{a}+\vec{b} = \langle 1, 3, 0\rangle$

② $\vec{a}-\vec{b} = \langle -2, 6, 1\rangle - \langle 3, -3, -1\rangle$

$\therefore \vec{a}-\vec{b} = \langle -5, 9, 2\rangle$

③ $5\vec{a}-4\vec{b} = 5\langle -2, 6, 1\rangle - 4\langle 3, -3, -1\rangle$

$\therefore 5\vec{a}-4\vec{b} = \langle -22, 42, 9\rangle$

④ $\|\vec{a}\| = \sqrt{4+36+1} = \sqrt{41}$

⑤ $3\vec{a} = 3\langle -2, 6, 1\rangle$

$\therefore 3\vec{a} = \langle -6, 18, 3\rangle$

$\|3\vec{a}\| = \sqrt{36+324+9} = \sqrt{369}$

$\therefore \|3\vec{a}\| = 3\sqrt{41}$

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